CSEE 3827

PS 1 (solutions)

Q. 1. Note: Conversion by using base 2 as intermediate base only!

convert:

a) $(673.6)_8$ to hexadecimal

converting to base 2, we have

$$(6 \quad 7 \quad 3 \quad . \quad 6)_8$$

$\Downarrow$ (Represent each digit by 3 binary bits since base is 8)

$$(110 \quad 111 \quad 011 \quad . \quad 110)_2$$

forming groups of 4 for hex conversion

$$(\underbrace{0001}\,\underbrace{1011}\,\underbrace{1011}.\underbrace{1100})_2$$

$\downarrow$

$$(1 \quad B \quad B \quad . \quad C)_{16}$$

$$\boxed{\text{Ans)} \quad (673.6)_8 \rightarrow (1BB.C)_{16}}$$

b) $(E7C.B)_{16}$ to octal

$$(E \quad 7 \quad C \quad . \quad B)_{16}$$

$\Downarrow$ (Represent each digit by 4 binary bits since base is 16)

$$(1110 \quad 0111 \quad 1100 \quad . \quad 1011)$$

forming groups of 3 for octal conversion

$$\underbrace{111}\,\underbrace{001}\,\underbrace{111}\,\underbrace{100}\,.\,\underbrace{101}\,\underbrace{100}$$

$\Downarrow$

$\Rightarrow (7174.54)_8$

$\boxed{\text{Ans)} \quad (E7C.B)_{16} \rightarrow (7174.54)_8}$

c) $(310.2)_4$ to octal

Base is 4 ie 2^{2}. Thus use 2 binary bits to represent every digit.

$$(3\ 1\ 0\ .\ 2)_4$$
$$\Downarrow$$
$$11\ 01\ 00\ .\ 10$$

Forming groups of 3 for octal conversion.

$$\underbrace{110}\underbrace{100}\ .\underbrace{100}$$
$$\Downarrow$$
$$(64\ .4)_8$$

$\boxed{\text{Ans)} \quad (310.2)_4 \rightarrow (64.4)_8}$

Q.2) To prove : $X \cdot Y + YZ + \overline{X} \cdot Z = (X \cdot Y) + (\overline{X} \cdot Z)$

by using truth table and Algebraic manipulation.

(i) Truth table:

X	Y	Z	$\overline{X}$	$X \cdot Y$	YZ	$\overline{X} \cdot Z$	* LHS	* RHS
0	0	0	1	0	0	0	0	0
0	0	1	1	0	0	1	1	1
0	1	0	1	0	0	0	0	0
0	1	1	1	0	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	1	0	0	1	1
1	1	1	0	1	1	0	1	1

* L.H.S = $XY + YZ + \overline{X} Z$
R.H.S = $XY + \overline{X} \cdot Z$

(ii) Algebraic manipulation.

T.P.T $XY + YZ + \overline{X} \cdot Z = X \cdot Y + \overline{X} \cdot Z$ — 1

By the theorem of duals, if the above expression is true, then their duals are also equivalent.

Thus for proving 1, it is sufficient to prove the duals to be equivalent ie

T.P.T: $(X + Y) \cdot (Y + Z) \cdot (\overline{X} + Z) = (X + Y) \cdot (\overline{X} + Z)$.

L.H.S = $(X+Y)\cdot(Y+Z)\cdot(\overline{X}+Z)$

$= (XY+XZ+Y+YZ)(\overline{X}+Z)$

$= (XYZ + XZ + \overline{X}Y + \overline{X}YZ + YZ)$.

$= (YZ(X+\overline{X}) + YZ + XZ + \overline{X}Y)$

$= YZ + XZ + \overline{X}Y$ — (2)

R.H.S $= (X+Y)\cdot(\overline{X}+Z)$

$= XZ + \overline{X}Y + YZ$ — (3)

From (2) & (3), L.H.S = R.H.S .

Hence proved

Q.3) Reduce to indicated number of literals.
{Important property for such type of problems:
→ Distributive property ⇒ $X + YZ = (X+Y)(X+Z)$ }

(a) $\bar{X}\bar{Y} + XYZ + \bar{X}Y$ to 3 literals

$= \bar{X}(Y+\bar{Y}) + XYZ$

$= \bar{X} + (X)(YZ)$

$= (\bar{X}+X)(\bar{X}+YZ)$ .... (distributive property)

$= \bar{X} + YZ$ .... (3 literals)

(b) $X + Y(Z + \overline{X+Z})$ to 2 literals.

$= X + Y(Z + \bar{X}\bar{Z})$ ... De morgan

$= X + Y(Z+\bar{X})\cdot(Z+\bar{Z})$ .. distributive law

$= X + YZ + \bar{X}Y$

$= (X+\bar{X})\cdot(X+Y) + YZ$

$= X + Y + YZ$

$= X + Y(1+Z)$

$= X + Y$ (2 literals).

ⓒ $\overline{W}X(\overline{Z}+\overline{Y}Z)+X(W+\overline{W}YZ)$ to 1 literal.

$=\overline{W}X(\overline{Z}+Z)(\overline{Z}+\overline{Y})+X(W+\overline{W})(W+YZ)$ ... distributive property

$=\overline{W}X(\overline{Z}+\overline{Y})+X(W+YZ)$

$=\overline{W}X\overline{Z}+\overline{W}X\overline{Y}+XW+XYZ$

$=\overline{W}X\overline{Z}+XYZ+X(W+\overline{W}\overline{Y})$

$=\overline{W}X\overline{Z}+XYZ+X(W+\overline{W})(W+\overline{Y})$

$=\overline{W}X\overline{Z}+XYZ+XW+X\overline{Y}$

$=X(W+\overline{W}\overline{Z})+X(\overline{Y}+YZ)$

$=X(W+\overline{W})(W+\overline{Z})+X(\overline{Y}+Y)(\overline{Y}+Z)$ .. distributive law

$=XW+X\overline{Z}+X\overline{Y}+XZ$

$=XW+X\overline{Y}+X(Z+\overline{Z})$

$=X+X\overline{Y}+XW$

$=X$ ... (one literal).

