## Personal Project: Shift-Reduce Dependency Parsing

## 1 Problem Statement

The goal of this project is to implement a shift-reduce dependency parser. This entails two subgoals:

- Inference: We must have a shift-reduce parser that finds the correct parse given an oracle.
- Learning: We must choose a model that approximates the oracle and train it with labeled data.


## 2 Framework

### 2.1 Projective Dependency Trees

Given a sentence $\underline{x}=x_{1} \ldots x_{n}$, we want to find its dependency tree structure. The $n$ words $x_{1} \ldots x_{n}$ correspond to $n$ nodes in the tree. A valid dependency tree $\underline{y}$ for $\underline{x}$ is a directed tree over the $n$ nodes rooted at a special node $*$. We will characterize it as a set of $\operatorname{arcs}(i, j, l)$ where the pair $(i, j)$ forms a directed edge and $l \in \mathcal{L}$ is the label of the edge. An example is worth a thousand words:

$$
\begin{aligned}
& \underline{x}=\mathrm{I} \quad \text { see } . \\
& \underline{y}=\{(0,2, \text { ROOT }),(2,1, \text { SBJ }),(2,3, \mathrm{PU})\}
\end{aligned}
$$



We will only consider projective dependency trees, in which for every edge $(i, j)$ there is a directed path from $i$ to all nodes between $i$ and $j$. As an illustration, tree (a) below is not projective since there is no path from 3 to 4 even though there is an edge $(3,5)$. In contrast, tree (b) is projective.


A projective dependency tree has the "nested property" that for every word $x$, all words that are reachable from $x$ form a contiguous subsequence of the sentence. Note that tree (a) violates this property.

### 2.2 Shift-Reduce Parsing

At any point in parsing sentence $\underline{x}$, a shift-reduce parser maintains a parser configuration $c=(S, Q, A)$ with respect to $\underline{x}$ where

- $S$ is a stack $[\ldots i]_{S}$ of nodes that are processed.
- $Q$ is a queue $[j \ldots]_{Q}$ of nodes that are yet to be processed.
- $A$ is a set of arcs at this point.

The parser moves from one configuration to the next by performing one of the following transitions:

- left-arc $(l):\left([\ldots i, j]_{S}, Q, A\right) \Rightarrow\left([\ldots j]_{S}, Q, A \cup\{(j, i, l)\}\right)$
- right-arc $(l):\left([\ldots i, j]_{S}, Q, A\right) \Rightarrow\left([\ldots i]_{S}, Q, A \cup\{(i, j, l)\}\right)$
- shift: $\left([\ldots]_{S},[i \ldots]_{Q}, A\right) \Rightarrow\left([\ldots i]_{S},[\ldots]_{Q}, A\right)$

Let $\mathcal{T}$ be the set of all transitions. Given $\underline{x}=x_{1} \ldots x_{n}$, the parser initializes the configuration as $c=\left([0]_{S},[1 \ldots n]_{Q},\{ \}\right)$ and applies a sequence of transitions $t \in \mathcal{T}$ to reach the goal configuration $c=\left([0]_{S},[]_{Q}, A\right)$ for some final set of arcs $A$. Then it returns $\underline{y}=A$ as the predicted dependency tree.

## 3 Inference

Let $o$ be an oracle that predicts the correct transition $o(\underline{x}, c)=t \in \mathcal{T}$ for any configuration $c$ with respect to sentence $\underline{x}$. Using this oracle, we can find the true dependency tree for any sentence with the algorithm Shift-ReduceParse. Note that the running time is linear in the length of the sentence $n$, since there can be at most $2 n$ transitions before reaching the goal configuration.

## Shift-ReduceParse

Input: a sentence $\underline{x}=x_{1} \ldots x_{n}$, an oracle $o$
Output: a dependency tree $y$, transition history $H$

- Initialize $c \leftarrow\left([0]_{S},[1 \ldots n]_{Q},\{ \}\right)$ and $H \leftarrow\}$.
- While $|S|>1$ or $Q \neq[]$,
$\diamond t \leftarrow o(\underline{x}, c)$
$\diamond H \leftarrow H \cup\{(c, t)\}$
$\diamond c=(S, Q, A) \leftarrow t(c)$
- Return $\underline{y}=A$ and $H$.


## 4 Learning

Given $Q$ training examples $\left(\underline{x}^{(1)}, \underline{y}^{(1)}\right) \ldots\left(\underline{x}^{(Q)}, \underline{y}^{(Q)}\right)$ where $\underline{x}^{(q)}$ is a sentence and $\underline{y}^{(q)}$ is the dependency tree associated with it, we want to train a predictor $\hat{o}$ that approximates the oracle $o$.

### 4.1 Sample Extraction

Since the oracle receives a sentence $\underline{x}$ and a parser configuration $c$ with respect to $\underline{x}$ as the input and returns a transition $t \in \mathcal{T}$ as the output, we need to prepare samples of form $((q, c), t)$ where $q \in[Q]$ points to the relevant sentence $\underline{x}^{(q)}$. For this purpose, we make use of an auxiliary function NextTransition.

