CSEE 3827: Fundamentals of Computer Systems

Lecture 2

January 26, 2009

Martha Kim
mak2191@columbia.edu

Agenda

- TA office hours
- Boolean algebra
- Logic gates
- Circuit fabrication

TA Office Hours

TA Room, first floor of Mudd (see: http://ta.cs.columbia.edu/tamap.shtml)

Roopa Kakarlapudi	Tuesdays 5-6:30PM
Harsh Parekh	Mondays 11-12:20PM; Tuesdays 3:30-5PM
Nishant Shah	Wednesdays 10-11:30AM

Boolean Logic

- Binary digits (or bits) have two values: {1,0}
- All logical functions can be implemented in terms of three logical operations:

NOT

x	$\overline{x}$
0	1
1	0

AND

x	y	x·y
0	0	0
0	1	0
1	0	0
1	1	1

OR

x	y	x + y
0	0	0
0	1	1
1	0	1
1	1	1

Boolean Logic 2

- Precedence rules just like decimal system
- Implied precedence: NOT > AND > OR
- Use parentheses as necessary

$$AB + C = (AB) + C$$

$$(\overline{A} + B)C = ((\overline{A}) + B)C$$

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Boolean Logic: Example

D	X	A	$L = \overline{DX} + A$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Boolean Logic: Example

D	X	A	$\overline{X}$	$D\overline{X}$	$L=D\overline{X} + A$
0	0	0	1	0	0
0	0	1	1	0	1
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	0	0	1

(M&K Table 2-2)

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Boolean Logic: Example 2

X	Y	$XY + \overline{X}\,\overline{Y}$
0	0	
0	1	
1	0	
1	1	

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Boolean Logic: Example 2

X	Y	$\overline{X}$	$\overline{Y}$	XY	$\overline{X}\,\overline{Y}$	$XY + \overline{X}\,\overline{Y}$
0	0	1	1	0	1	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	1	0	0	1	0	1

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Boolean Algebra: Identities and Theorems

OR	AND	NOT	
X+0 = X	X1 = X		(identity)
X+1 = 1	X0 = 0		(null)
X+X = X	XX = X		(idempotent)
$X+\overline{X} = 1$	$X\overline{X} = 0$		(complementarity)
		$\overline{\overline{X}} = X$	(involution)
X+Y = Y+X	XY = YX		(commutativity)
X+(Y+Z) = (X+Y)+Z	X(YZ) = (XY)Z		(associativity)
X(Y+Z) = XY + XZ	X+YZ = (X+Y)(X+Z)		(distributive)
$\overline{X+Y} = \overline{X}\ \overline{Y}$	$\overline{XY} = \overline{X} + \overline{Y}$		(DeMorgan's theorem)

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Boolean Algebra: Example

Simplify this equation using algebraic manipulation.

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

Boolean Algebra: Example

Simplify this equation using algebraic manipulation.

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

$\overline{X}Y(Z + \overline{Z}) + XZ$ (by reverse distribution)

$\overline{X}Y1 + XZ$ (by complementarity)

$\overline{X}Y + XZ$ (by identity)

Boolean Algebra: Example 2

Find the complement of F.

$$F = A\overline{B} + \overline{A}B$$

$$\overline{F} =$$

Boolean Algebra: Example 2

Find the complement of F.

$$F = A\overline{B} + \overline{A}B$$

$$\overline{F} = \overline{A\overline{B} + \overline{A}B}$$

$$\overline{(A\overline{B})}\,\overline{(\overline{A}B)} \qquad \text{(by DeMorgan's)}$$

$$(\overline{A} + \overline{\overline{B}})(\overline{\overline{A}} + \overline{B}) \qquad \text{(by DeMorgan's)}$$

$$(\overline{A} + B)(A + \overline{B}) \qquad \text{(by involution)}$$

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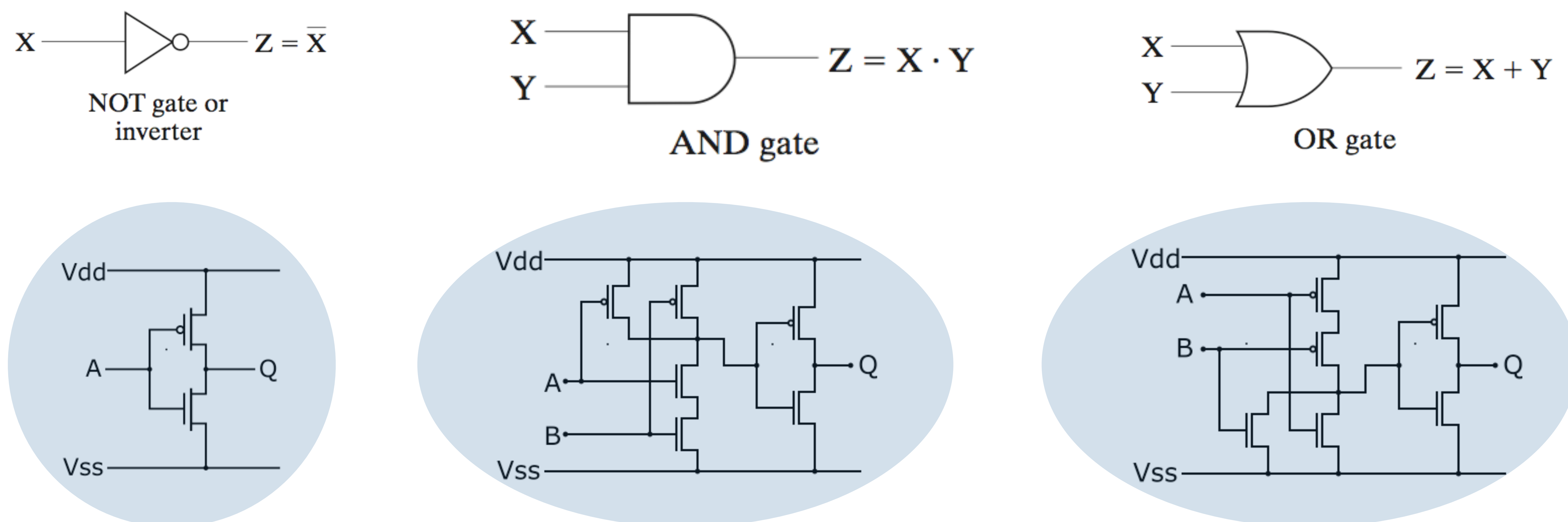
Boolean Algebra: Why?



