COMS 4160: Problems and Questions on Rendering

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Questions and Problems

1. Match the surface material to the formula (and goniometric diagram shown in class). Also, give an example of a real material that reasonably closely approximates the mathematical description. Not all materials need have a corresponding diagram. The materials are *ideal mirror, dark glossy, ideal diffuse, retroreflective*. The formulae for the BRDF f_r are $k_a(\vec{R} \cdot \vec{V}), k_b(\vec{R} \cdot \vec{V})^4, k_c/(\vec{N} \cdot \vec{V}), k_d\delta(\vec{R}), k_e$.

2. Consider a simplified skylight model, so the radiance along any direction is given by $A + B \sin \alpha$ where A and B are positive constants, and α is the elevation angle (i.e. the angle to the horizontal, being 0 degrees toward the horizontal and 90 degrees toward the zenith or top of the sky). That is, the radiance is more higher up in the sky. The lighting is isotropic; there is no variation with azimuthal angle (ϕ). Assume for this problem that there is no occlusion by trees, buildings etc., the sky hemisphere is the only source of illumination [no ground lighting, direct sunlight etc.], the surfaces are Lambertian with albedo 1, and the sky can be assumed to be a distant source. What is the irradiance on the ground, assumed to be a horizontal surface?. Now, assume we have a sphere suspended (or on the ground, if that makes things more logical for you). Remember the assumptions, i.e. lighting only from the (distant) sky, no occlusions etc. Which point on the sphere will be brightest? What will be the reflected radiance at this point? Which point will be the dimmest? What will be the reflected radiance at that point? Qualitatively, how will the brightness on the sphere vary as a function of location (parameterized by spherical coordinates for instance)? Extra credit for deriving an analytic quantitative formula for the brightness of the sphere as a function of the surface normal.

Answers

1. Materials An ideal mirror is something like a normal reflective mirror, and the BRDF corresponds to $k_d\delta\left(\vec{R}\right)$. A dark glossy surface is close to glossy plastic with a formula like $k_b\left(\vec{R} \cdot \vec{V}\right)^4$. An ideal diffuse surface is Lambertian, close to wall paint or ideally, spectralon, with BRDF a constant k_e. A retroreflective surface is something like a highway reflector, that reflects light back toward the viewer and would have a formula using $\delta\left(\vec{L}\right)$ instead of $\vec{R}$ as in an ordinary reflector.

2. Irradiance The irradiance is given by

$$E = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} L(\theta, \phi) \cos\theta \sin\theta \, d\theta, \tag{1}$$

where θ stands for the angle with respect to the zenith, or normal to the ground plane. In this coordinate frame, the elevation angle = $\pi/2 - \theta$, and so the incident radiance $L = A + B\cos\theta$. Removing the azimuthal dependence in the above integral, we obtain

$$E = 2\pi \int_0^{\pi/2} (A + B\cos\theta)\cos\theta \sin\theta \, d\theta. \tag{2}$$

To evaluate this integral, we put $u = \cos\theta$, to obtain

$$E = 2\pi \int_0^1 Au + Bu^2 \, du = \pi \left(A + \frac{2B}{3} \right). \tag{3}$$

The brightest point is clearly the top of the sphere, or the point facing directly upward, which receives the same irradiance as the ground. The dimmest point will be at the bottom of the sphere, which will be completely dark (since it sees only the lower hemisphere that is dark, since we assume not interreflection from the ground). The radiance will decrease from top to bottom, remaining azimuthally symmetric.

Finding an analytic formula for the radiance as a function of angle is an advanced topic, well outside the scope of this course. One can try to use the spherical harmonic irradiance formula in my thesis.