Computer Graphics (Fall 2008)

COMS 4160, Lecture 5: Viewing

http://www.cs.columbia.edu/~cs4160

To Do

- Questions/concerns about assignment 1?
- Remember it is due Thu. Ask me or TA if any problems.

Motivation

- We have seen transforms (between coord systems)
- But all that is in 3D
- We still need to make a 2D picture
- Project 3D to 2D. How do we do this?
- This lecture is about viewing transformations

Demo (Projection Tutorial)

- Nate Robbins OpenGL tutors
- Projection.exe
- Download others

What we've seen so far

- Transforms (translation, rotation, scale) as 4x4 homogeneous matrices
- Last row always 0 0 0 1. Last w component always 1
- For viewing (perspective), we will use that last row and w component no longer 1 (must divide by it)

Outline

- *Orthographic projection (simpler)*
- Perspective projection, basic idea
- Derivation of gluPerspective (handout: glFrustum)
- Brief discussion of nonlinear mapping in z

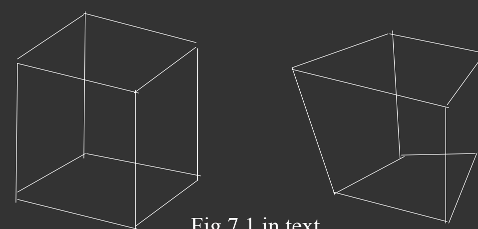
Not well covered in textbook chapter 7. We follow section 3.5 of real-time rendering most closely. Handouts on this will be given out.

Projections

- To lower dimensional space (here 3D -> 2D)
- Preserve straight lines
- Trivial example: Drop one coordinate (Orthographic)

Orthographic Projection

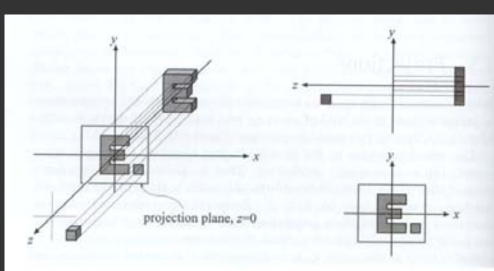
- Characteristic: Parallel lines remain parallel
- Useful for technical drawings etc.



Orthographic Fig 7.1 in text Perspective

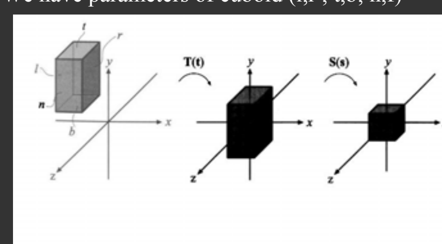
Example

- Simply project onto xy plane, drop z coordinate



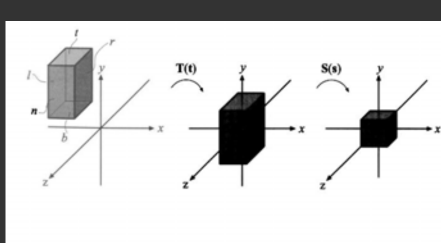
In general

- We have a cuboid that we want to map to the normalized or square cube from [-1, +1] in all axes
- We have parameters of cuboid (l,r ; t,b; n,f)



Orthographic Matrix

- First center cuboid by translating
- Then scale into unit cube



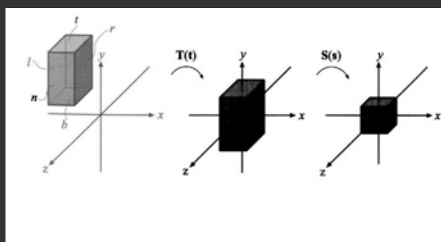
Transformation Matrix

Scale Translation (centering)

$$M = \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{f-n} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -\frac{l+r}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{f+n}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Caveats

- Looking down –z, f and n are negative (n > f)
- OpenGL convention: positive n, f, negate internally



Final Result

$$M = \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad glOrtho = \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \boxed{\frac{-2}{f-n}} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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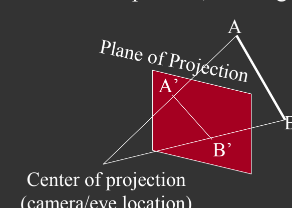
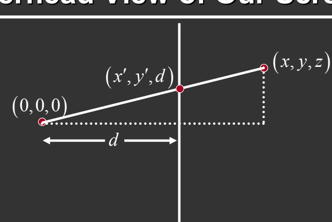
Perspective Projection

- Most common computer graphics, art, visual system
- Further objects are smaller (size, inverse distance)
- Parallel lines not parallel; converge to single point



Overhead View of Our Screen



Looks like we've got some nice similar triangles here?

$$\frac{x}{z} = \frac{x'}{d} \Rightarrow x' = \frac{d*x}{z} \qquad \frac{y}{z} = \frac{y'}{d} \Rightarrow y' = \frac{d*y}{z}$$

In Matrices

- Note negation of z coord (focal plane –d)
- (Only) last row affected (no longer 0 0 0 1)
- w coord will no longer = 1. Must divide at end