These circuits consume area, power, and time

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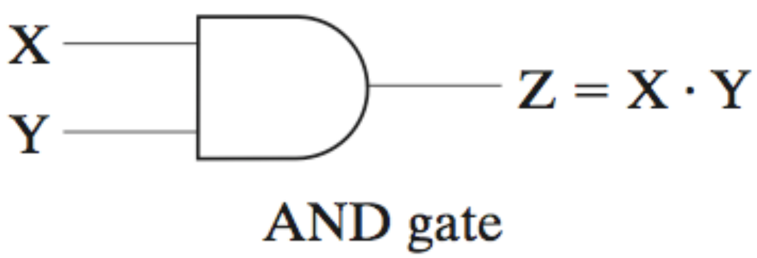
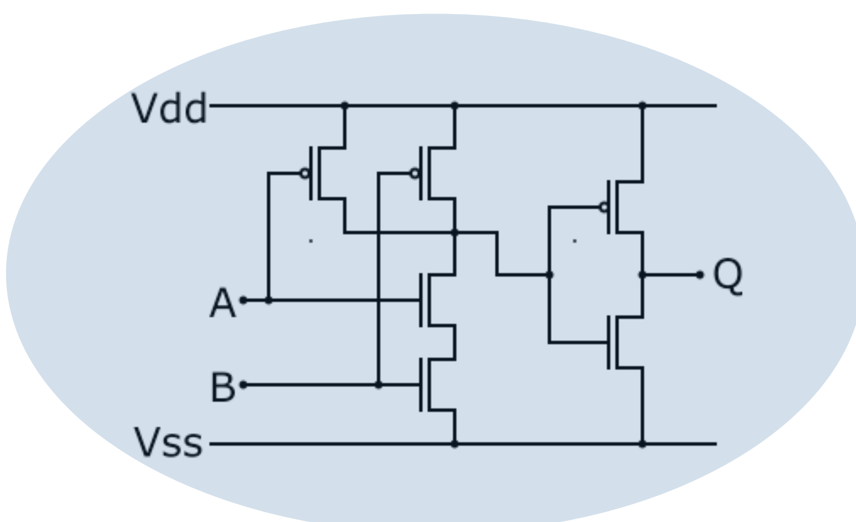
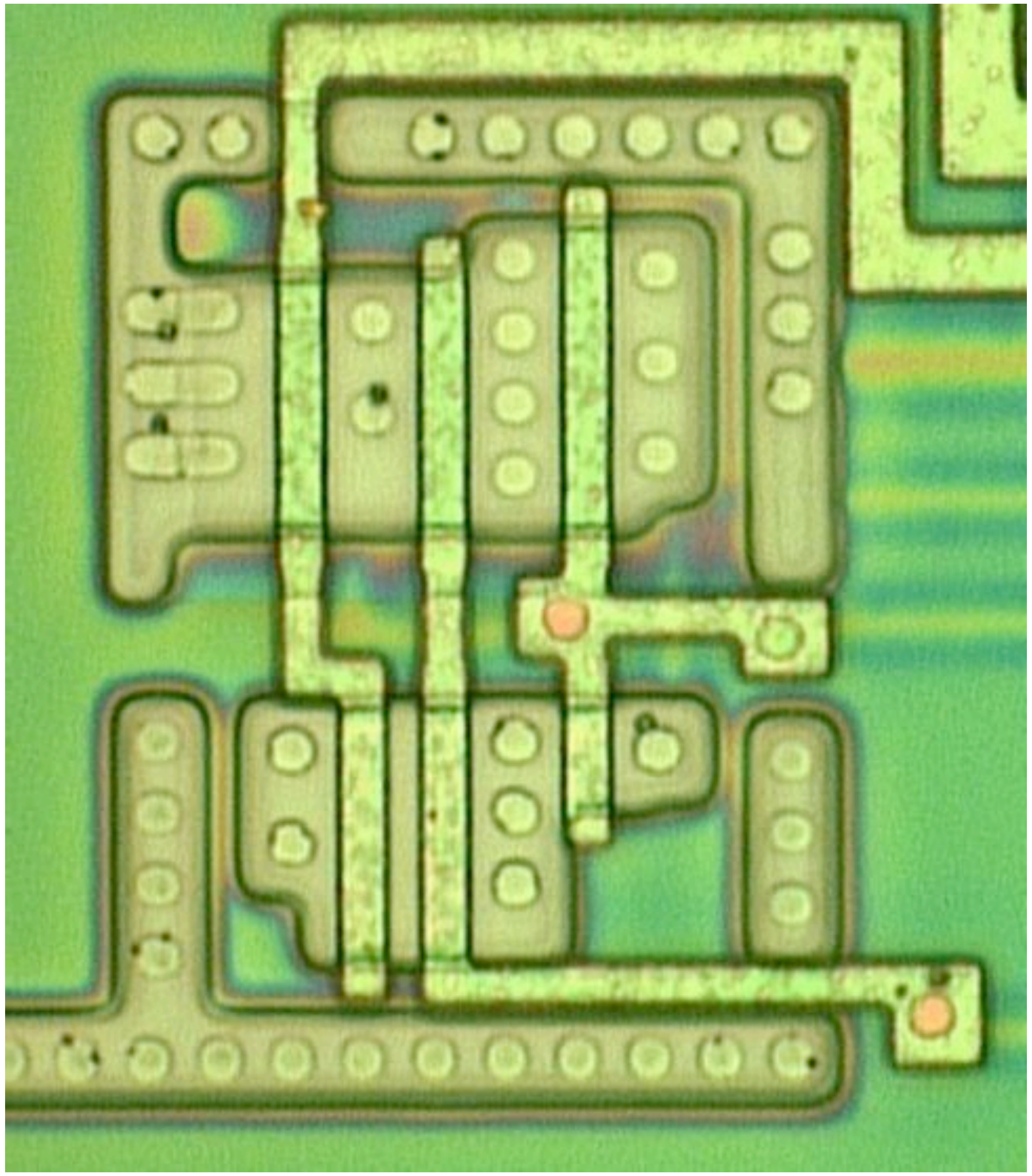
Logic gate area







