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Graph computation language based on the BLAS* specification

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*Basic Linear Algebra Subprograms

Let's hop right in

01 Motivation

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04 Other features

02 Semirings

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× 05 Dirty Details

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03 Selection

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06 Demo 👘

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Motivation

Graphs as matrices

Graphs can be represented as matrices.

Graph operations can be written as matrix operations. Matrix operations are highly optimized, fully realizing parallel computation.

Benefits

GraphBLAS API



Graph Algorithms in the Language of Linear Algebra



CONTRELITORS Rader, Blier, Bord, Balvar, Daularay, Eddman, Falouscoe, Fineman, Gilbert, Hettech, Hendrickens, Espolmeyer, Koper, Kobla, Leskovet, Maddari, Mohindta, Ngayen, Rader, Reinhardt, Robinson & Shah

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The GraphBLAS C API Specification [†]: Version 1.30

Aydın Buluç, Timothy Mattson, Scott McMillan, José Moreira, Carl Yang

Generated on 2019/09/25 at 15:32:56 EDT

[†]Based on GraphBLAS Mathematics by Jeromy Kepner

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BFS using the	C GraphBLAS	Library
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#include <stdlib.h> #include <stdio.h> #include <stdint.h> #include <stdbool.h> #include "GraphBLAS.h" 7 8 * Given a boolean n x n adjacency matrix A and a source vertex s, performs a BFS traversal * of the graph and sets v[i] to the level in which vertex i is visited (v[s] = 1). 9 * If i is not reacheable from s, then v/i = 0. (Vector v should be empty on input.) 10 11 */ GrB_Info BFS(GrB_Vector *v, GrB_Matrix A, GrB_Index s) 12 13 14 GrB_Index n: GrB_Matrix_nrows(&n,A); // n = # of rows of A15 16 17 GrB_Vector_new(v,GrB_INT32,n); // Vector<int32_t> v(n)18 // vertices visited in each level 19 GrB_Vector q: 20 GrB_Vector_new(&g,GrB_BOOL,n); // Vector<bool> q(n) 21 GrB_Vector_setElement(q,(bool)true,s); // q[s] = true, false everywhere else 22 23 /* 24 * BFS traversal and label the vertices. 25 */ 26 int32.t d = 0;// d = level in BFS traversal 27 28 bool succ = false; // succ == true when some successor found do { 29 // next level (start with 1) ++d: 30 $GrB_assign(*v,q,GrB_NULL,d,GrB_ALL,n,GrB_NULL); // v[q] = d$ 31 GrB_vxm(q,*v,GrB_NULL,GrB_LOR_LAND_SEMIRING_BOOL, 32 33 q.A.GrB_DESC_RC): $// q[!v] = q \parallel \mathcal{BB} A$; finds all the // unvisited successors from current q 34 GrB_reduce(&succ, GrB_NULL, GrB_LOR_MONOID_BOOL. 35 q, GrB_NULL); // succ = ||(q)|36 while (succ); // if there is no successor in q, we are done. 37 38 GrB_free(&g): // g vector no longer needed 39 40 return GrB_SUCCESS:

From "The GraphBLAS C API Specification", Buluç, et al

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Can we do better?

BFS in BLAStoff

```
def BFS(G, frontier) {
    #logical;
    N = |G|[0];
    levels = Zero(N : 1);
    maskedGT = G^T:
    depth = 0;
    while (plusColumnReduce(frontier)) {
        #arithmetic;
        depth = depth + 1;
        #logical;
        levels[rangeFromVector(frontier)] = depth;
        mask = !(levels)[0, Zero(N:1), N, 1];
        maskedGT = maskedGT @ mask;
        frontier = maskedGT * frontier;
    #arithmetic;
    return levels + One(|levels|)~(-1);
```

There's a lot going on here. Let's talk about some of these features!

BLAStoff Overview

- Every object is a matrix
- Imperative
- Wide offering of primitive matrix operations
- Versatile matrix selection operator
- Semiring semantics

A set of two binary operators: addition and multiplication.

A set of two binary operators: addition and multiplication.

- (R, +) is a commutative monoid with identity element 0
- (R, *) is a monoid with identity element 1
- Multiplication left and right distributes over addition
- Multiplication by 0 annihilates R

A set of two binary operators: addition and multiplication.

Arithmetic semiring:

- 3 + 7 = 10
- 3 * 7 = 21
- etc.

A set of two binary operators: addition and multiplication.

Arithmetic semiring:

- 3 + 7 = 10
- 3 * 7 = 21
- etc.

+ • 3 + 0 = 1• 3 + 7 = 1

3+7=1
0+0=0

Logical semiring:

- 3 * 0 = 0
- etc.

A set of two binary operators: addition and multiplication.

Arithmetic semiring:

- 3 + 7 = 10
- 3 * 7 = 21
- etc.

+ • + + .

Logical semiring:

- 3 + 0 = 1
- 3 + 7 = 1
- 0 + 0 = 0
- 3 * 0 = 0
- etc.

t**c**. ₊ .

Maxmin semiring:

- 3 + 7 = 7
- 3 * 7 = 3
- etc.

