Audio and Other Waveforms

Stephen A. Edwards

Columbia University

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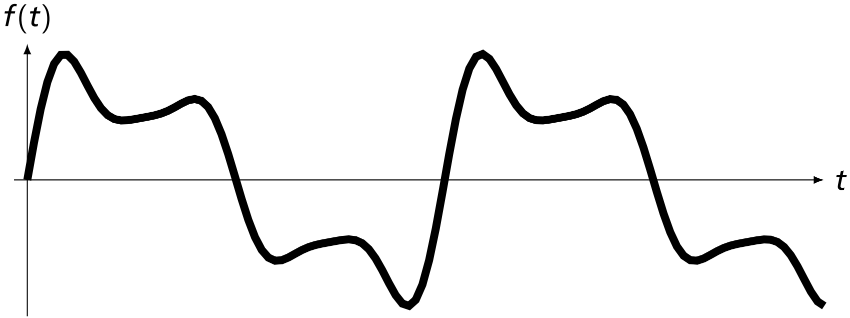
Waveforms

Time-varying scalar value

Commonly called a "signal" in the control-theory literature

Sound: air pressure over time



Raster video: brightness over time

Speed over time, position over time, etc.

The Fourier Series

Any periodic function can be expressed as a sum of harmonics

For a smooth function $f(t)$ with period T, i.e.,

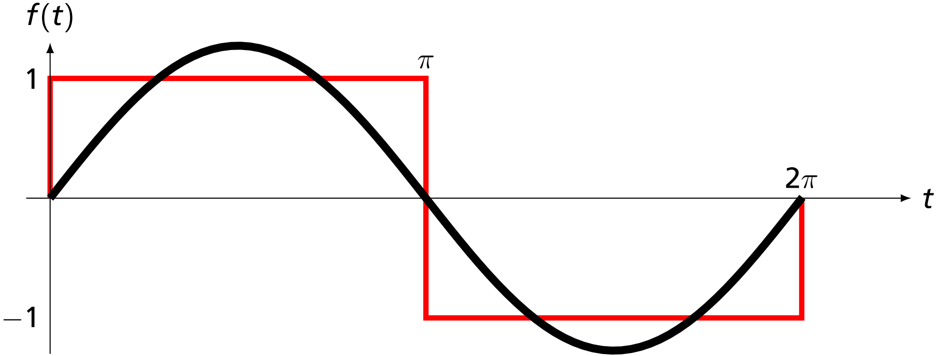
$$f(t) = f(t + T),$$

there exists coefficients a_n, b_n such that

$$f(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos \frac{2\pi mt}{T} + b_m \sin \frac{2\pi mt}{T}$$

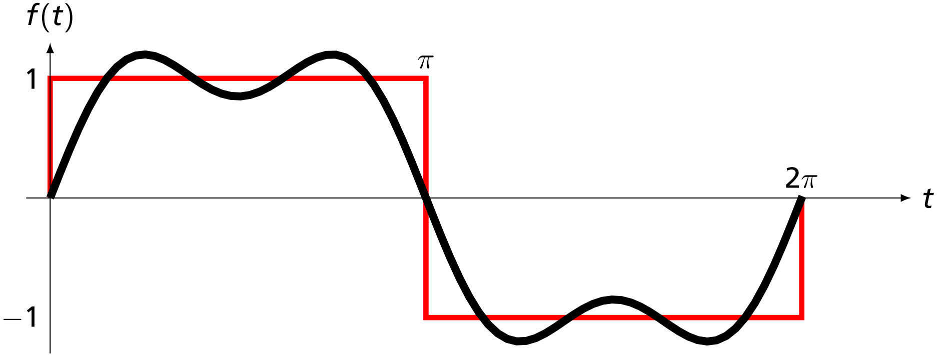
The Fourier Series for a Square Wave

$$f(t) = \frac{4}{\pi}\left(\sin t \qquad\qquad\qquad\qquad\qquad\qquad\qquad\right)$$