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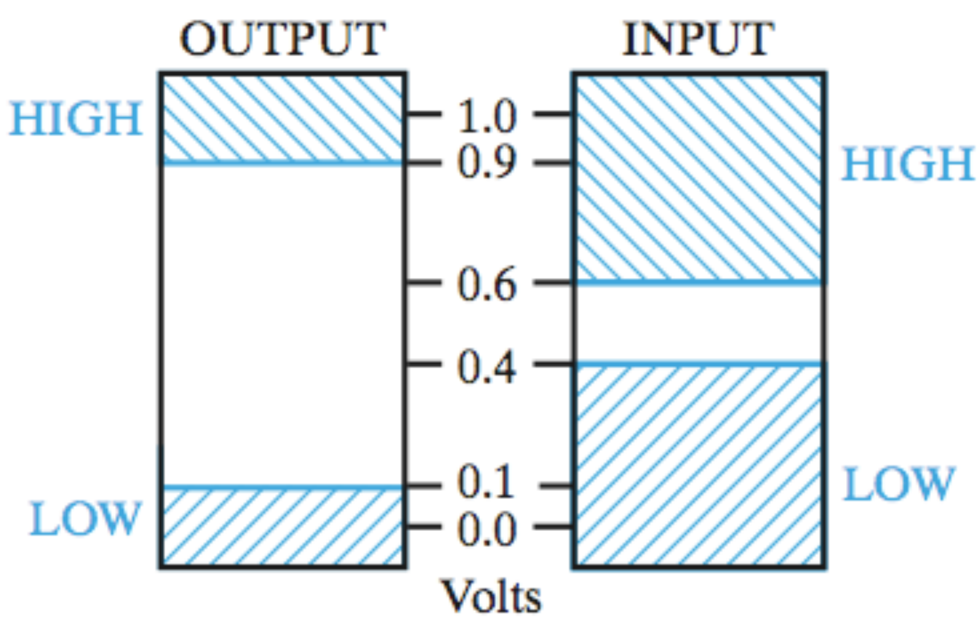
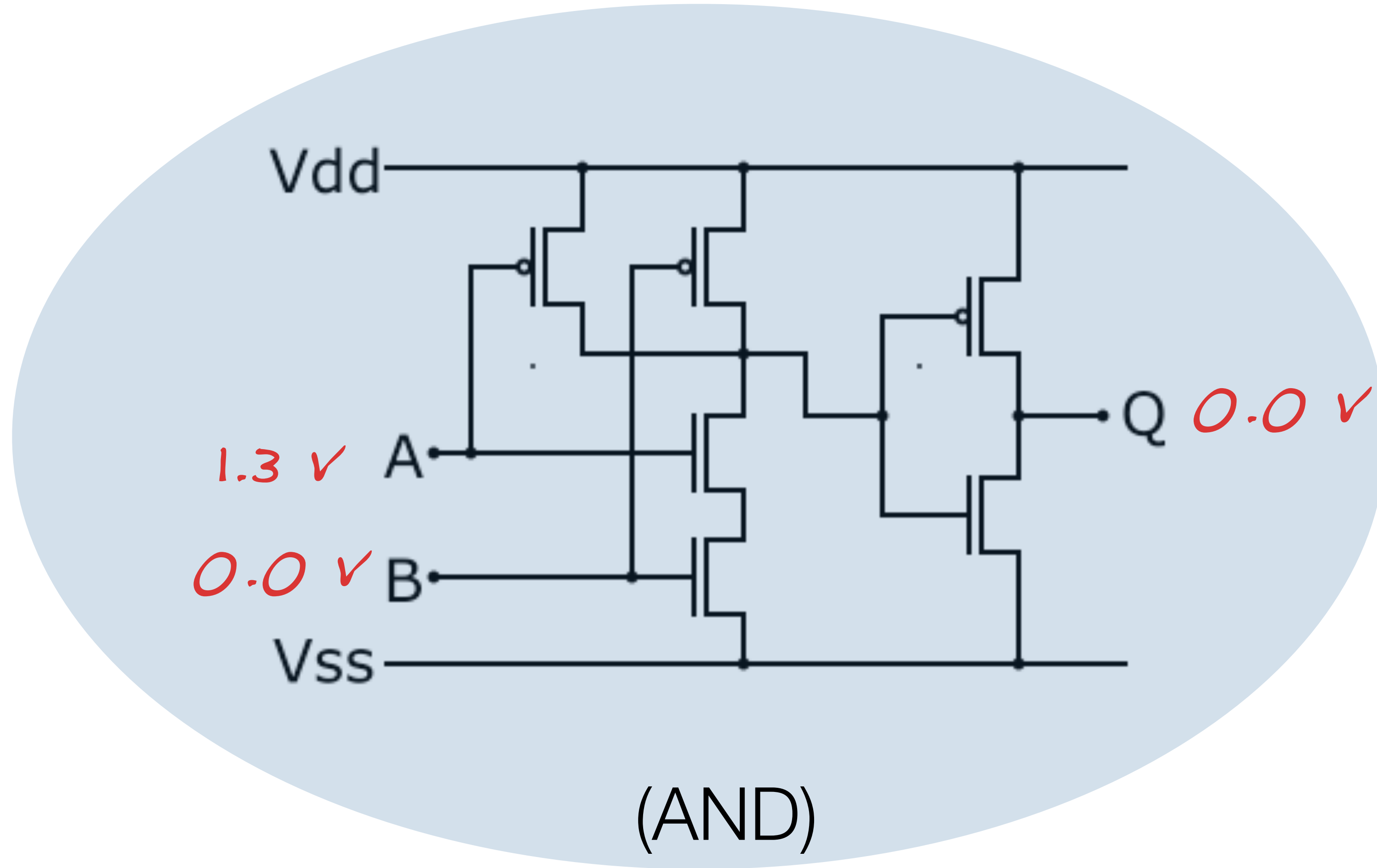
Information signaled through voltage level