#semiring-name; to change semiring

#semiring-name; to change semiring

A = [1, 2;3, 4]; 4 B = [0, -1];-2, 5]; 6 7 #maxmin; printm(A + B); // prints: $1 2 \setminus n3 5$ 8 printm(A * B); // prints: -2 2\n-2 4 9 10 #arithmetic; 11 printm(A + B);12 // prints: 1 1\n1 9 printm(A * B); // prints: -4 9\n-8 17 13

```
def addThree(A, B, C) {
         sum = A + B + C;
         return sum;
 4
 5
 6
     def f(A, B, C) {
         #maxmin;
         printm(addThree(A, B, C)); // prints 6
 8
         printm(A + B + C);
 9
                                      // prints 3
10
     }
11
12
     A = 1;
13
     B = 2;
14
     C = 3;
15
16
     printm(A + B + C); // prints 6
     f(A, B, C);
17
```

```
def addThree(A, B, C) {
         #maxmin;
 2
         sum = A + B + C;
         return sum;
 4
 5
 6
     def f(A, B, C) {
 7
         #maxmin;
 8
         printm(addThree(A, B, C)); // prints 3
 9
         printm(A + B + C);
                                      // prints 3
10
11
     }
12
13
     A = 1;
14
     B = 2;
15
     C = 3;
16
     printm(A + B + C); // prints 6
17
     f(A, B, C);
18
```



```
def addThree(A, B, C) {
 2
         #_;
         sum = A + B + C;
 3
         return sum;
 4
 5
 6
     def f(A, B, C) {
 7
         #maxmin;
 8
         printm(addThree(A, B, C)); // prints 3
 9
         printm(A + B + C);
                                      // prints 3
10
11
     }
12
13
     A = 1;
14
     B = 2;
15
     C = 3;
16
     printm(A + B + C); // prints 6
17
     f(A, B, C);
18
```

[•]Selection

"There is wisdom in the **selection** of wisdom."

– Dr. Bergen Evans, English professor and TV host

• Robust

- Expressive
- Powerful
- But concise
- In other words, matrix.get(i,j) won't cut it

How should selection work?

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Our selection operator

- M[A, B, c, d]
- A: row indices, B: column indices
- c, d: size of the submatrices
- A is the only required argument
- B, c, d default to [0], [1], [1], respectively

Example v = [1;2;3;4];M = [1, 2, 3;2 4, 5, 6; 3 7, 8, 9]; 4 v[2]; // gets [3] 6 M[2,1]; // gets [8] v[[0;3]]; // gets [1;4] 8 M[1, 1, 1, 2]; // gets [5,6] 9

row indices

 $13 \sim M = [1, 2, 3;$ 4, 5, 6; 7, 8, 9];

7, 8, 8, 9;

1, 2, 2, 3;

4, 5, 5, 6;]*/

17 18 19

14

15

21

22

23

16 M[[1;0], [0;1], 2, 2]; /* gets: [4, 5, 5, 6; 20

Example

size

column indices

row indices

13

14

15

16

17

18

 $M = \begin{bmatrix} 1, & 2, & 3; \\ 4, & 5, & 6; \\ 7, & 8, & 9 \end{bmatrix};$

* * * * * *

 M[[1;0], [0;1], 2, 2];

Example

/* gets: 5, 6; 4, 5, 7, 8, 8, 9; 1, 2, 2, 3; 6;]*/ 4, 5, 5,

.

size

column indices

+ * + * *

Example M = [1, 2, 3;4, 5, 6; 7, 8, 9]; M[0, [0;2], 2, 1] = [-1;-1];/* sets M to [-1, 2, -1;-1, 5, -1; 7, 8, 9];*/

Operators

"It's not the **operation** itself that is the concern, it's the anesthesia."

– Sanjay Gupta, neurosurgeon

Convolution ~

A ~ B: slide B across A like so...



...where each windowed view becomes just one entry in the resulting matrix.

Why is this useful for us?

Convolution ~

- Can be used to emulate other typical operators, most notably scalar multiplication.
- BLAStoff has no scalars. To achieve this, we just use a sliding window of size 1x1!

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A = [1, 2, 3; 4, 5, 6]; k = 2; B = A ~ k; // B is now [2, 4, 6; 8, 10, 12];

Size ||

- For an m x n matrix A, |A| returns a 2 x 1 column vector with values m and n.
- For instance, to make an m x n matrix of zeros would simply be:

A = [1, 2, 3; 4, 5, 6]; B = Zero(|A|);

// B is now [0, 0, 0; 0, 0, 0];



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Size | |: Nifty Example

If we isolate the values into separate variables, we can use selection to replace all values of A!

| m = A [0];
n = A [1]; | |
|----------------------------|---------------------------|
| A[range(m), | <pre>range(n)] = 3;</pre> |

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Reduce Rows %

Row-reductions with either summation or product.

A = [1, 2; 3, 4]; B = +%A; // B is [3; 7] C = *%A; // C is [2; 12]

(And this works with semirings!!)

Another Feature: Graph Literals

Graphs can be declared just like matrices

[0->1; 2->3; 3->0]

These create equivalent matrices

0,0,0,1; 0,0,0,0; 1,0,0,0]

G

M =

[0,1,0,0;

Other basic operators

- Matrix multiplication (*)
- Element-wise multiplication (@) and addition (+)
- Exponent: (b | T) where b is a 1x1 matrix and $b \ge 0$
- Vertical concatenation (:)

Dirty details; Are we proud?

O1 Our Process

- Excellent division of labor, everyone specialized while still interacting with all the code
- Github issues for feature tracking

What We're Proud Of



02 Our Project

- Implemented our full LRM, save stretch goals
- Learned linear algebra and abstract algebra

03 Our Code

- Removed SAST
 while keeping
 type-checking of int
 vs float matrices
- Programmatically created function types, definitions, and calls
- Lazy evaluation
- Semiring stack

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What We're Not Proud Of: Our commit messages

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BFS Demo

Questions?

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