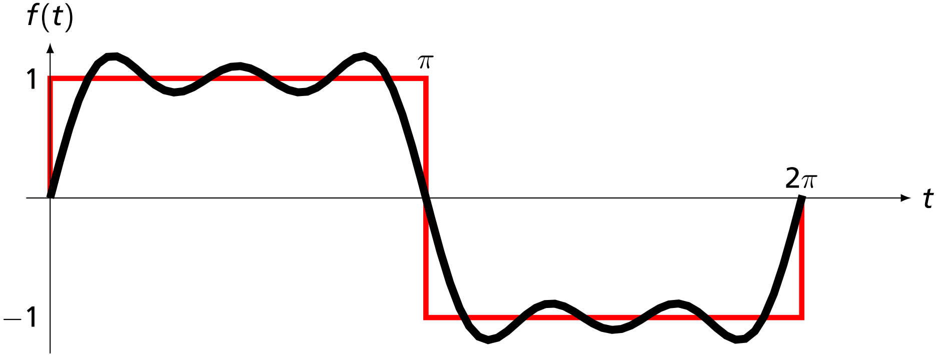
The Fourier Series for a Square Wave

$$f(t) = \frac{4}{\pi}\left(\sin t + \frac{\sin 3t}{3}\right)$$



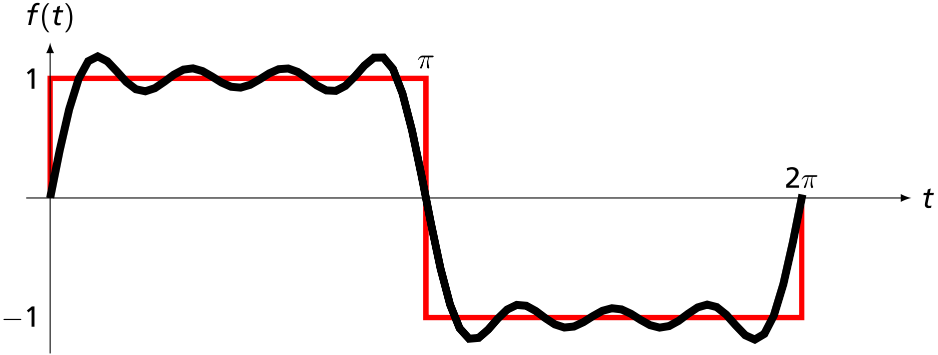
The Fourier Series for a Square Wave

$$f(t) = \frac{4}{\pi}\left(\sin t + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} \qquad\qquad\qquad\qquad\right)$$



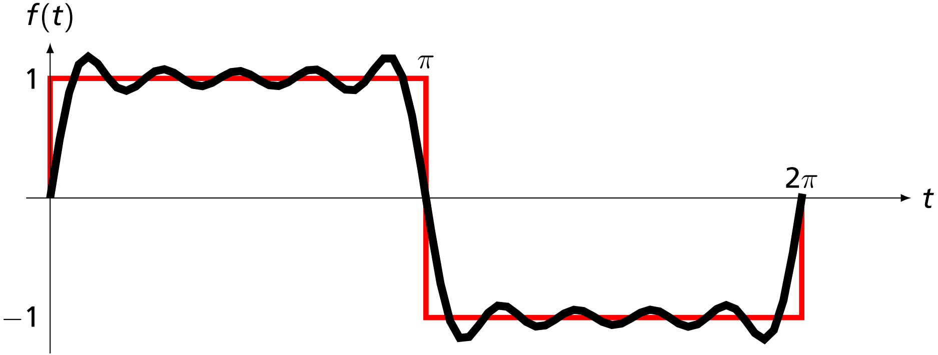
The Fourier Series for a Square Wave

$$f(t) = \frac{4}{\pi}\left(\sin t + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \frac{\sin 7t}{7} \qquad\qquad\right)$$



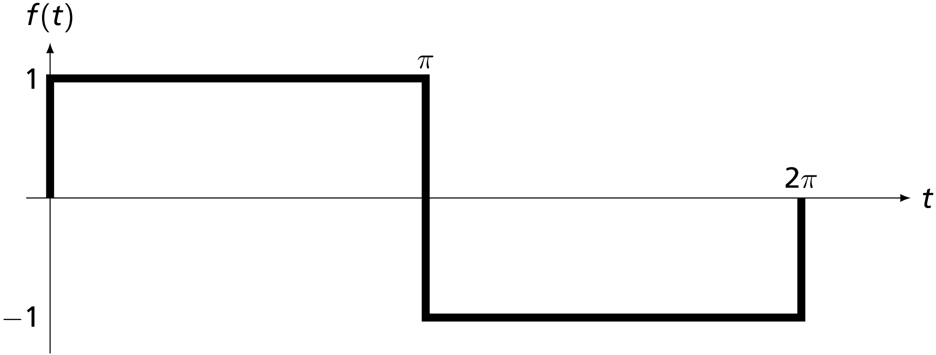
The Fourier Series for a Square Wave

$$f(t) = \frac{4}{\pi}\left(\sin t + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \frac{\sin 7t}{7} + \frac{\sin 9t}{9} \qquad \right)$$



The Fourier Series for a Square Wave

$$f(t) = \frac{4}{\pi}\left(\sin t + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \frac{\sin 7t}{7} + \frac{\sin 9t}{9} + \cdots\right)$$