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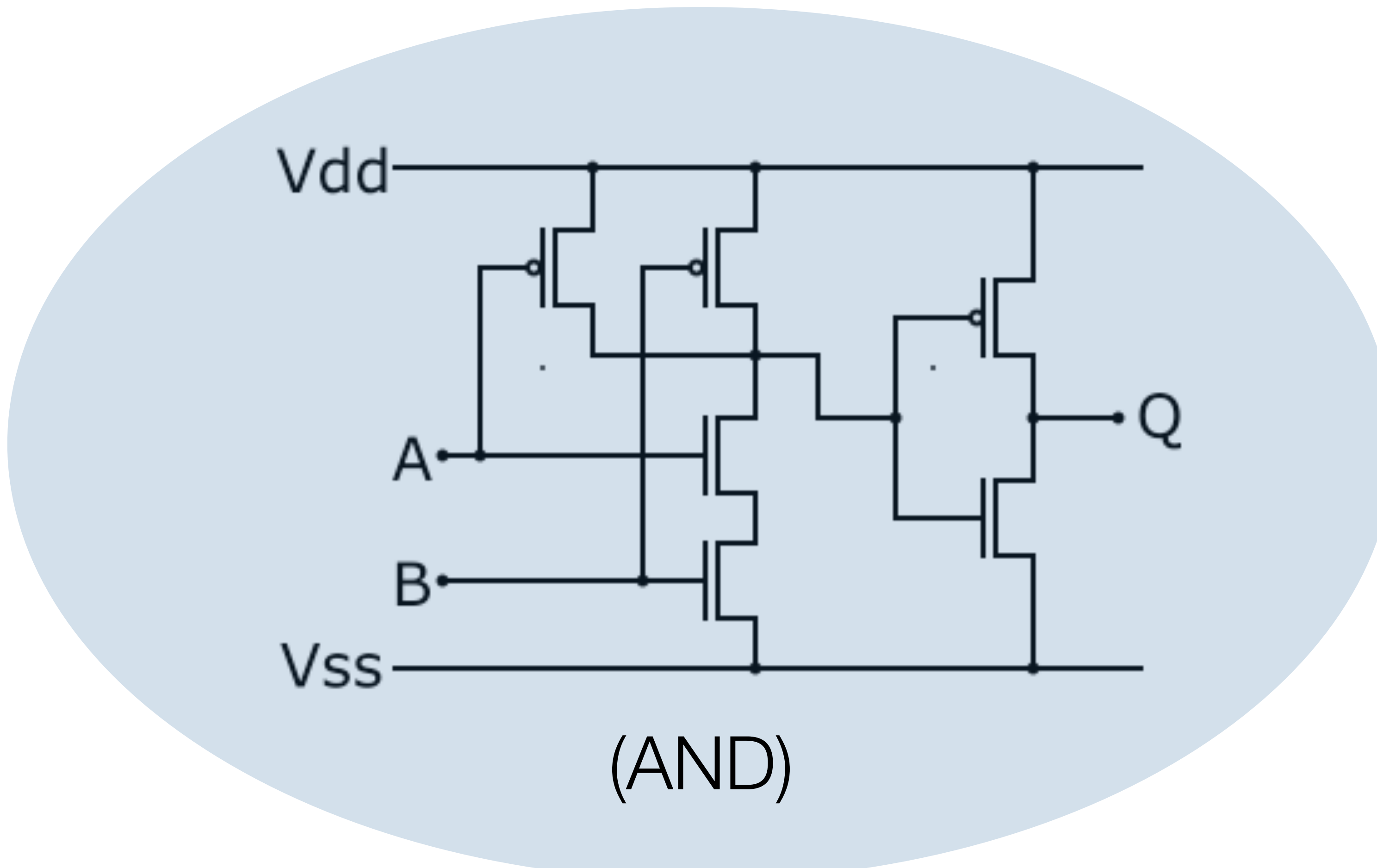
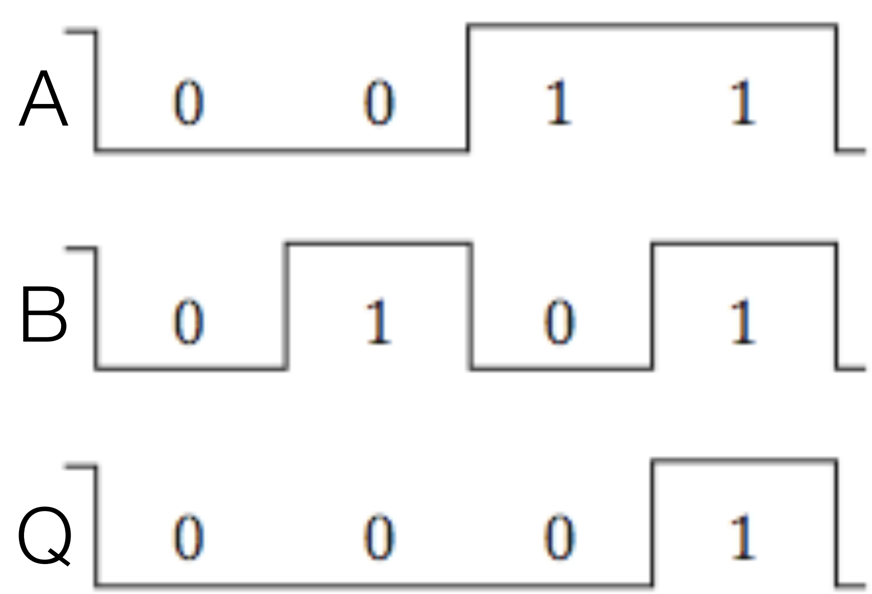
Idealized timing diagram of AND gate





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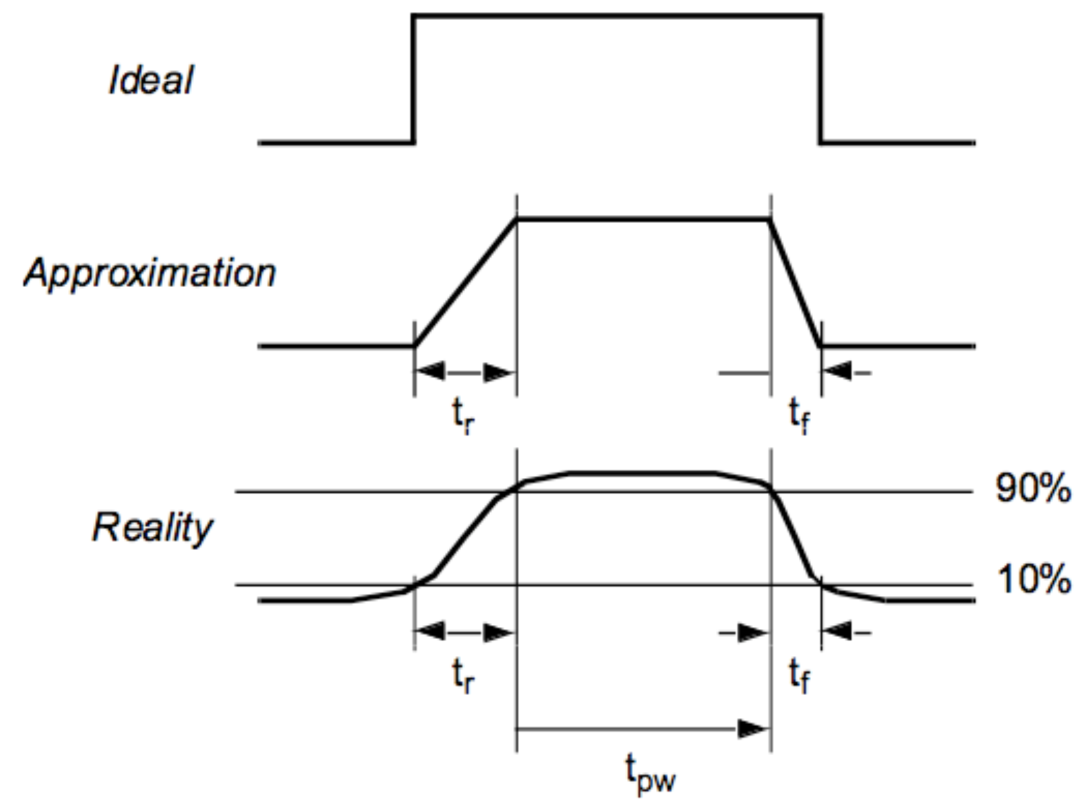
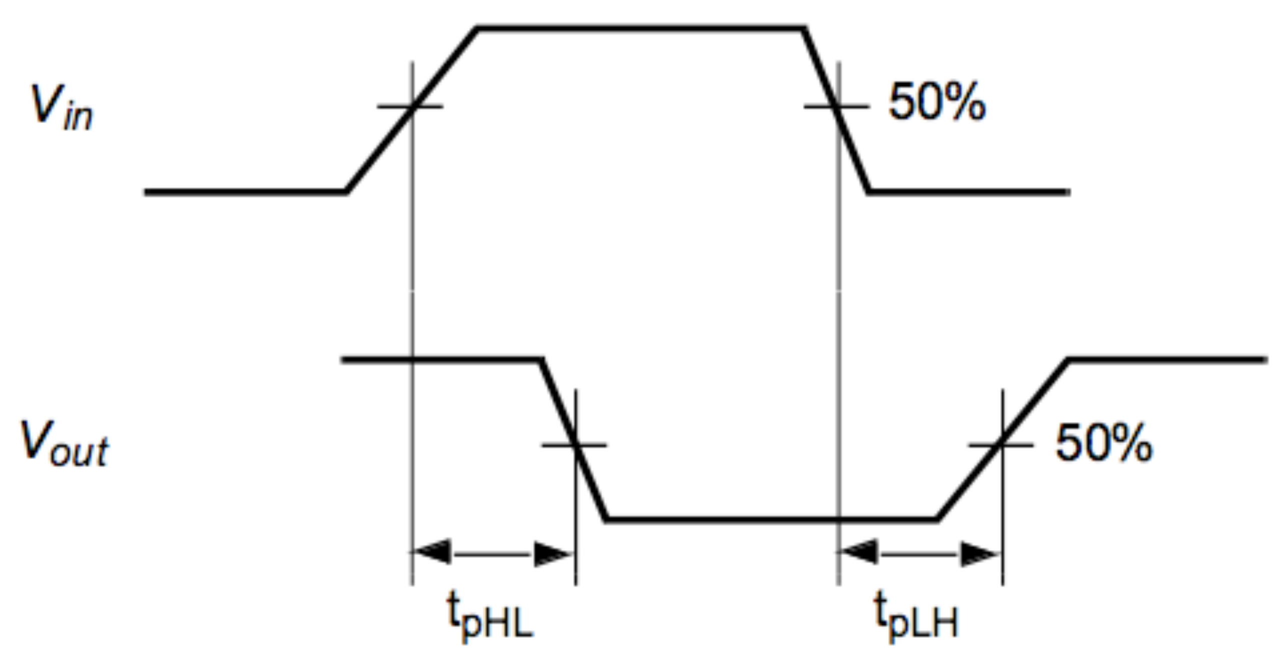
Actual signal timing has delays

