

Fundamentals of Computer Systems

Caches

Stephen A. Edwards

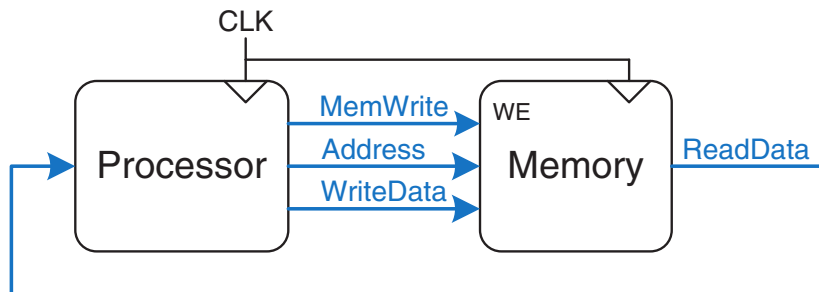
Columbia University

Summer 2020

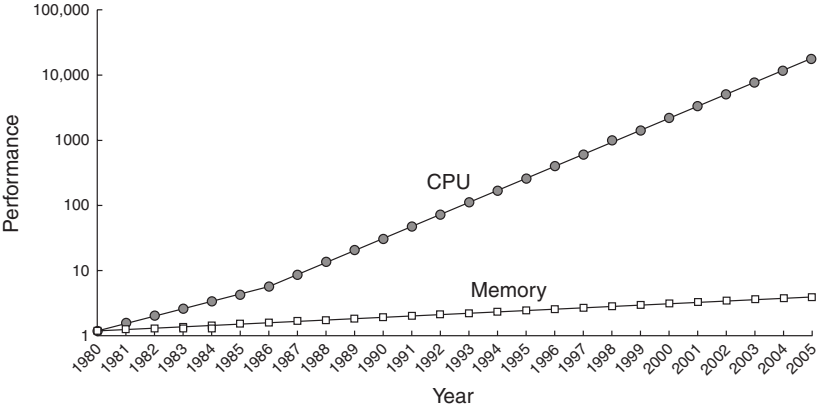
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Computer Systems

Performance depends on which is slowest:
the processor or the memory system



Memory Speeds Haven't Kept Up

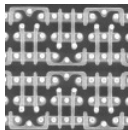


Our single-cycle memory assumption has been wrong since 1980.

Hennessy and Patterson. *Computer Architecture: A Quantitative Approach*. 3rd ed., Morgan Kaufmann, 2003.

Your Choice of Memories

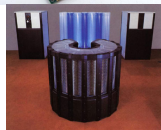
Fast Cheap Large



On-Chip SRAM



Commodity DRAM



Supercomputer



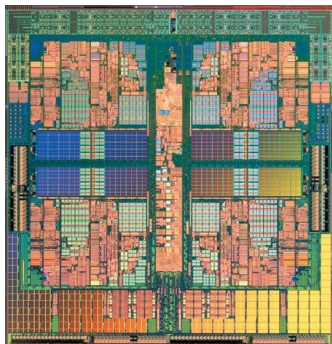
Memory Hierarchy

Fundamental trick to making a big memory appear fast

Technology	Cost (\$/Gb)	Access Time (ns)	Density (Gb/cm²)
SRAM	30 000	0.5	0.00025
DRAM	10	100	1 – 16
Flash	2	300*	8 – 32
Hard Disk	0.1	10 000 000	500 – 2000

* Read speed; writing much, much slower

A Modern Memory Hierarchy



AMD Phenom 9600

Quad-core

2.3 GHz

1.1–1.25 V

95 W

65 nm

A desktop machine:

Level	Size	Tech.
L1 Instruction*	64 K	SRAM
L1 Data*	64 K	SRAM
L2*	512 K	SRAM
L3	2 MB	SRAM
Memory	4 GB	DRAM
Disk	500 GB	Magnetic

*per core

Temporal Locality

FIRST BOOK



DEFINITIONS.