ⓓ $(AB+\overline{A}\overline{B})(\overline{C}\overline{D}+CD)+\overline{AC}$ to 4 literals.

$=AB\overline{C}\overline{D}+ABCD+\overline{A}\overline{B}\overline{C}\overline{D}+\overline{A}\overline{B}CD+\overline{A}+\overline{C}$ .. (Demorgans)

$=\overline{A}(1+\overline{B}CD+\overline{B}\overline{C}\overline{D})+\overline{C}(1+AB\overline{D})+ABCD$

$=\overline{A}+\overline{C}+ABCD$

$=(\overline{A}+A)(\overline{A}+BCD)+\overline{C}$ – distributive law

$=\overline{A}+BCD+\overline{C}$

$=\overline{A}+(\overline{C}+C)(\overline{C}+BD)$ – distributive law

$=\overline{A}+\overline{C}+BD$ .. (4 literals)

Q-4)

$$F = A\bar{B}C + \bar{A}\bar{C} + AB$$

Complementing twice we have

$$F = \overline{\overline{A\bar{B}C + \bar{A}\bar{C} + AB}}$$

$$= \overline{\overline{(A\bar{B}C)} \cdot \overline{(\bar{A}\bar{C})} \cdot \overline{(AB)}} \quad \ldots (1) \text{ De-morgan's}$$

↓

This has only AND and complement functions

(solution for 'b' part)

From (1)

$$F = \overline{\overline{(A\bar{B}C)} \cdot \overline{(\bar{A}.\bar{C})} \cdot \overline{(AB)}}$$

$$= \overline{(\bar{A}+B+\bar{C}) \cdot (A+C) \cdot (\bar{A}+\bar{B})}$$

$$= \overline{(\bar{A}+B+\bar{C})} + \overline{(A+C)} + \overline{(\bar{A}+\bar{B})}$$

} De-morgan's

↓

This has only OR and complement functions

(solution for part 'a').

Q.5)

a)

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

b)

$F = A'BC + AB'C + ABC' + ABC$ - SOP

$= BC(A+A') + AB'C + ABC'$

$= BC + AB'C + ABC'$

$= C(B + AB') + ABC'$

$= C(B+B')(B+A) + ABC'$

$= BC + AC + ABC'$

$= BC + A[C + BC']$

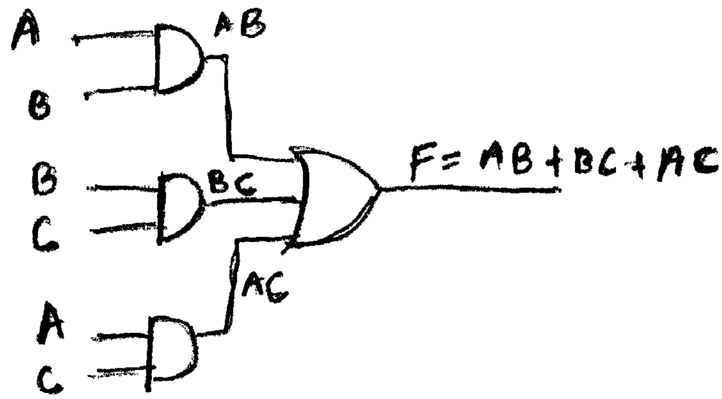
$= BC + A[C+C'][C+B]$ - distributive law

c) $\boxed{F = BC + AC + AB}$

↳ minimized expression.

Q. 5)

d)



Q. 6)

a) $(AB + C)(B + \bar{C}D)$

$AB + AB\bar{C}D + BC$

$AB + BC$ — SOP

Now <u>FOR</u> <u>POS</u>

we have from SOP

$AB + BC$

$= (A + C) \cdot B$

↳ Required POS

Q-6)

b) $\overline{X} + X(X+\overline{Y})(Y+\overline{Z})$

$= \overline{X} + X(XY + X\overline{Z} + \overline{Y}\,\overline{Z})$

$= \overline{X} + XY + X\overline{Z} + X\overline{Y}\,\overline{Z}$

$= (\overline{X}+X)(\overline{X} + Y + \overline{Z} + \overline{Y}\,\overline{Z})$

$= \overline{X} + Y + \overline{Z}$ → SOP as well as POS.

Q.6) c) $(A + B\overline{C} + CD)(\overline{B} + EF)$

$= A\overline{B} + ABEF + B\overline{C}EF + \overline{B}CD + CDEF$ — SOP.

POS

$= (A + B\overline{C} + CD)(\overline{B} + EF)$

$= [(A+B)(A+\overline{C}) + CD](\overline{B}+E)(\overline{B}+F)$

$= [(A+B+CD)(A+\overline{C}+CD)](\overline{B}+E)(\overline{B}+F)$

$= [((A+C)(A+D)+B)(A+(\overline{C}+C)(\overline{C}+D))](\overline{B}+E)(\overline{B}+F)$

$= (A+B+C)(A+B+D)(A+D+\overline{C})\cdot(\overline{B}+E)(\overline{B}+F)$

↳ POS