- **transition time**: *time required for output to change (RC delay: ohms x farads = time*



- **propagation time**: *time from input change to output change*



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Returning to boolean algebra...

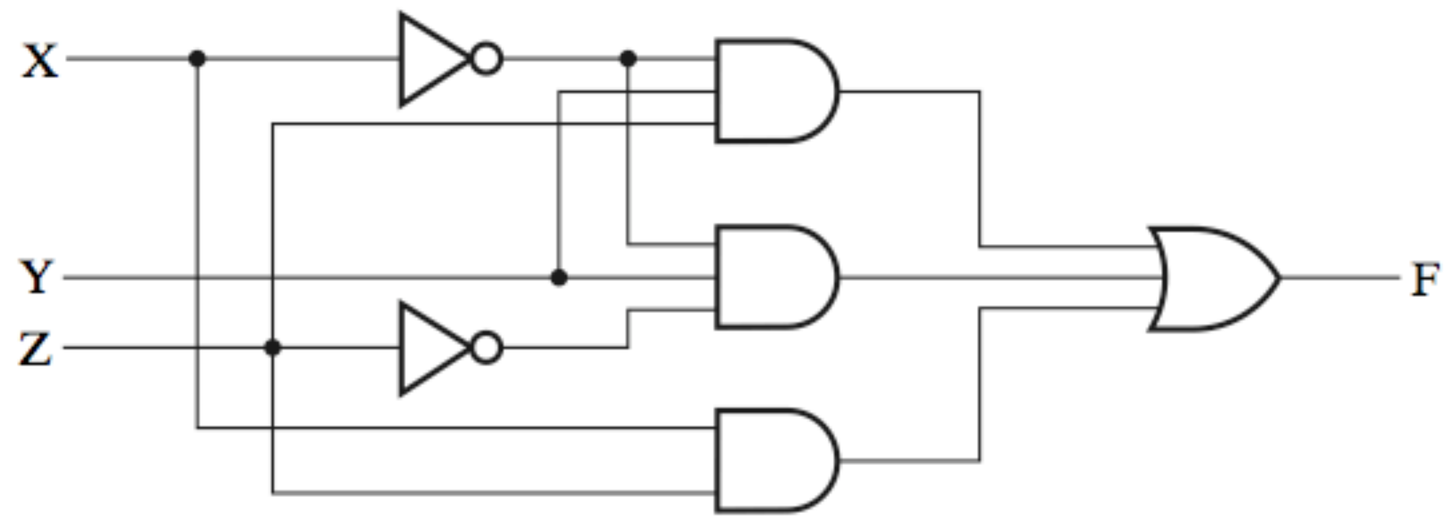
$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$

$\overline{X}Y(Z + \overline{Z}) + XZ$ (by reverse distribution)

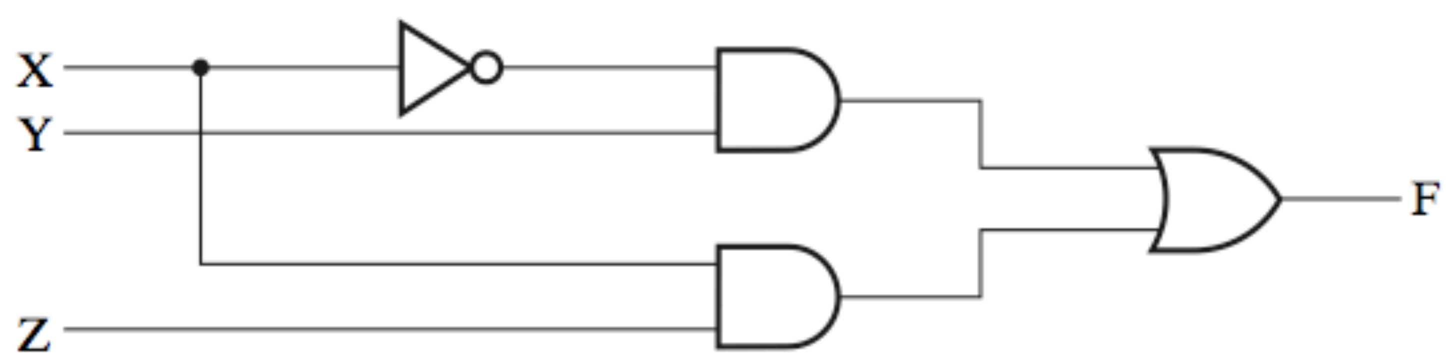
$\overline{X}Y1 + XZ$ (by complementarity)

$\overline{X}Y + XZ$ (by identity)

Returning to boolean algebra...