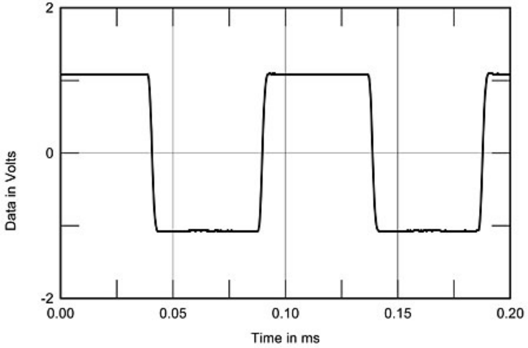


Bandwidth-Limited Signals

Basic observation: nothing changes infinitely fast

Bounding the rate of change sets the bandwidth of a signal

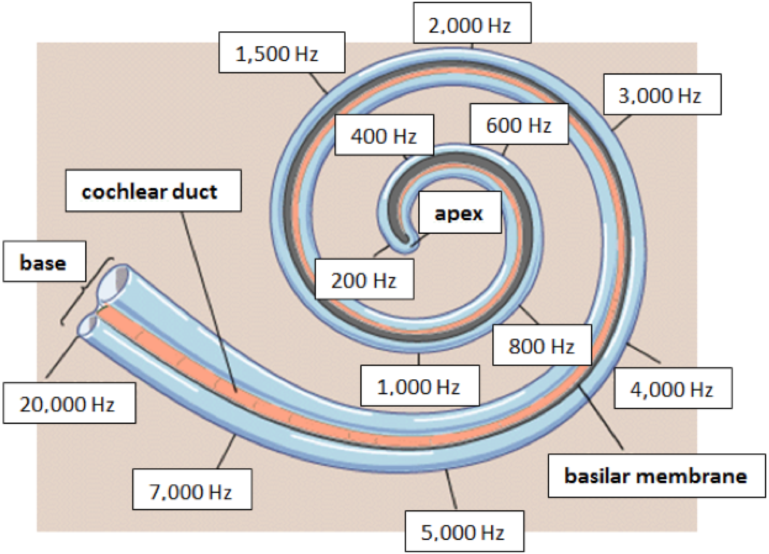
Hertz or Hz: “per second”



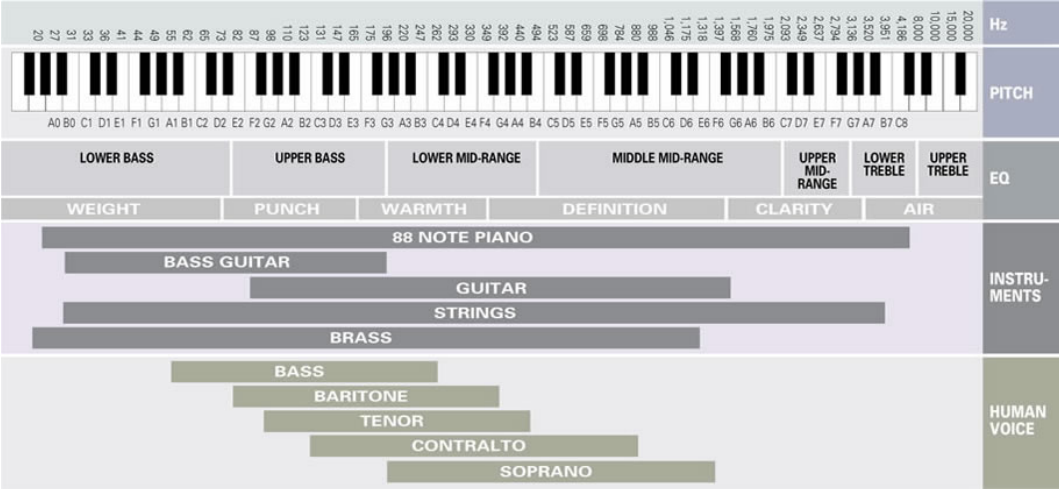
Source: Stereophile magazine: Marantz SM-11S1, A $4000 audiophile amplifier rated 5 Hz–120 kHz. Small-signal 10kHz squarewave into 8 ohms.