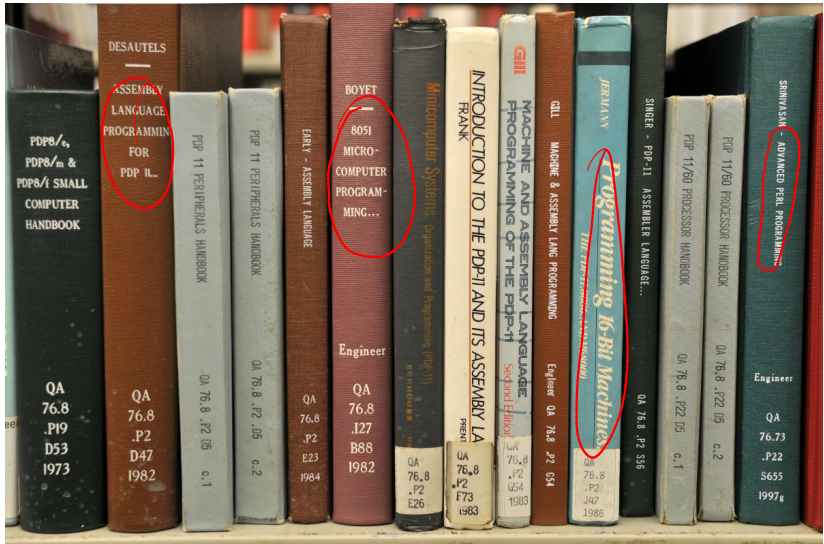
1. A point is that which is without parts.
2. A line is length without breadth.
3. The extremities of a line are points.
4. A right line, is that which lies evenly between its extremities.
5. A superficies, is that which has only length and breadth.
6. The boundings of a superficies are lines.
7. A plane superficies, is that which lies evenly between its extreme right lines.
8. A rectilinear angle, is the inclination of two right lines to each other, which touch, but do not form one straight line.
An angle is designated either by one letter at the vertex; or three, of which the middle one is at the vertex, the remaining two any place on the legs.
9. The legs of an angle, are the lines which make the angle.
10. The vertex of an angle is the point in which the legs mutually touch each other.



What path do your eyes take when you read this?

Did you look at the drawings more than once?

Spatial Locality

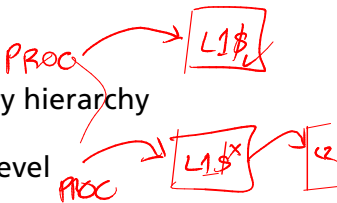


If you need something, you may also need something nearby

Memory Performance

Hit: Data is found in the level of memory hierarchy

Miss: Data not found; will look in next level



$$\text{Hit Rate} = \frac{\text{Number of hits}}{\text{Number of accesses}} \quad 0-1$$

$$\text{Miss Rate} = \frac{\text{Number of misses}}{\text{Number of accesses}} \quad 0-1$$



$$\text{Hit Rate} + \text{Miss Rate} = 1 \quad \text{Expected latency}$$

The expected access time E_L for a memory level L with latency t_L and miss rate M_L :

$$E_L = \underbrace{t_L}_{\text{Hits}} + \underbrace{M_L \cdot E_{L+1}}_{\text{Misses}} \quad \text{Expected latency}$$

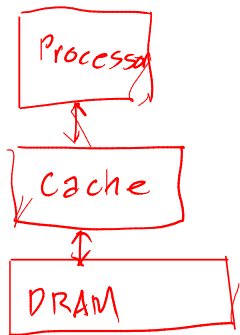
Memory Performance Example

Two-level hierarchy: Cache and main memory

Program executes 1000 loads & stores ← Accesses

750 of these are found in the cache

What's the cache hit and miss rate?



$$\frac{750 \text{ Hits}}{1000 \text{ Accesses}} = 75\% \text{ Hit Rate}$$

$$1 - \text{Hit} = 25\% \text{ Miss Rate}$$

Memory Performance Example

Two-level hierarchy: Cache and main memory

Program executes 1000 loads & stores

750 of these are found in the cache

What's the cache hit and miss rate?

$$\text{Hit Rate} = \frac{750}{1000} = 75\%$$
$$\text{Miss Rate} = 1 - 0.75 = 25\%$$

Aggregate

stall for 100

If the cache takes 1 cycle and the main memory 100,

What's the expected access time?

dominating

Average

$$E_0 = t_0 + (1 - t_0) E_1 = 1 + 25 \times 100 = 26 \text{ cycles}$$

cache access time 0.25 100 Main memory access time

Memory Performance Example

Two-level hierarchy: Cache and main memory

Program executes 1000 loads & stores

750 of these are found in the cache

What's the cache hit and miss rate?

$$\text{Hit Rate} = \frac{750}{1000} = 75\%$$

$$\text{Miss Rate} = 1 - 0.75 = 25\%$$

If the cache takes 1 cycle and the main memory 100,

What's the expected access time?