(a) $F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$



(b) $F = \overline{X}Y + XZ$

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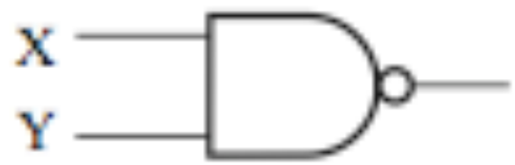
Universal gates: NAND, NOR

x	y	$z = \overline{xy}$
0	0	1
0	1	1
1	0	1
1	1	0



$\overline{XY}$

x	y	$z = \overline{x+y}$
0	0	1
0	1	0
1	0	0
1	1	0

$\overline{X+Y}$

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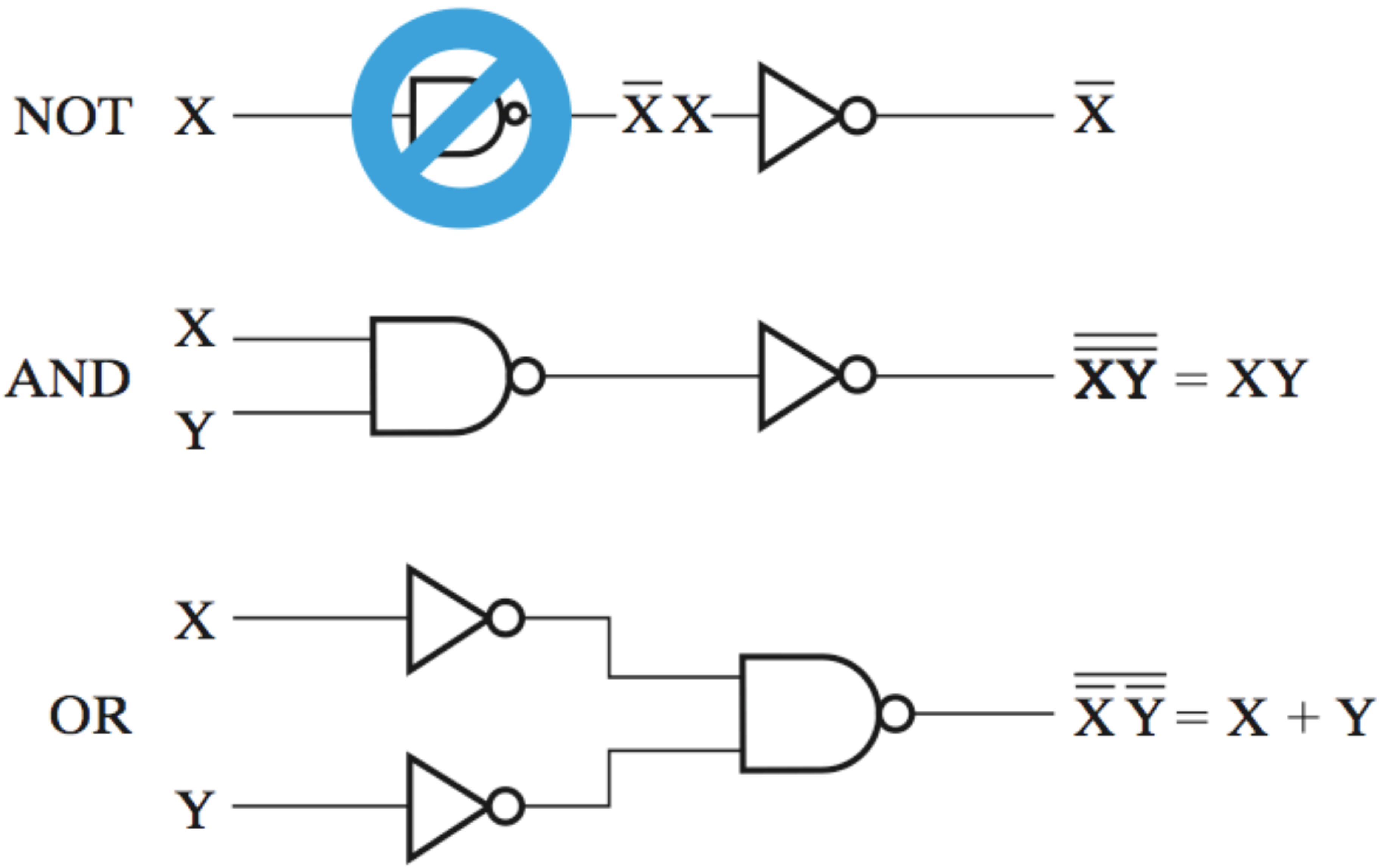
Universal how?



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Boolean algebra practice 1

Prove that this boolean equation is true using algebraic manipulation.

$$1 = \overline{A}B + \overline{B}\,\overline{C} + AB + \overline{B}C$$

$B(\overline{A} + A) + \overline{B}(\overline{C} + C)$ (by distribution)

$B + \overline{B}$ (by complementarity)

1 (by complementarity)

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Boolean algebra practice 2

Prove that this boolean equation is true using algebraic manipulation.

$$\overline{X} + Y = \overline{X}\,\overline{Y} + \overline{X}Y + XY$$

$\overline{X}\,\overline{Y} + \overline{X}Y + \overline{X}Y + XY$ (by idempotence)

$\overline{X}(\overline{Y} + Y) + Y(\overline{X} + X)$ (by distribution)

$\overline{X}\,1 + Y\,1$ (by null)

$\overline{X} + Y$ (by identity)

Boolean algebra practice 3

Find the complement of F.