The Bandwidth of Sound

Human ears are almost a Fourier transform



Human Hearing

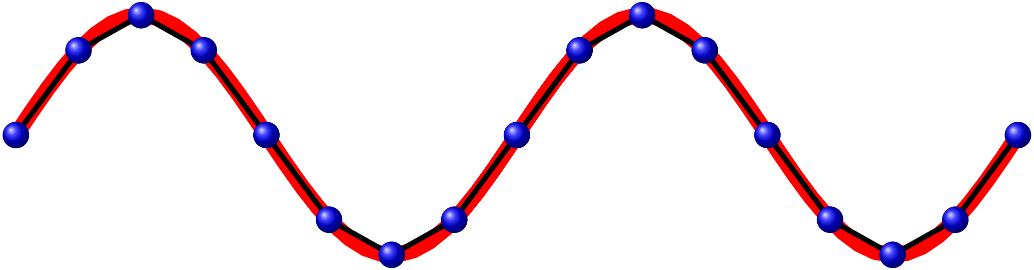


Empirically, humans hear 20 Hz–20 kHz

Highest frequency limit tends to decrease with age

Nyquist Theorem

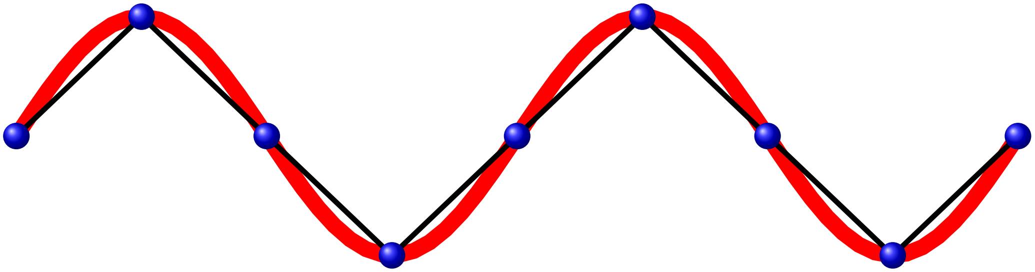
To reconstruct a bandwidth-limited signal from samples, you need to sample at least twice the maximum frequency.



Sampling at $8 \times f$

Nyquist Theorem

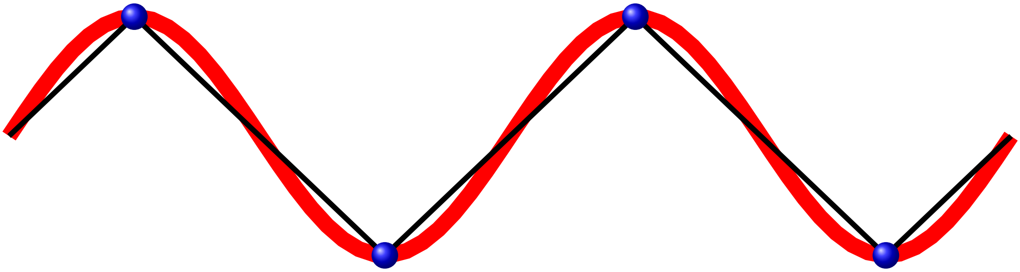
To reconstruct a bandwidth-limited signal from samples, you need to sample at least twice the maximum frequency.



Sampling at $4 \times f$

Nyquist Theorem

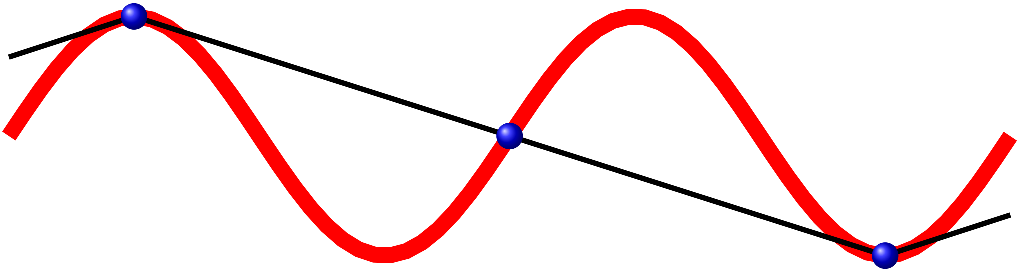
To reconstruct a bandwidth-limited signal from samples, you need to sample at least twice the maximum frequency.



Sampling at $2 \times f$

Nyquist Theorem

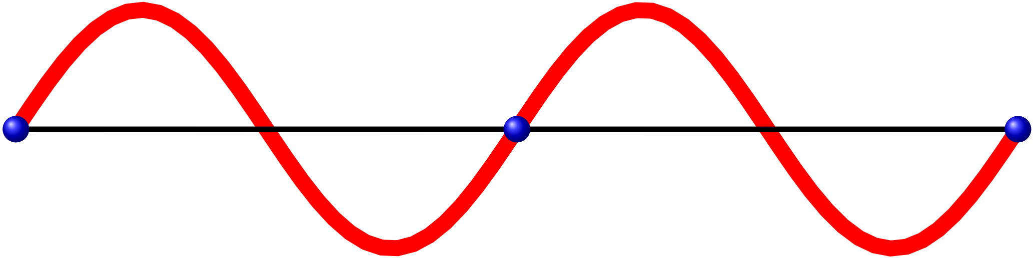
To reconstruct a bandwidth-limited signal from samples, you need to sample at least twice the maximum frequency.



Sampling at $4/3 \times f$

Nyquist Theorem

To reconstruct a bandwidth-limited signal from samples, you need to sample at least twice the maximum frequency.