## NextTransition

Input: a dependency tree $\underline{y}$, a configuration $c=(S, Q, A)$
Output: the next transition to be applied to $c$ based on $\underline{y}$

1. Return shift if $|S|<2$.
2. Otherwise, $S=[\ldots i, j]_{S}$ for some $i<j$.
(a) Return left-arc $(l)$ if $(j, i, l) \in \underline{y}$.
(b) Return right-arc $(l)$ if $(i, j, l) \in \underline{y}$ and every $\left(j, j^{\prime}, l^{\prime}\right) \in \underline{y}$ is also in $A$.
(c) Return shift otherwise.

The extra condition in $2(\mathrm{~b})$ makes sure that node $j$ parents all its children before it is removed from the stack. This is not necessary in 2(a) since if the tree $\underline{y}$ is projective, node $i$ must parent all its children before reaching $j$ in order to satisfy the nested property.

Now we can extract a set of samples $E=\left\{\left(\left(q^{(z)}, c^{(z)}\right), t^{(z)}\right)\right\}_{z=1}^{Z}$ of some size $Z$ using the algorithm ExtractSamples.

## ExtractSamples

Input: training examples $\left(\underline{x}^{(1)}, \underline{y}^{(1)}\right) \ldots\left(\underline{x}^{(Q)}, \underline{y}^{(Q)}\right)$
Output: a set of samples $E=\left\{\left(\left(q^{(z)}, c^{(z)}\right), t^{(z)}\right)\right\}_{z=1}^{Z}$

- $E \leftarrow\}$
- For $q=1 \ldots Q$,
$\diamond$ Define oracle $o_{q}$ for $\underline{x}^{(q)}$ as follows. Given configuration $c$, the oracle will predict

$$
o_{q}\left(\underline{x}^{(q)}, c\right)=\operatorname{NextTransition}\left(\underline{y}^{(q)}, c\right)
$$

$\diamond \underline{y}_{q}, H_{q} \leftarrow \operatorname{Shift-ReduceParse}\left(\underline{x}^{(q)}, o_{q}\right) / / \underline{y}_{q}=\underline{y}^{(q)}$ must hold
$\diamond E \leftarrow E \cup\left\{((q, c), t):(c, t) \in H_{q}\right\}$

- Return $E$.


### 4.2 Feature Representation

Now that we have labeld samples $((q, c), t)$, it is straightforward to train a multiclass classifier that mimics the oracle. But first, we must decide on how to represent the input $(q, c)$. Let $\phi$ be a feature function that maps a sentence-configuration pair $(\underline{x}, c)$ to a $d$-dimensional vector $\phi(\underline{x}, c) \in \mathbb{R}^{d}$. We can use any features in $\underline{x}$ and $c=(S, Q, A)$ useful for making prediction, such as

- Part-of-speech tags of the nodes on the stack
- Word identities of the nodes on the stack
- Labels of the arcs originating from the nodes on the stack

For example, suppose we extract a sample $((q, c), t)$ where

$$
\begin{aligned}
\underline{x}^{(q)} & =\mathrm{I} \text { see } . \\
c & =\left([0,2,3]_{S},[]_{Q},\{(2,1, \mathrm{SBJ})\}\right) \\
t & =\text { right-arc(PU })
\end{aligned}
$$

We can use a binary vector $v=\phi\left(\underline{x}^{(q)}, c\right) \in \mathbb{R}^{d}$ to encode the following information:

$$
\begin{aligned}
\operatorname{ROOT} & =\text { True } \\
\operatorname{POS}(3) & =\mathrm{SYM} \\
\operatorname{POS}(2) & =\mathrm{VB} \\
\mathrm{WORD}(3) & = \\
\mathrm{WORD}(2) & =\text { see } \\
\mathrm{ARC}-\mathrm{L}(3) & =\varnothing \\
\mathrm{ARC}-\mathrm{R}(3) & =\varnothing \\
\mathrm{ARC}-\mathrm{L}(2) & =\mathrm{SBJ} \\
\mathrm{ARC}-\mathrm{R}(2) & =\varnothing
\end{aligned}
$$

For notational cleanness, we will use $E^{\prime}=\left\{\left(v^{(z)}, t^{(z)}\right)\right\}_{z=1}^{Z}=\left\{\left(\phi\left(\underline{x}^{\left(q^{(z)}\right)}, c^{(z)}\right), t^{(z)}\right)\right\}_{z=1}^{Z}$ to denote the set of feature-transformed samples.

