Basic Idea

For my project I would like to implement an N-body (gravitational only) modelling program with the intention to either apply it to predicting planetary orbits within the solar system ($N = 8$ or 9), or the orbits of the Jovian moons ($N \approx 79$). In either case my plan is to implement a post-Newtonian force relation to include at least part of General Relativistic back-reaction terms. Due to the nature of such simulations, usually requiring forward integration, the main parallelisation opportunity will be in each single time step for the N objects. That is, in order to evolve each object forward the current forces acting upon it must be updated for each time step, which may be done entirely in parallel *per time step*[1].

Implementation Ideas

In practice the implementation would most likely be a Runge-Kutta model, perhaps RK4 or Rk5, implementing the solution to the following modified force law:

$$\mathbf{F} = m\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = -\frac{GMm\mathbf{r}}{|\mathbf{r}|^3} + \frac{GMm}{c^2|\mathbf{r}|^3}\left(4GM\hat{\mathbf{r}} - \mathbf{v}^2\mathbf{r} + 4(\mathbf{r}\cdot\mathbf{v})\mathbf{v}\right) \tag{1}$$

Where M is the mass of the other body, c is the propagation speed of gravity (the speed of light in vacuum), $\hat{\mathbf{r}}$ is $\frac{\mathbf{r}}{|\mathbf{r}|}$, and $\mathbf{r}$ and $\mathbf{v}$ are the position and velocity of the object respectively. In the literature on this matter often times further accuracy is had by including the J_2 multipole term. However any reasonable calculation of that involves some approximations about the general rotational "shape" of the solar system which hold true for all the planets but *not* for Pluto or the Jovian moons, which would be cool to include in our calculations.

A big issue for a solar system model will be stiffness arising from the differing orbital periods of the planets, on the order of two magnitudes between Mercury and Neptune, and any tidal forces as I am not including those at all. Tidal forces would factor most severely for Earth-Moon orbits, but less so for Jovian orbits usually[2]. The stiffness issue can be handled somewhat by including an adaptive time step, which could help parallelise a tiny bit too where the shorter time step objects can be stepped many times per each step of the longer time step objects, however N is still fairly small in that case, giving a low number of total calculations required per step. To my knowledge the adaptive time step method is the main optimisation used in the literature across "small" time scales for the stiffness issue, for larger scales symplectic integration methods are used.

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[1] In reality for small enough time steps (relative to the light-spanning time of the simulated volume) previous time steps *could* be ignored due to the finite propagation speed of gravity, however not only would accurately representing that require a field theory consistent computational model, which is very hard (none worth mentioning exist in the world), but in our case the distances in question are at *most* about five light-hours (distance to Pluto). So not too useful here.

[2] While the moons *are* hit by a lot of tidal forces, their mass difference with Jupiter makes their orbits more "stable" with regards to tidal perturbations