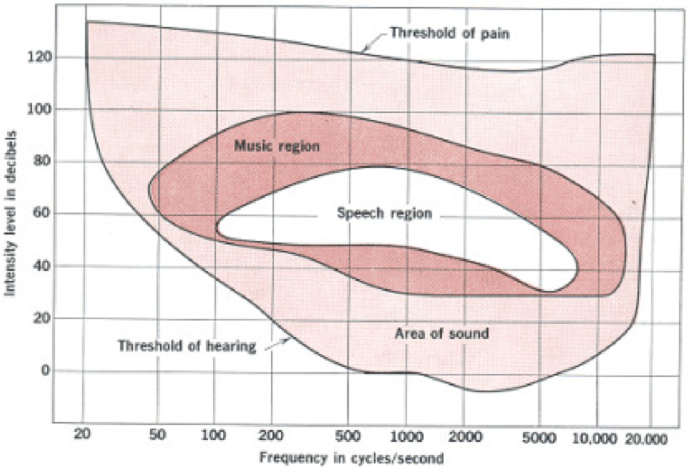
Sampling at $1 \times f$

Audio Sampling Rates

CD-quality audio: 44.1 kHz

Telephone-quality audio: 8 kHz

Dull, Metcalfe, Brooks. Modern Physics



Signal-to-Noise Ratio

You can't always get what you want
But if you try sometimes you might find
You get what you need

—The Rolling Stones

Signals are never pure: there's always something that makes them deviate from the ideal.

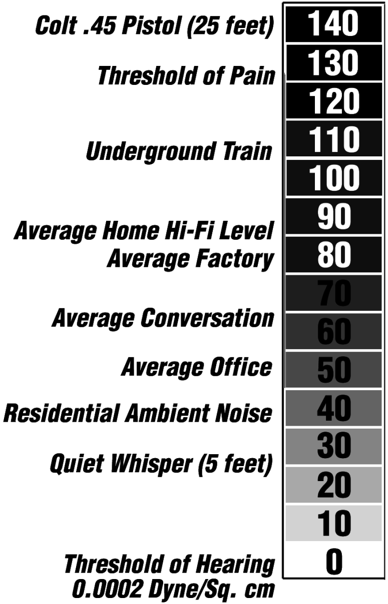
Signal-to-Noise ratio:

$$SNR = \frac{Signal\ Power}{Noise\ Power}$$

Usually measured using a log scale, i.e.,

$$dB = 10 \log_{10} \frac{P_{signal}}{P_{noise}}$$

Human Hearing dB, SNR, and bits



$$n \times 6.02 + 1.76 = SNR\ in\ dB$$

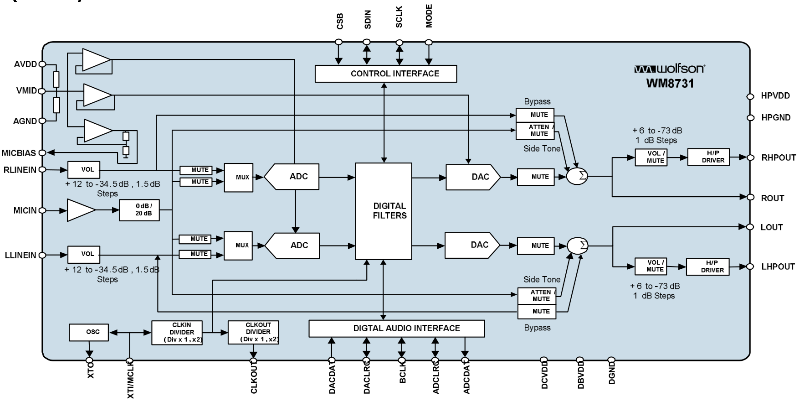
CD samples:

16 bits = 98 dB

Near the limit of human hearing

The CODEC on the DE1-SoC: Wolfson WM8731

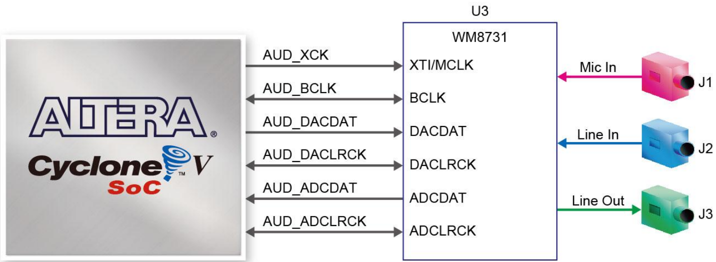
enCOder/DECoder: analog-to-digital converter (ADC) + digital-to-analog converter (DAC)