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{d} & 0 \end{pmatrix}$$

Verify

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{d} & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = ? \qquad \begin{pmatrix} x \\ y \\ z \\ -\frac{1}{d} \end{pmatrix} = \begin{pmatrix} -\frac{d*x}{z} \\ -\frac{d*y}{z} \\ -d \\ 1 \end{pmatrix}$$

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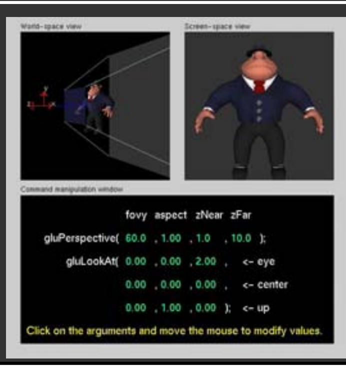
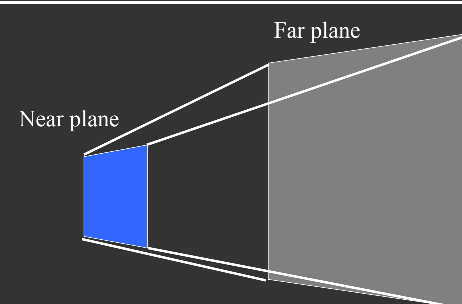
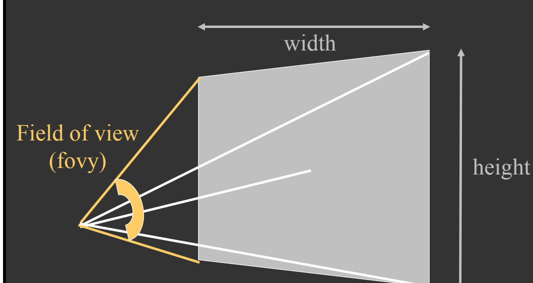
Remember projection tutorial



Viewing Frustum



Screen (Projection Plane)

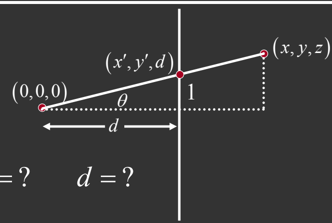


Aspect ratio = width / height

gluPerspective

- gluPerspective(fovy, aspect, zNear > 0, zFar > 0)
- Fovy, aspect control fov in x, y directions
- zNear, zFar control viewing frustum

Overhead View of Our Screen



$\theta = ? \qquad d = ?$

$$\theta = \frac{fovy}{2} \qquad d = \cot\theta$$

In Matrices

- Simplest form: $P = \begin{pmatrix} \frac{1}{aspect} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{d} & 0 \end{pmatrix}$
- Aspect ratio taken into account
- Homogeneous, simpler to multiply through by d
- Must map z values based on near, far planes (not yet)

In Matrices

$$P = \begin{pmatrix} \frac{1}{aspect} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{d} & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{d}{aspect} & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & A & B \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

- A and B selected to map n and f to -1, +1 respectively

Z mapping derivation

$$\begin{pmatrix} A & B \\ -1 & 0 \end{pmatrix}\begin{pmatrix} z \\ 1 \end{pmatrix} = ? \qquad \begin{pmatrix} Az+B \\ -z \end{pmatrix} = -A - \frac{B}{z}$$

- Simultaneous equations?

$$-A + \frac{B}{n} = -1 \qquad A = -\frac{f+n}{f-n}$$

$$-A + \frac{B}{f} = +1 \qquad B = -\frac{2fn}{f-n}$$

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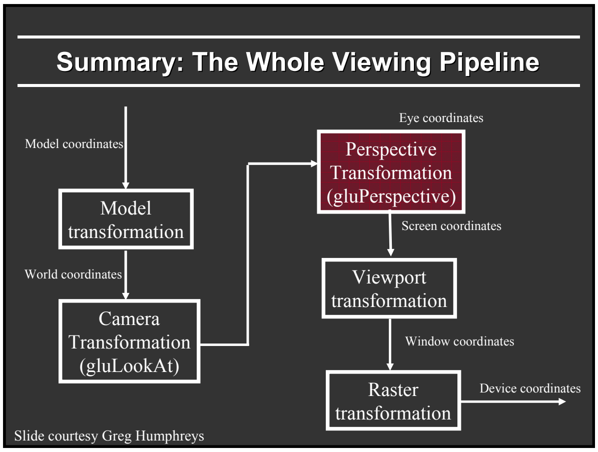
Mapping of Z is nonlinear

$$\begin{pmatrix} Az+B \\ -z \end{pmatrix} = -A - \frac{B}{z}$$

- Many mappings proposed: all have nonlinearities
- Advantage: handles range of depths (10cm – 100m)
- Disadvantage: depth resolution not uniform
- More close to near plane, less further away
- Common mistake: set near = 0, far = infty. Don't do this. Can't set near = 0; lose depth resolution.
- We discuss this more in review session


Summary: The Whole Viewing Pipeline
Model coordinates
Model transformation
World coordinates
Camera Transformation (gluLookAt)
Eye coordinates
Perspective Transformation (gluPerspective)
Screen coordinates
Viewport transformation
Window coordinates
Raster transformation
Device coordinates
Slide courtesy Greg Humphreys