Expected access time of main memory: $E_1 = 100$ cycles

Access time for the cache: $t_0 = 1$ cycle

Cache miss rate: $M_0 = 0.25$

$$E_0 = t_0 + M_0 \cdot E_1 = 1 + 0.25 \cdot 100 = 26 \text{ cycles}$$

Cache

Highest levels of memory hierarchy

Fast: level 1 typically 1 cycle access time

With luck, supplies most data

Cache design questions:

What data does it hold? *Recently accessed*

How is data found? *Simple address hash*

What data is replaced? *Often the oldest*

Temporal locality



What Data is Held in the Cache?

Ideal cache: always correctly guesses what you want before you want it.

Real cache: never that smart

Caches Exploit

Temporal Locality

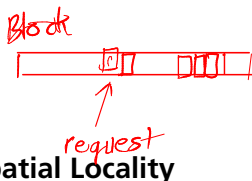
Copy newly accessed data into cache, replacing oldest if necessary



Spatial Locality

Copy nearby data into the cache at the same time

Specifically, always read and write a block at a time (e.g., 64 bytes), never a single byte.



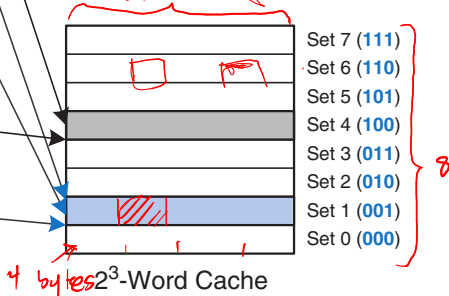
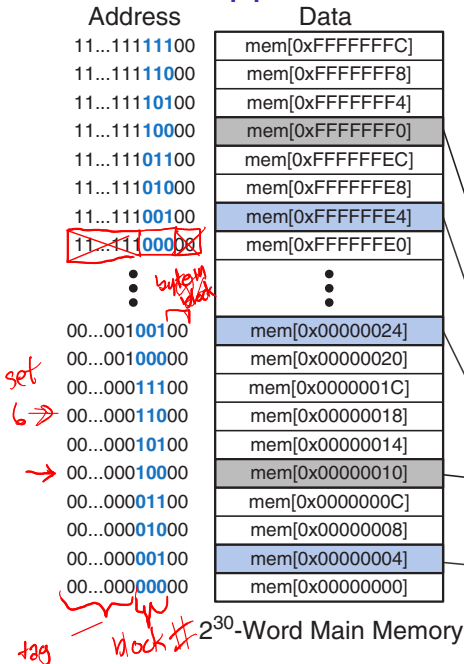
A Direct-Mapped Cache

This simple cache has

- ▶ 8 sets
 - ▶ 1 *block* per set
 - ▶ 4 bytes per block
- block = set (DM)*
64 bytes / block

To simplify answering "is this memory in the cache?," each byte is mapped to exactly one set.

block = 1 word = 4 bytes



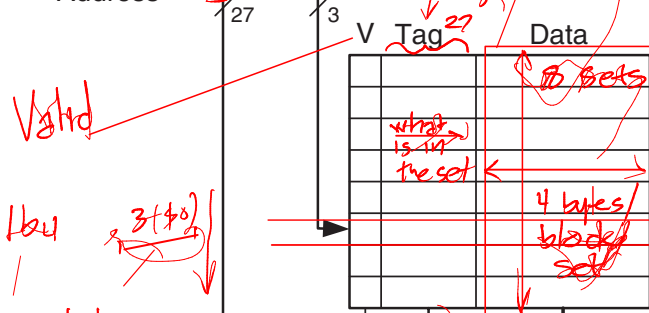
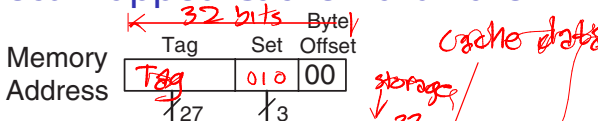
Direct-Mapped Cache Hardware

Address bits:

0-1: byte within block

2-4: set number

5-31: block "tag"



Valid

How

3 (40)

8 bit byte offset read = 3

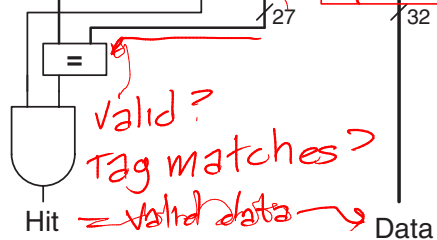
valid?