Two 24-bit ADCs; two 24-bit DACs + headphone amp

Sampling rates: 8 kHz – 96 kHz, 16–24 bit words

DE1-SoC Interface to the Audio Codec



Pin Assignment of Audio CODEC

Signal Name	*FPGA Pin No.*	*Description*	*I/O Standard*
AUD_ADCLRCK	PIN_K8	Audio CODEC ADC LR Clock	3.3V
AUD_ADCDAT	PIN_K7	Audio CODEC ADC Data	3.3V
AUD_DACLRCK	PIN_H8	Audio CODEC DAC LR Clock	3.3V
AUD_DACDAT	PIN_J7	Audio CODEC DAC Data	3.3V
AUD_XCK	PIN_G7	Audio CODEC Chip Clock	3.3V
AUD_BCLK	PIN_H7	Audio CODEC Bit-stream Clock	3.3V
I2C_SCLK	PIN_J12 or PIN_E23	I2C Clock	3.3V
I2C_SDAT	PIN_K12 or PIN_C24	I2C Data	3.3V

I^2C bus for configuration: data format, volume levels, etc.

Synchronous serial protocol (data + L/R + bit clock) for data

WM8731 Serial Protocol

One of four communication modes, set by I^2C registers:

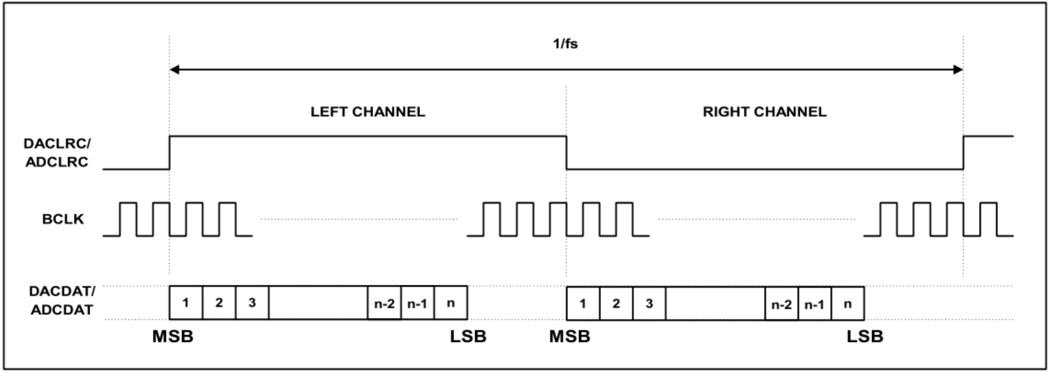
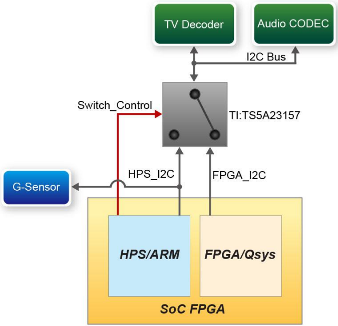


Figure 26 Left Justified Mode

DE1-SoC I^2C Multiplexer



Control mechanism for the I2C multiplexer

Pin Assignment of I2C Bus

Signal Name	*FPGA Pin No.*	*Description*	*I/O Standard*
FPGA_I2C_SCLK	PIN_J12	FPGA I2C Clock	3.3V
FPGA_I2C_SDAT	PIN_K12	FPGA I2C Data	3.3V
HPS_I2C1_SCLK	PIN_E23	I2C Clock of the first HPS I2C concontroller	3.3V
HPS_I2C1_SDAT	PIN_C24	I2C Data of the first HPS I2C concontroller	3.3V
HPS_I2C2_SCLK	PIN_H23	I2C Clock of the second HPS I2C concontroller	3.3V
HPS_I2C2_SDAT	PIN_A25	I2C Data of the second HPS I2C concontroller	3.3V

Storing Waveforms

If you store each sample,

$$\frac{\mathit{samples}}{\mathit{second}} \times \frac{\mathit{bits}}{\mathit{sample}} \times \mathit{channels} = \frac{\mathit{bits}}{\mathit{second}}$$

Total memory consumption:

$$\frac{\mathit{bits}}{\mathit{seconds}} \times \mathit{seconds} = \mathit{bits}$$

E.g., CD-quality audio: 44.1 kHz, 16 bits/sample, 2 channels

$$44.1\ \mathit{kHz} \times 16 \times 2 = 1.4\ \mathit{Mbps} = 175\ \mathit{KB/s}$$

A 74-minute CD:

$$1.4\ \mathit{Mbps} \times 60\ \frac{\mathit{seconds}}{\mathit{minute}} \times 74\ \mathit{minutes} \times \frac{\text{byte}}{8\ \mathit{bits}} = 783\ \mathit{MB}$$

Reducing Memory: Sample Less; Use Fewer Bits

74 minutes of CD-quality audio

(16 bits/sample, stereo, 44.1 kHz)

$$44.1\ kHz \times 32\ bits \times 60\ sec/min \times 74\ min \div 8\ bits/byte = 783\ MB$$

