Fundamentals of Computer Systems Thinking Digitally

Stephen A. Edwards

Columbia University

Summer 2021

The Subject of this Class

0

The Subjects of this Class

1

But let your communication be, Yea, yea; Nay, nay: for

— Matthew 5:37

whatsoever is more than these cometh of evil.



SUPERCODER 2000 Air cooled coding keyboard for professional use.





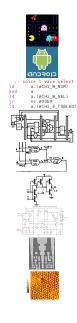








Engineering Works Because of Abstraction



Application Software

Operating Systems

Architecture

Micro-Architecture

Logic

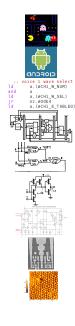
Digital Circuits

Analog Circuits

Devices

Physics

Engineering Works Because of Abstraction



Application Software COMS 3157, 4156, et al.

Operating Systems COMS W4118

Architecture Second Half of 3827

Micro-Architecture Second Half of 3827

Logic First Half of 3827

Digital Circuits First Half of 3827

Analog Circuits ELEN 3331

Devices ELEN 3106

Physics ELEN 3106 et al.

Boring Stuff

http://www.cs.columbia.edu/~sedwards/classes/2021/3827-summer/

Prof. Stephen A. Edwards sedwards@cs.columbia.edu

Lectures 4:10 – 6:40 PM, Mondays and Wednesdays May 3–June 14

Weight	What	When
40%	Homeworks	See Webpage
60%	Final exam	June 18th

Submit homework online via Courseworks

Software You Need



Circuit design problems: download (class website) .zip file with .dig files, edit with Digital, upload to Courseworks

SPIM: A MIPS32 Simulator spimsimulator.sourceforge.net

MIPS assembly coding:, download .zip file with .s files, edit in favorite text editor, test and debug in SPIM, upload to Courseworks



The Inkscape SVG File Editor inkscape.org

INKSCAPE

You can do homework by downloading an SVG file from the class website, editing it in Inkscape, and uploading it to Courseworks

Rules and Regulations

Each assignment turned in must be unique; work must ultimately be your own.

Don't cheat: Columbia Students Aren't Cheaters

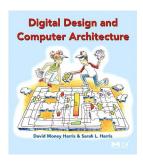
Test will be closed-book; you may use a single sheet of your own notes

Optional Texts: Alternative 1

No required text. One option:

David Harris and Sarah Harris. Digital Design and Computer Architecture. Either 1st or 2nd ed.

Almost precisely right for the scope of this class: digital logic and computer architecture.





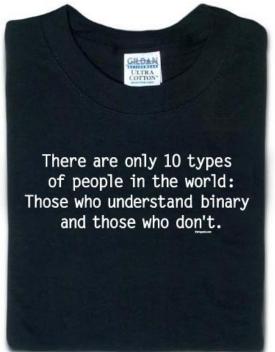
Optional Texts: Alternative 2

M. Morris Mano and Charles Kime. Logic and Computer Design Fundamentals. 4th ed.



David A. Patterson and John L. Hennessy. Computer Organization and Design, The Hardware/Software Interface. 4th ed.





thinkgeek.com

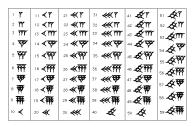
Which Numbering System Should We Use?



Roman: I II III IV V VI VII VIII IX X



Mayan: base 20, Shell = 0



Babylonian: base 60

The Decimal Positional Numbering System



Ten figures: 0 1 2 3 4 5 6 7 8 9

$$730_{10} = 7 \times 10^2 + 3 \times 10^1 + 0 \times 10^0$$

$$990_{10} = 9 \times 10^2 + 9 \times 10^1 + 0 \times 10^0$$

Why base ten?



Hexadecimal, Decimal, Octal, and Binary

Hex Dec Oct

HICA	Dec	Oct	וווט
0	0	0	0
1	1	1	1
2	2	2	10
3	3	3	11
4	4	4	100
5	5	5	101
6	6	6	110
7	7	7	111
8	8	10	1000
9	9	11	1001
Α	10	12	1010
В	11	13	1011
C	12	14	1100
D	13	15	1101
Ε	14	16	1110
F	15	17	1111

Bin

Binary and Octal: Electronics Likes Powers of Two



	Oct	Bin
~	0	0
1968	1	1
	2	10
ر.	3	11
φ	4	100
PDP-8/	5	101
	6	110
DEC.	7	111

PC =
$$0101101111101_2$$

= $0 \times 2^{11} + 1 \times 2^{10} + 0 \times 2^9 + 1 \times 2^8 + 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
= 2675_8
= $2 \times 8^3 + 6 \times 8^2 + 7 \times 8^1 + 5 \times 8^0$
= 1469_{10}

Hexadecimal Numbers

Base 16: 0 1 2 3 4 5 6 7 8 9 A B C D E F
Instead of groups of 3 bits (octal), Hex uses groups of 4.