Tag matches?

= valid data

Cache hit if
in the set of the address,

- ▶ block is valid (V=1)
- ▶ tag (address bits 5-31) matches



Direct-Mapped Cache Behavior

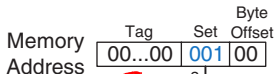
00100 4
01100 C
00000 8

A dumb loop:

repeat 5 times

load from 0x4;
load from 0xC;
load from 0x8.

```
li    $t0, 5
l1:  beq    $t0, $0, done
     lw     $t1, 0x4($0)
     lw     $t2, 0xC($0)
     lw     $t3, 0x8($0)
     addiu  $t0, $t0, -1
     j     l1
done:
```



address 4

V	Tag	Data	Set
0			Set 7 (111)
0			Set 6 (110)
0			Set 5 (101)
0			Set 4 (100)
1	00...00	mem[0x00...0C]	Set 3 (011)
1	00...00	mem[0x00...08]	Set 2 (010)
1	00...00	mem[0x00...04]	Set 1 (001)
0			Set 0 (000)

Cache when reading 0x4 last time

Assuming the cache starts empty, what's the miss rate?

4 C 8 4 C 8
M M M H H H

$\frac{3}{15}$

1b

0x9(\$0)

set 2
byte offset 1

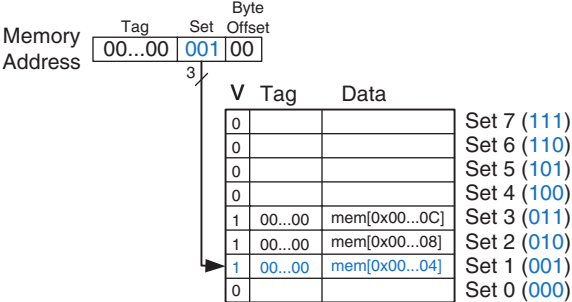
20%

Direct-Mapped Cache Behavior

A dumb loop:
 repeat 5 times
 load from 0x4;
 load from 0xC;
 load from 0x8.

```

li    $t0, 5
l1:  beq    $t0, $0, done
     lw     $t1, 0x4($0)
     lw     $t2, 0xC($0)
     lw     $t3, 0x8($0)
     addiu $t0, $t0, -1
     j     l1
done:
  
```



Cache when reading 0x4 last time

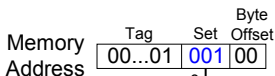
Assuming the cache starts empty, what's the miss rate?

4 C 8 4 C 8 4 C 8 4 C 8 4 C 8

M M M H H H H H H H H H H H H

3/15 = 0.2 = 20%

Direct-Mapped Cache: Conflict



A dumber loop:

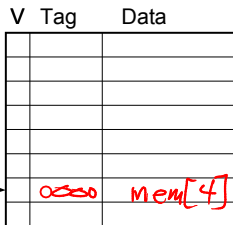
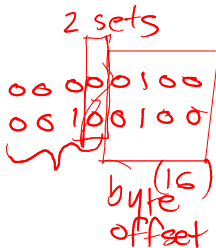
repeat 5 times

load from 0x4;

load from 0x24

```

li    $t0, 5
l1:  beq    $t0, $0, done
      lw     $t1, 0x4($0)
      lw     $t2, 0x24($0)
      addiu $t0, $t0, -1
      j     l1
done:
    
```



- Set 7 (111)
- Set 6 (110)
- Set 5 (101)
- Set 4 (100)
- Set 3 (011)
- Set 2 (010)
- Set 1 (001)
- Set 0 (000)

Cache State

Assuming the cache starts empty, what's the miss rate?

4 → 24 → 4 → 24 100%

M M M M

16-byte block?
How many sets?