$$F = (\overline{V}W + X)Y + \overline{Z}$$

$$\overline{F} = \overline{(\overline{V}W + X)Y + \overline{Z}}$$

$$\overline{((\overline{V}W + X)Y)}\,\overline{\overline{Z}} \qquad \text{(by DeMorgan's)}$$

$$(\overline{(\overline{V}W + X)} + \overline{Y})Z \qquad \text{(by DeMorgan's \& involution)}$$

$$(\overline{\overline{V}W}\,\overline{X} + \overline{Y})Z \qquad \text{(by DeMorgan's)}$$

$$((\overline{\overline{V}} + \overline{W})\overline{X} + \overline{Y})Z \qquad \text{(by DeMorgan's)}$$

$$((V + \overline{W})\overline{X} + \overline{Y})Z \qquad \text{(by null)}$$

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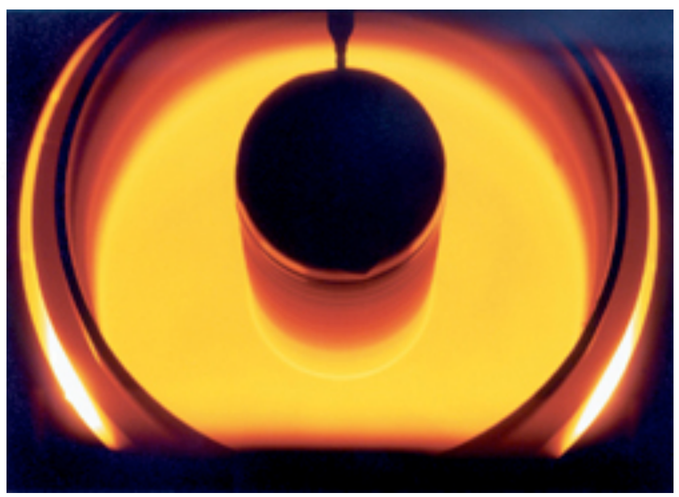
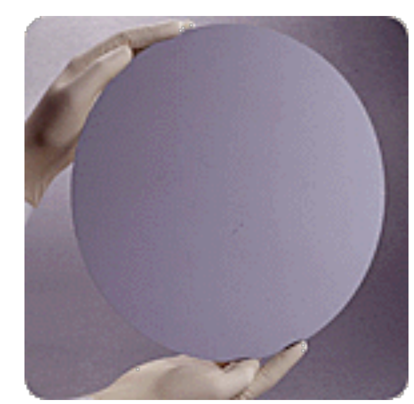
Integrated circuit fabrication

raw silicon



crystallization
of molten silicon

silicon ingots



wafer

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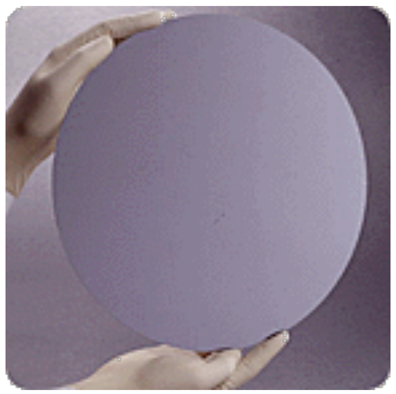
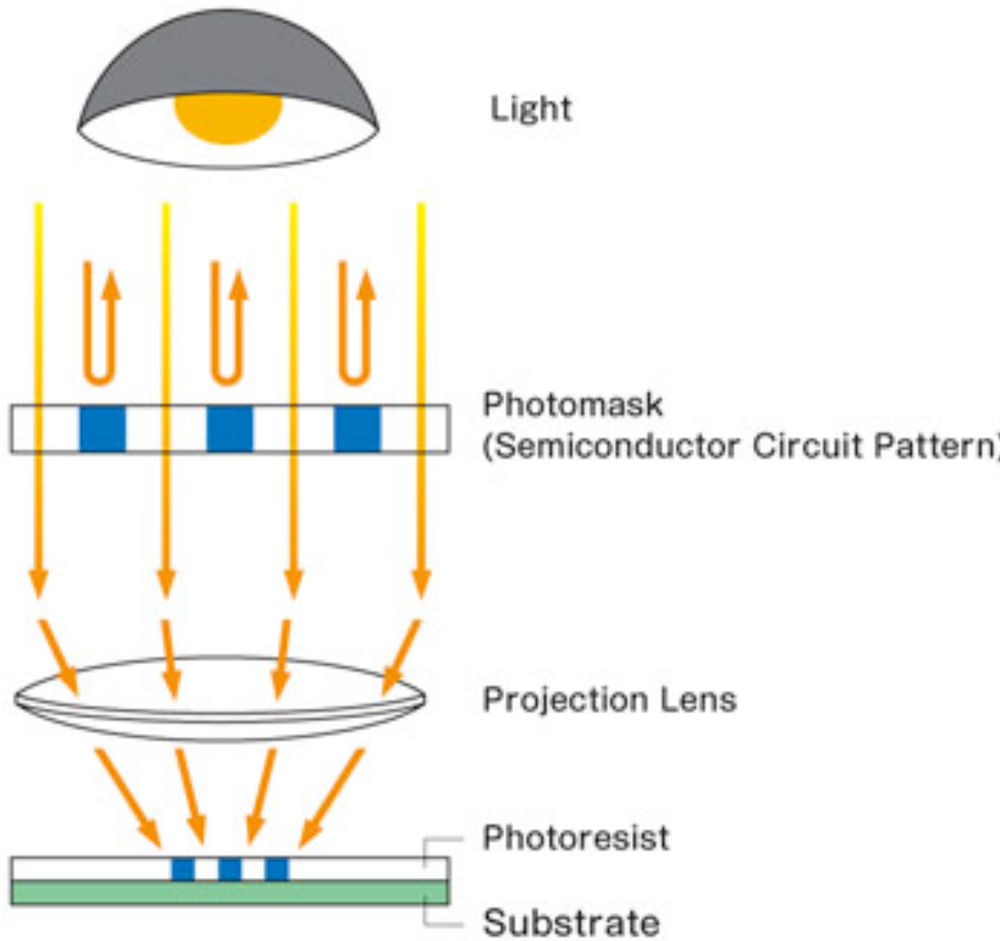
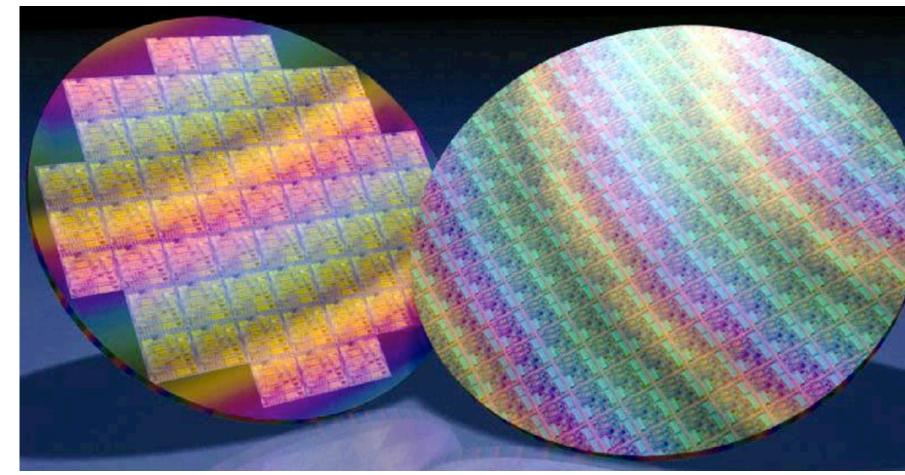
Integrated circuit fabrication 2