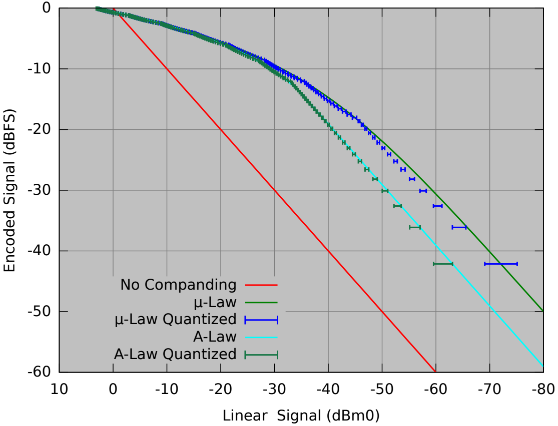
74 minutes of telephone-quality audio:

(8 bits/sample, mono, 8 kHz)

$$8\ kHz \times 8\ bits \times 60\ sec/min \times 74\ min \div 8\ bits/byte = 35\ MB$$

Reducing Memory: Lossy Compression (Companding)

μ-law and A-law compression
Logarithmic encoding of 12 bit samples in 8 bits
Trades dynamic range for quantization noise

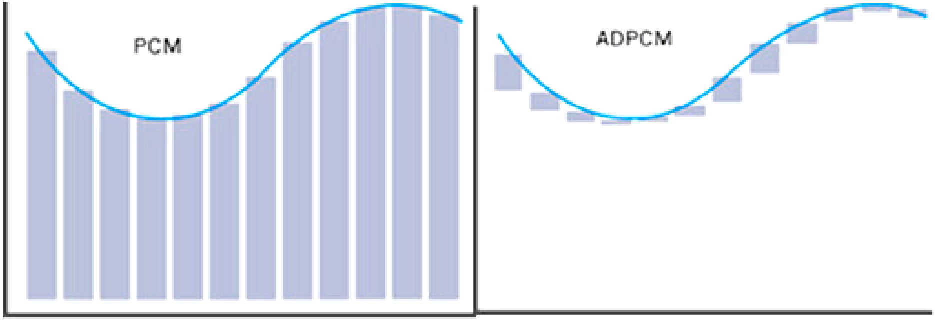


Source: Ozhiker, Wikimedia commons

ADPCM: Adaptive Predictive Pulse Code Modulation

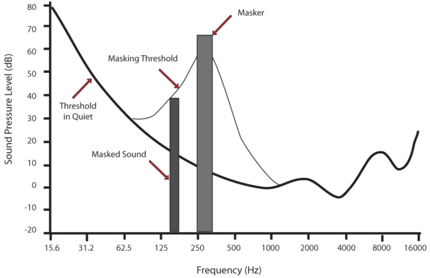
Uses 4 bits/sample to reconstruct 8-bit samples

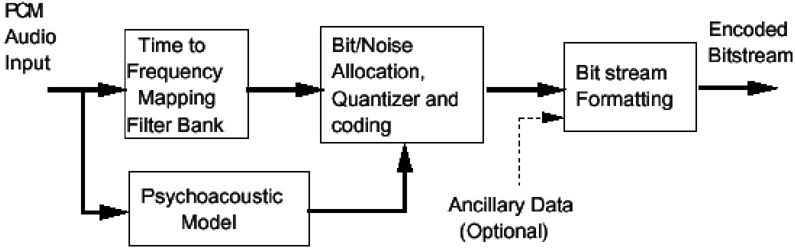
Encodes the *difference* between the next sample and its predicted value



MPEG Layer 3 Compression: Perceptual Coding

Carefully reproduce what we hear well and worry less about what we can't (soft sounds masked by loud ones)





