COMS 4231: Analysis of Algorithms I, Fall 2012

Problem Set 1, due Thursday September 20, in class

Please follow the homework submission guidelines posted on the web

Problem 1. For the following pairs of functions f, g write all equations that hold of the form \( f = \Theta(g) \), where \( \Theta \in \{ o, O, \Theta, \Omega, \omega \} \). Justify your answer in part (c). You do not need to justify it in the other parts.

(a) \( f(n) = 5n^3 + 8n + 2 \), \( g(n) = n^3 - 3n^2 + 5n + 4 \)
(b) \( f(n) = 2^n \), \( g(n) = 2^{n+5} \)
(c) \( f(n) = 2^{3n} \), \( g(n) = 3^{2n} \)

Problem 2. Let \( f, g \) be two nonnegative functions on the positive integers.
(a) Show that if \( f(n) = \Theta(g(n)) \) then \( (f(n))^3 = O((g(n))^3) \).
(b) Give an example to show that \( f(n) = \Theta(g(n)) \) does not imply \( 2^{f(n)} = O(2^{g(n)}) \).

Problem 3. Order the following functions by order of growth, that is, if a function \( f \) is listed before a function \( g \) then \( f(n) = O(g(n)) \). Group together functions that have the same order of growth, i.e. \( f(n) = \Theta(g(n)) \). All logarithms are to the base 2, unless explicitly specified otherwise. Justify only the functions that you have grouped together (if any). You do not need to justify the rest of your ordering.

\[ n, \sqrt{n}, (2n+1)^2, \log n, \log \log n, (\log n)^2, n(\log n)^2, n!, 2^{\log n}, \]
\[ 8\sqrt{n}, \left( \frac{3}{2} \right)^n, 35, n^{\log n}, \log_{10} n, n^n \]

Problem 4.
1. We are given two sorted arrays \( A \) and \( B \) with \( n \) integers each, and wish to determine if there is an element \( a \) of \( A \) and an element \( b \) of \( B \) such that \( a = 2b \).
Give a linear-time algorithm for this problem.

2. We are given an unsorted array \( C \) of \( n \) integers and wish to determine if there are two elements \( a, b \) of \( C \) such that \( a = 2b \).
Give a \( O(n \log n) \)-time algorithm for this problem.

Problem 5. Problem 2.3 in CLRS, page 41 (Horner’s rule).