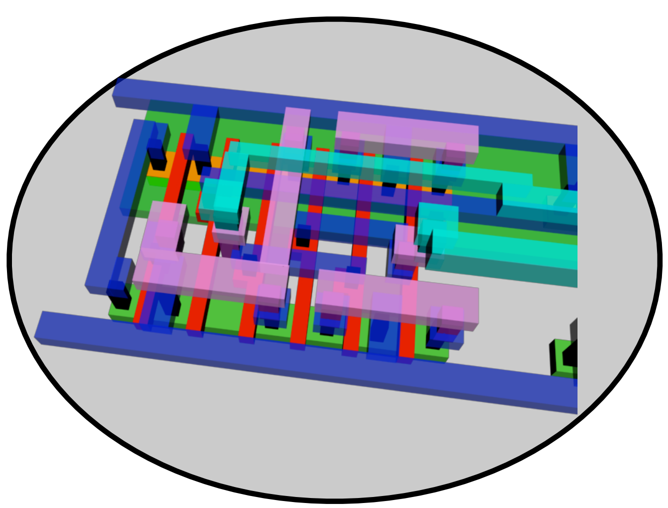


wafer





processed wafer



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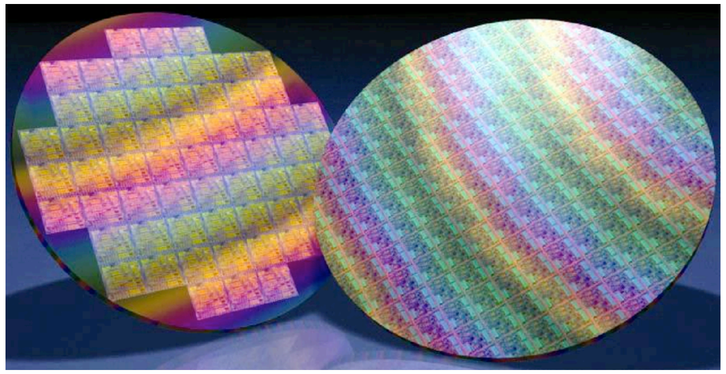
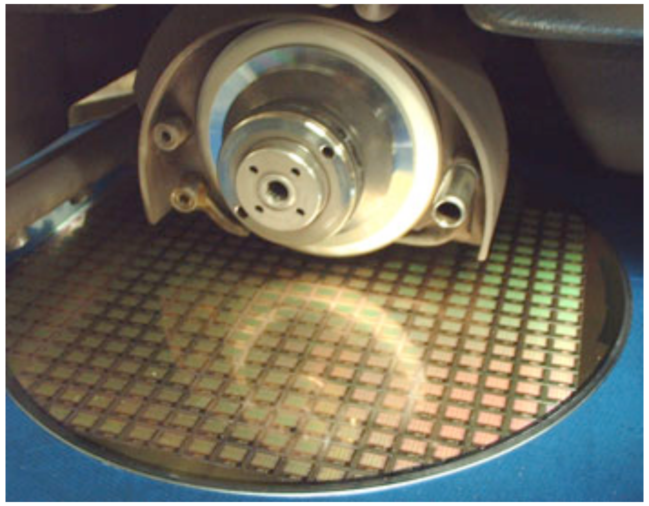
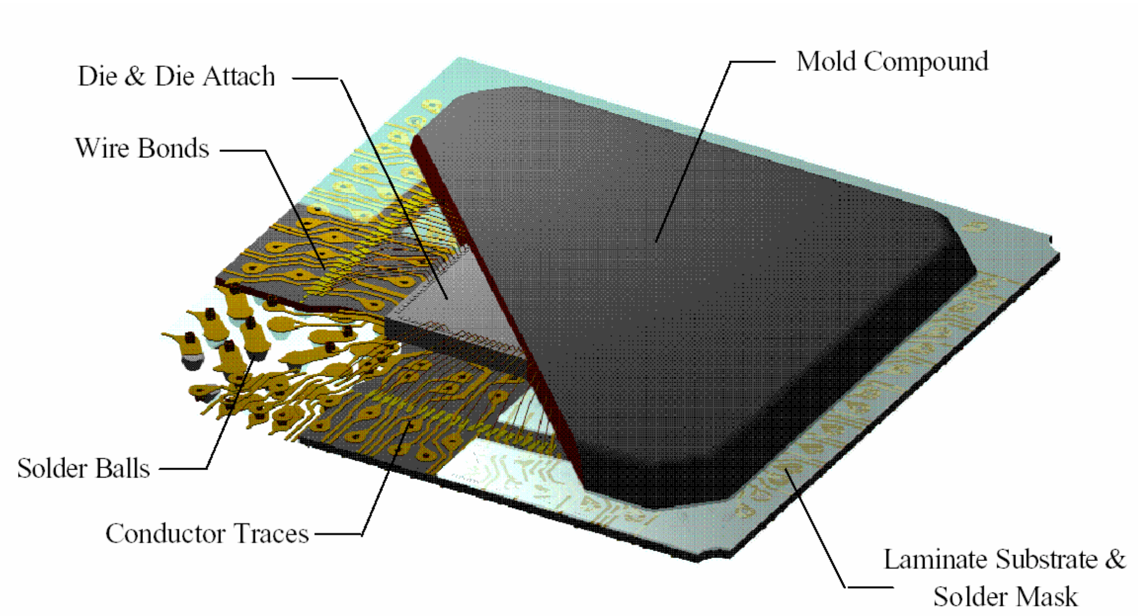
Integrated circuit fabrication 3



processed wafer



dicing



packaging

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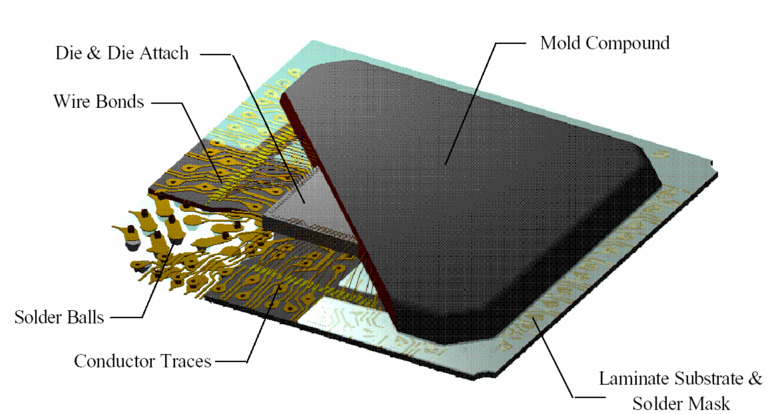
Integrated circuit fabrication 4



packaged die

test

$$$

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A more detailed tutorial on integrated circuit fabrication:

http://www.necel.com/fab/en/flow.html

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Next class: more boolean algebra, duals