CAFEFOOD₁₆ =
$$12 \times 16^7 + 10 \times 16^6 + 15 \times 16^5 + 14 \times 16^4 + 15 \times 16^3 + 0 \times 16^2 + 0 \times 16^1 + 13 \times 16^0$$

= $3,405,705,229_{10}$

| C | A | F | E | F | 0 | 0 | D | Hex 1100101011111111011111000000001101 Binary | 3 | 1 | 2 | 7 | 7 | 5 | 7 | 0 | 0 | 1 | 5 | Octal

Computers Rarely Manipulate True Numbers

Infinite memory still very expensive

Finite-precision numbers typical

32-bit processor: naturally manipulates 32-bit numbers

64-bit processor: naturally manipulates 64-bit numbers

How many different numbers can you

binary
octal
represent with 5 decimal
hexadecimal

Jargon



434		l 0	1	2	2	1	_	c	7	0	0	
620		U	_'			4		-				
+628	0	0	1	2	3	4	5	6	7	8	9	
1 020	1	1	2	3	4	5	6	7	8	9	10	
	2	2	3	4	5	6	7	8	9	10	11	
	3	3	4	5	6	7	8	9	10	11	12	
	4	4	5	6	7	8	9	10	11	12	13	
	5	5	6	7	8	9	10	11	12	13	14	
	6	6	7	8	9	10	11	12	13	14	15	
	7	7	8	9	10	11	12	13	14	15	16	
4+8 = 12	8	8	9	10	11	12	13	14	15	16	17	
4 + 0 - 12	9	9	10	11	12	13	14	15	16	17	18	
	10	10	11	12	13	14	15	16	17	18	19	

$$4 + 8 = 12$$

$$1+3+2 = 6$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

$$4+8 = 12$$
 $1+3+2 = 6$
 $4+6 = 10$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
2 3	3	4	5	6	7	8	9	10	11	12
4 5	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19
	'									

$$4+8 = 12$$
 $1+3+2 = 6$
 $4+6 = 10$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
2 3	3	4	5	6	7	8	9	10	11	12
4 5	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

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+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
2 3	3	4	5	6	7	8	9	10	11	12
4 5 6	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

$$\begin{array}{c|ccccc}
0011 \\
10011 \\
+11001 \\
\hline
1100 \\
& & + & 0 & 1 \\
\hline
1+1 & = & 10 & & 0 & 00 & 01 \\
1+1+0 & = & 10 & & 1 & 01 & 10 \\
1+0+0 & = & 01 & & 10 & 10 & 11 \\
0+0+1 & = & 01 & & 0+1 & 10 & 11
\end{array}$$

Signed Numbers: Dealing with Negativity

How should we represent negative numbers?



Binary Signed Magnitude Numbers

The familiar notation: negative numbers have a leading – Binary signed-magnitude encoding: leading 1 indicates negative; remaining bits treated as binary.

والمراهد المراجع المراجع المراجع والمرجع والمرجع والمراجع المراجع

0000 ₂ = 0	annoying:
$0010_2 = 2$	If the signs match, add the magnitudes
$1010_2 = -2$	and use the same sign.
$1111_2 = -7$	If the signs differ, subtract the smaller number from the larger; return the sign
$1000_2 = -0$?	of the larger.

One's Complement Numbers

Like Signed Magnitude, a leading 1 indicates a negative One's Complement number. However, number magnitude is *complement* of remaining bits interpreted as binary.

To negate a number, complement (flip) each bit.

0000_{2}	=	0
------------	---	---

 $0010_2 = 2$

 $1101_2 = -2$

 $1000_2 = -7$

 $1111_2 = -0$?

Addition is nicer: just add the one's complement numbers as if they were

normal binary.

Really annoying having a -0: two numbers are equal if their bits are the same or if one is 0 and the other is -0.



NOTALL ZEROS ARE CREATED EQUAL

ZERO CALORIES. MAXIMUM PEPSI TASTE.



Two's Complement Numbers



Really neat trick: just make only the most significant bit represent a *negative* number instead of positive; treat the rest as binary.

$$1101_2 = -8 + 4 + 1 = -3$$

 $1111_2 = -8 + 4 + 2 + 1 = -1$

$$0111_2 = 4 + 2 + 1 = 7$$

$$1000_2 = -8$$

Easy addition: just add in binary and discard any carry.