### 4.3 Averaged Perceptron

A linear classifier keeps a weight vector $w_{t} \in \mathbb{R}^{d}$ for each $t \in \mathcal{T}$ and defines a score function $f\left(w_{t}, \phi(c)\right) \in \mathbb{R}$. Given a sentence $\underline{x}$ and a parser configuration $c$ with respect to $\underline{x}$, an oracle approximator $\hat{o}$ using this classifier will predict

$$
\hat{o}(\underline{x}, c)=\underset{t \in \mathcal{T}}{\arg \max } f\left(w_{t}, \phi(\underline{x}, c)\right)
$$

We will choose the averaged perceptron as our classifier, which defines $f\left(w_{t}, \phi(c)\right)=w_{t} \cdot \phi(c)$. The weight vector $w_{t} \in \mathbb{R}^{d}$ is learned from feature-transformed samples $E^{\prime}=\left\{\left(v^{(z)}, t^{(z)}\right)\right\}_{z=1}^{Z}$ using the algorithm TrainAveragedPerceptron. Two remarks on this specific installment of the algorithm:

- Averaging: Instead of storing a vector $w_{t}^{r, z} \in \mathbb{R}^{d}$ for all $t \in \mathcal{T}, r \in[R], z \in[Z]$ and then averaging

$$
w_{t}=\frac{\sum_{r=1}^{R} \sum_{z=1}^{Z} w_{t}^{r, z}}{R Z}
$$

we keep distinct weights $w_{t}^{\prime}$ only once and record how many examples it endures without making a mistake by a dictionary $s_{t}$. Then the final weights are given by

$$
w_{t} \leftarrow \frac{\sum_{w_{t}^{\prime} \in s_{t}} w_{t}^{\prime} \times s_{t}\left(w_{t}^{\prime}\right)}{\sum_{w_{t}^{\prime} \in s_{t}} s_{t}\left(w_{t}^{\prime}\right)}
$$

- Update: The update scheme here is called "ultraconservative": $w_{t} \leftarrow w_{t}+\gamma_{t} \phi\left(c^{(z)}\right)$ where $\gamma_{t^{(z)}}=1$, $\sum_{t \neq t^{(z)}} \gamma_{t}=-1$, and $\gamma_{t}=0$ for $t \in \mathcal{T}$ on which no mistake is made. The normalization contraint is necessary for the percetron convergence guarantee.


## TrainAveragedPerceptron

Input: $E^{\prime}=\left\{\left(v^{(z)}, t^{(z)}\right)\right\}_{z=1}^{Z}$, number of rounds $R \in \mathbb{N}$
Data Structure: a dictionary $s_{t}$ for each $t \in \mathcal{T}$
Output: $w_{t} \in \mathbb{R}^{d}$ for each $t \in \mathcal{T}$

- $w_{t} \leftarrow(0, \ldots, 0) \in \mathbb{R}^{d}$ for all $t \in \mathcal{T}$
- For $r=1 \ldots R$, for $z=1 \ldots Z$,
$\diamond$ For $t \in \mathcal{T}, s_{t}\left(w_{t}\right) \leftarrow s_{t}\left(w_{t}\right)+1$ if $w_{t} \in s_{t}, s_{t}\left(w_{t}\right) \leftarrow 1$ otherwise
$\diamond$ Find a set of transitions that incorrectly scored higher than the true transition:

$$
\Psi=\left\{t \in \mathcal{T}-\left\{t^{(z)}\right\}: w_{t} \cdot v^{(z)}>w_{t(z)} \cdot v^{(z)}\right\}
$$

$\diamond$ If $|\Psi|>0$,

* Update $w_{t^{(z)}} \leftarrow w_{t^{(z)}}+v^{(z)}$
* For $t \in \Psi$, update $w_{t} \leftarrow w_{t}-\frac{1}{|\Psi|} v^{(z)}$
- Return

$$
w_{t} \leftarrow \frac{\sum_{w_{t}^{\prime} \in s_{t}} w_{t}^{\prime} \times s_{t}\left(w_{t}^{\prime}\right)}{\sum_{w_{t}^{\prime} \in s_{t}} s_{t}\left(w_{t}^{\prime}\right)} \text { for all } t \in \mathcal{T}
$$

### 4.4 Summary

Here we summarize the procedure of estimating the oracle developed in this section. The inputs are training data of dependency trees, a feature function $\phi$ to represent any sentence-configuration pair $(\underline{x}, c)$, and the number of training rounds $R$.

## EstimateOracle

Input: $\left(\underline{x}^{(1)}, \underline{y}^{(1)}\right) \ldots\left(\underline{x}^{(Q)}, \underline{y}^{(Q)}\right)$, feature function $\phi$, number of rounds $R$
Output: an oracle appoximator $\hat{o}$

- $E \leftarrow \operatorname{ExtractSamples}\left(\left(\underline{x}^{(1)}, \underline{y}^{(1)}\right) \ldots\left(\underline{x}^{(Q)}, \underline{y}^{(Q)}\right)\right)$
- $E^{\prime} \leftarrow\left\{\left(\phi\left(\underline{x}^{(q)}, c\right), t\right):((q, c), t) \in E\right\}$
- $\left\{w_{t}\right\}_{t \in \mathcal{T}} \leftarrow$ TrainAveragedPerceptron $\left(E^{\prime}, R\right)$
- Return an oracle approximator $\hat{o}$ that predicts for any sentence $\underline{x}$ and a configuration $c$ with respect to $\underline{x}$

$$
\hat{o}(c)=\underset{t \in \mathcal{T}}{\arg \max } w_{t} \cdot \phi(\underline{x}, c)
$$

## References

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