These are conflict misses

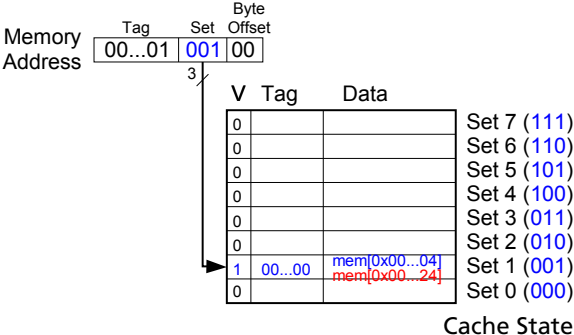
— conflicting locations

Direct-Mapped Cache: Conflict

A dumber loop:
 repeat 5 times
 load from 0x4;
 load from 0x24

```

    li    $t0, 5
l1: beq  $t0, $0, done
    lw   $t1, 0x4($0)
    lw   $t2, 0x24($0)
    addiu $t0, $t0, -1
    j    l1
done:
    
```



Assuming the cache starts empty, what's the miss rate?

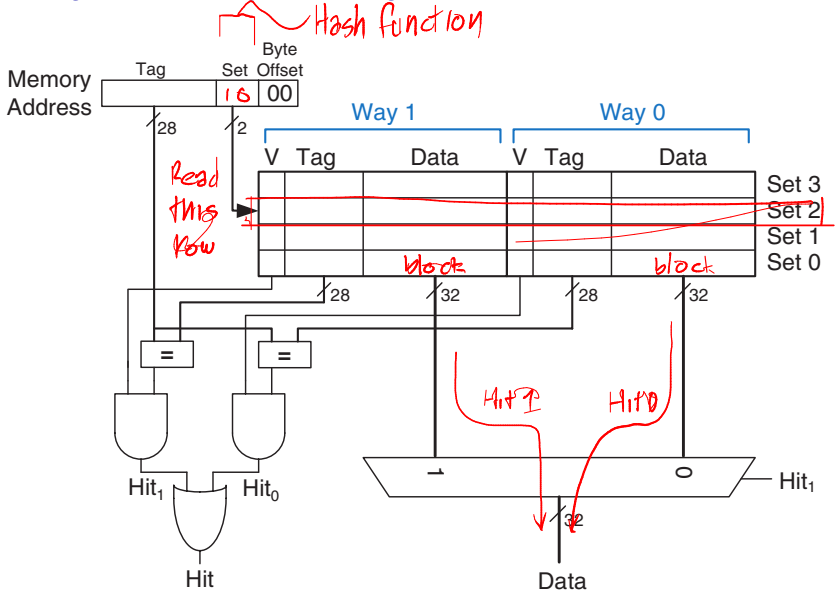
4 24 4 24 4 24 4 24 4 24

M M M M M M M M M M

10/10 = 1 = 100% Oops

These are *conflict misses*

No Way! Yes Way! 2-Way Set Associative Cache



2-Way Set Associative Behavior

```

li    $t0, 5
l1:  beq    $t0, $0, done
      lw     $t1, 0x4($0)
      lw     $t2, 0x24($0)
      addiu  $t0, $t0, -1
      j     l1
done:

```

Assuming the cache starts empty, what's the miss rate?

4 24 4 24 4 24 4 24 4 24
 M M H H H H H H H H

$2/10 = 0.2 = 20\%$

Associativity reduces conflict misses

Compulsory

0x44 }
 set 1

Way 1			Way 0		
V	Tag	Data	V	Tag	Data
0			0		
0			0		
1	00...00	mem[0x00...04]	1	00...10	mem[0x00...24]
0			0		