Negation: complement each bit (as in one's complement) then add 1.

Subtraction done with negation and addition.

Very good property: no −0

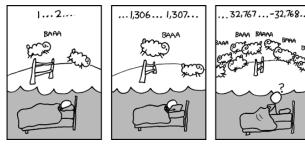
Two's complement numbers are equal if and only if all their bits are the same.

Number Representations Compared

Code	Binary	Signed Mag.	One's Comp.	Two's Comp.
0000	0	0	0	0
0001	1	1	1	1
÷				
0111	7	7	7	7
1000	8	-0	-7	-8
1001	9	-1	-6	-7
÷				
1110	14	-6	-1	-2
1111	15	-7	-0	-1

Smallest number

Largest number



https://xkcd.com/571/

How many bits in his brain?

Fixed-point Numbers

How to represent fractional numbers? In decimal, we continue with negative powers of 10:

$$31.4159 = 3 \times 10^{1} + 1 \times 10^{0} + 4 \times 10^{-1} + 1 \times 10^{-2} + 5 \times 10^{-3} + 9 \times 10^{-4}$$

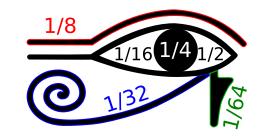
Also works in binary:

$$1011.0110_{2} = 1 \times 2^{3} + 0 \times 2^{2} + 1 \times 2^{1} + 1 \times 2^{0} + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4}$$
$$= 8 + 2 + 1 + 0.25 + 0.125$$
$$= 11.375$$

Addition and subtraction algorithms the same.

F a u c Interesting

The ancient Egyptians used binary fractions:



The Eye of Horus

Binary-Coded Decimal



thinkgeek.com

Humans prefer
reading decimal
numbers;
computers prefer
binary.
BCD is a
BCD is a compromise: every
compromise: every
compromise: every four bits

Dec	BCD
0	0000 0000
1	0000 0001
2	0000 0010
i	:
8	0000 1000
9	0000 1001
10	0001 0000
11	0001 0001
:	:
18	0001 1000
19	0001 1001
20	0010 0000
:	:

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

000101011000 +001001000010

158

+242

1010 First group

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

000101011000 +001001000010

1010 First group + 0110 Correction

158 +242

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

•	101000
1	
158	
+242	
0	

1 000101011000 +001001000010

1010 First group + 0110 Correction 0100000 Second group

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

1
000101011000
+001001000010
1010
+ 0110
10100000
+ 0110

First group Correction Second group Correction

Binary addition followed by a possible correction.

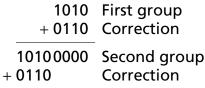
Any four-bit group greater than 9 must have 6 added to it.

Example:

11	
158	
+242	
00	

1	1
0001010	11000
+0010010	00010
	1010

01000000



Third group

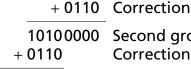
Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

1 1	
000101011000	
+001001000010	
1010	First group

01000000



Second group Correction

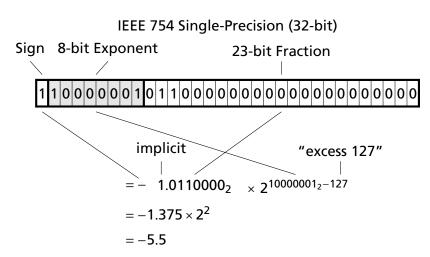
Third group

(No correction) 010000000000

Result

Floating-Point Numbers: "Scientific Notation"

Greater dynamic range at the expense of precision Excellent for real-world measurements



ASCII For Representing Characters and Strings

	Ü	1	2	3	4	5	6	/
0	NUL	DLE	SP	0	@	P	6	р
1	SOH	DC1	!	1	Α	Q	a	q
2	STX	DC2	"	2	В	R	b	${\tt r}$
3	ETX	DC3	#	3	C	S	С	s
4	EOT	DC4	\$	4	D	T	d	t
5	ENQ	NAK	%	5	E	U	e	u
6	ACK	SYN	&	6	F	V	f	v
7	BEL	ETB	,	7	G	W	g	W
8	BS	CAN	(8	Н	X	h	x
9	HT	EM)	9	I	Y	i	Y
Α	LF	SUB	*	:	J	Z	j	Z
В	VT	ESC	+	;	K	[k	{
C	FF	FS	,	<	L	\	1	
D	CR	GS	_	=	M]	m	}
Ε	SO	RS		>	N	٨	n	~
F	SI	US	/	?	0		0	DEL