Sound Synthesis: Analog

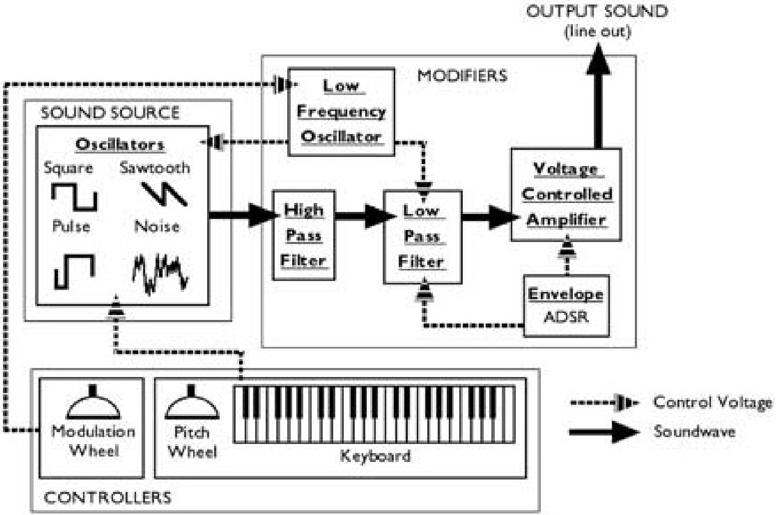
Modular analog sound synthesis c. 1968

Oscillators + noise sources + envelope generators + filters

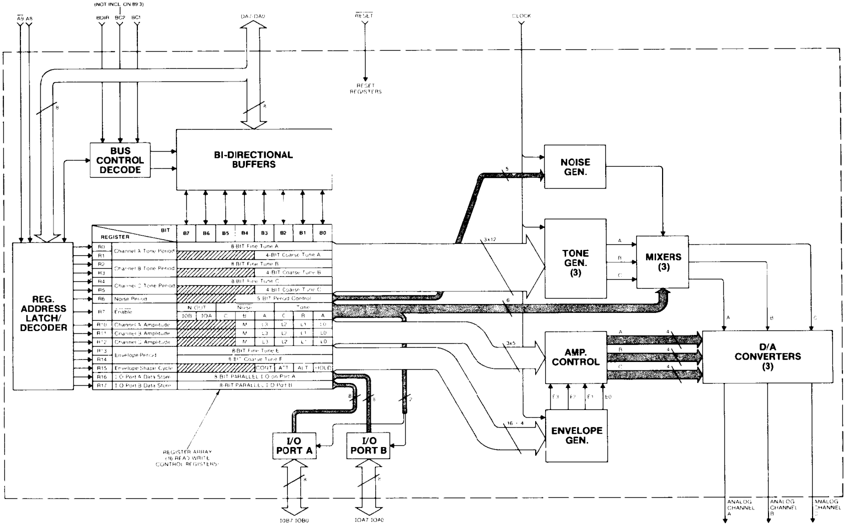
Moog synthesizer

Subtractive Synthesis

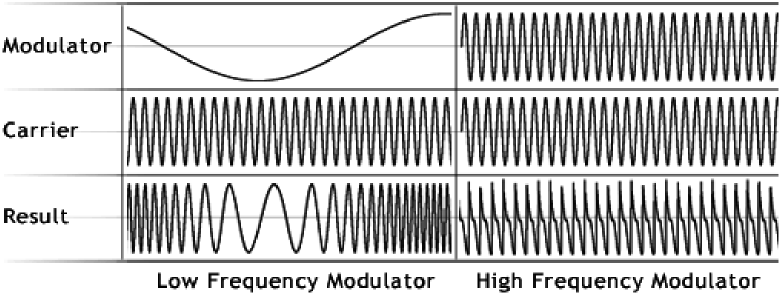
Start with a saw, square, or triangle wave, then filter



The AY-3-8912 Programmable Sound Generator



FM Synthesis



What does it sound like? Any pop music from the 1980s

Summary of Audio Waveform Generation

- Direct sampling (Pulse Code Modulation)

  Consider sampling frequency, bits/sample

- Lossy Compression

  Companding (μ-law, A-law)

  ADPCM

  Perceptual Coding (MP3 et al.)

- Synthesis

  Subtractive (oscillators, filters, envelopes)

  FM (Carrier $\times$ modulator, envelopes)

  Wavetable/sampling (sound snippets + note events)

Representing Images

Same story; two dimensional waveforms

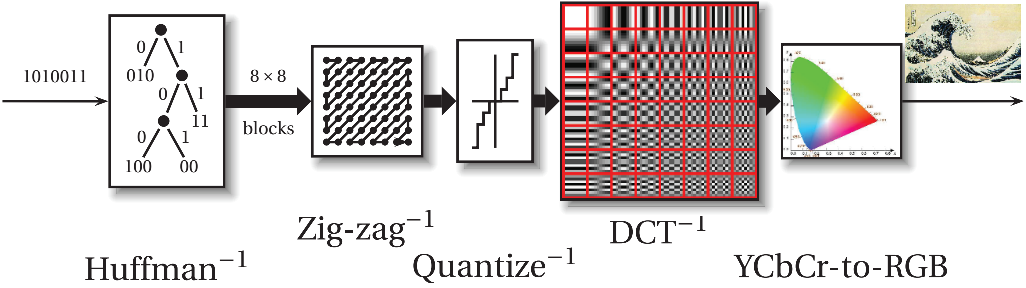
E.g., a single frame of VGA/standard definition television:

$$640 \times 480 \times 24 \frac{bits}{pixel} = 900\ KB$$

HD is terrifying:

$$1920 \times 1080 \times 24 \frac{bits}{pixel} = 5.9\ MB$$

JPEG: Still Image Compression



Colorspace conversion

Space-to-frequency domain conversion

Quantization

Zig-zag encoding

Huffman encoding