byte offset # bytes here

Address 4 maps here,

Size of byte offset = \log_2 block size

block

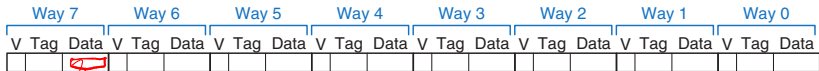
24

24, too size

An Eight-way Fully Associative Cache

Impractical

1 set only
all blocks in
the same set



block

No conflict misses: only compulsory or capacity misses

Haven't tested it yet

cache overflowed

Either very expensive or slow because of all the associativity

32 address bits

8 ways

8 byte blocks



tag

set offset

$$2^3 = 8$$

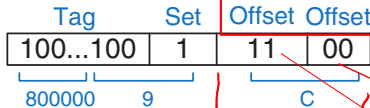
byte in
the block

Exploiting Spatial Locality: Larger Blocks

Homework

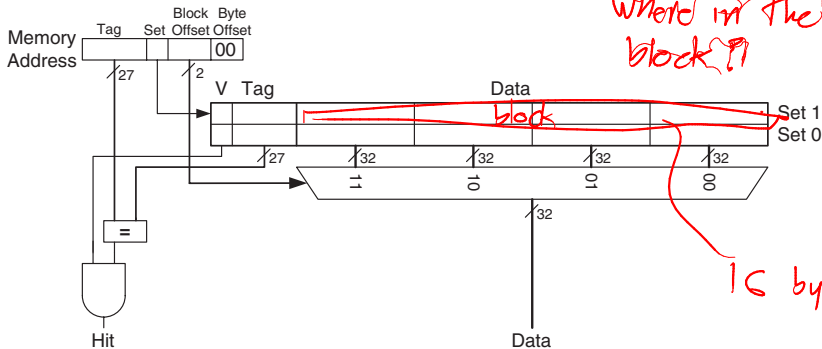
0x8000 0009C:

Memory Address



Block Byte Offset Offset

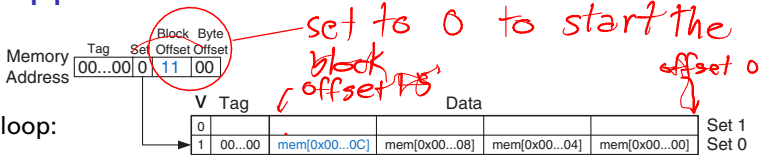
by key word which word
Where in the block?!



16 bytes

- 2 sets
- 1 block per set (Direct Mapped)
- 4 words per block

Direct-Mapped Cache Behavior w/ 4-word block



The dumb loop:

repeat 5 times

load from 0x4;
load from 0xC;
load from 0x8.

set 0, same tag

Read whole block into set 0

```

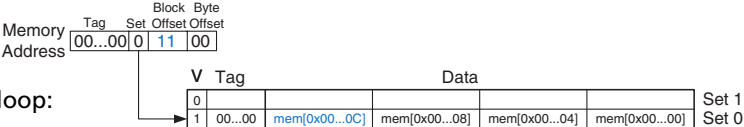
li    $t0, 5
l1:  beq    $t0, $0, done
      lw     $t1, 0x4($0)
      lw     $t2, 0xC($0)
      lw     $t3, 0x8($0)
      addiu $t0, $t0, -1
      j     l1
done:
    
```

Assuming the cache starts empty, what's the miss rate?

4 C 8 4 ...
M H H H

$$\frac{1}{15} = 6.7\% \text{ miss rate}$$

Direct-Mapped Cache Behavior w/ 4-word block



The dumb loop:

repeat 5 times

- load from 0x4;
- load from 0xC;
- load from 0x8.

Cache when reading 0xC

```

li    $t0, 5
l1:  beq    $t0, $0, done
      lw     $t1, 0x4($0)
      lw     $t2, 0xC($0)
      lw     $t3, 0x8($0)
      addiu $t0, $t0, -1
      j     l1
done:
    
```

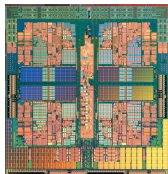
Assuming the cache starts empty, what's the miss rate?

4	C	8	4	C	8	4	C	8	4	C	8	4	C	8
M	H	H	H	H	H	H	H	H	H	H	H	H	H	H

1/15 = 0.0666 = 6.7%

Larger blocks reduce compulsory misses by exploiting spatial locality

The Desktop Machine Revisited



AMD Phenom
9600
Quad-core
2.3 GHz
1.1–1.25 V
95 W
65 nm

On-chip caches:

Cache	Size	Sets	Ways	Block
L1I*	64 K	512	2-way	64-byte
L1D*	64 K	512	2-way	64-byte
L2*	512 K	512	16-way	64-byte
L3	2 MB	1024	32-way	64-byte

* per core










Unified I & D

Increases
Increases

minimize the
conflict misses

same

Intel On-Chip Caches

Chip	Year	Freq. (MHz)	L1		L2	
			Data	Instr		
	80386	1985	16–25	off-chip		none
	80486	1989	25–100	8K unified		off-chip
	Pentium	1993	60–300	8K	8K	off-chip
	Pentium Pro	1995	150–200	8K	8K	256K–1M (MCM)
	Pentium II	1997	233–450	16K	16K	256K–512K (Cartridge)
	Pentium III	1999	450–1400	16K	16K	256K–512K
	Pentium 4	2001	1400–3730	8–16K	12k op trace cache	256K–2M
	Pentium M	2003	900–2130	32K	32K	1M–2M
	Core 2 Duo	2005	1500–3000	32K per core	32K per core	2M–6M

2nd die
multiple dice