Computer Graphics (Fall 2008)

COMS 4160, Lecture 4: Transformations 2

http://www.cs.columbia.edu/~cs4160

To Do

- Start doing assignment 1
  - Time is short, but needs only little code [Due Thu Sep 25, 11:59pm]
  - Ask questions or clear misunderstandings by next lecture
- Specifics of HW 1
  - Last lecture covered basic material on transformations in 2D. You likely need this lecture though to understand full 3D transformations
  - Last lecture had some complicated stuff on 3D rotations. You only need final formula (actually not even that, setrot function available)
  - gluLookAt derivation this lecture should help clarifying some ideas
- Read bulletin board and webpage!!

Outline

- *Translation: Homogeneous Coordinates*
- Transforming Normals
- Rotations revisited: coordinate frames
- gluLookAt (quickly)

Exposition is slightly different than in the textbook

Translation

- E.g. move x by +5 units, leave y, z unchanged
- We need appropriate matrix. What is it?

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} & & \\ & ? & \\ & & \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+5 \\ y \\ z \end{pmatrix}$$

transformation_game.jar

Homogeneous Coordinates

- Add a fourth homogeneous coordinate (w=1)
- 4x4 matrices very common in graphics, hardware
- Last row always 0 0 0 1 (until next lecture)

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x+5 \\ y \\ z \\ 1 \end{pmatrix}$$

Representation of Points (4-Vectors)

Homogeneous coordinates

$$P = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x/w \\ y/w \\ z/w \\ 1 \end{pmatrix}$$

- Divide by 4^{th} coord (w) to get (inhomogeneous) point
- Multiplication by $w > 0$, no effect
- Assume $w \geq 0$. For $w > 0$, normal finite point. For $w = 0$, point at infinity (used for vectors to stop translation)

Advantages of Homogeneous Coords

- Unified framework for translation, viewing, rot…
- Can concatenate any set of transforms to 4x4 matrix
- No division (as for perspective viewing) till end
- Simpler formulas, no special cases
- Standard in graphics software, hardware

General Translation Matrix

$$T = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & T \\ 0 & 1 \end{pmatrix}$$

$$P' = TP = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + T_x \\ y + T_y \\ z + T_z \\ 1 \end{pmatrix} = P + T$$

Combining Translations, Rotations

- Order matters!! TR is not the same as RT (demo)
- General form for rigid body transforms
- We show rotation first, then translation (commonly used to position objects) on next slide. Slide after that works it out the other way

transformation_game.jar

simplestGlut.exe

Combining Translations, Rotations

$$P' = (TR)P = MP = RP + T$$

$$M = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc|c} R_{11} & R_{12} & R_{13} & T_x \\ R_{21} & R_{22} & R_{23} & T_y \\ R_{31} & R_{32} & R_{33} & T_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} R & T \\ 0 & 1 \end{pmatrix}$$

transformation_game.jar

Combining Translations, Rotations

$$P' = (RT)P = MP = R(P + T) = RP + RT$$

$$M = \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} R_{3\times 3} & R_{3\times 3}T_{3\times 1} \\ 0_{1\times 3} & 1 \end{pmatrix}$$

transformation_game.jar

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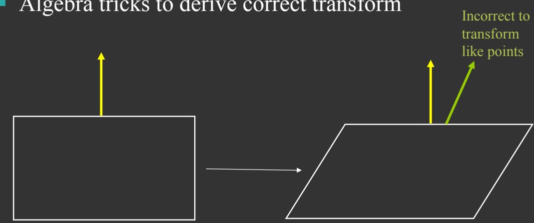
Normals

- Important for many tasks in graphics like lighting
- Do not transform like points e.g. shear
- Algebra tricks to derive correct transform



Finding Normal Transformation

$$t \to Mt \qquad n \to Qn \qquad Q = ?$$

$$n^T t = 0$$

$$n^T Q^T M t = 0 \quad \Rightarrow \quad Q^T M = I$$

$$Q = (M^{-1})^T$$

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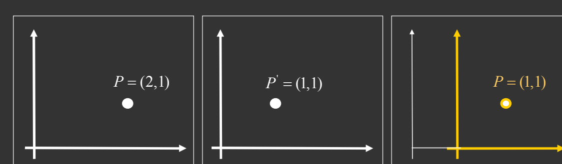
Section 6.5 of textbook

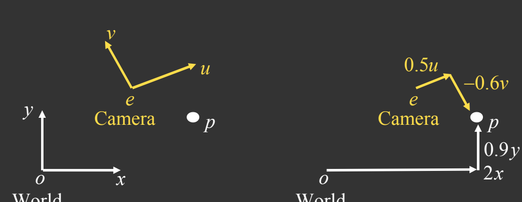
Coordinate Frames

- All of discussion in terms of operating on points
- But can also change coordinate system
- Example, motion means either point moves backward, or coordinate system moves forward

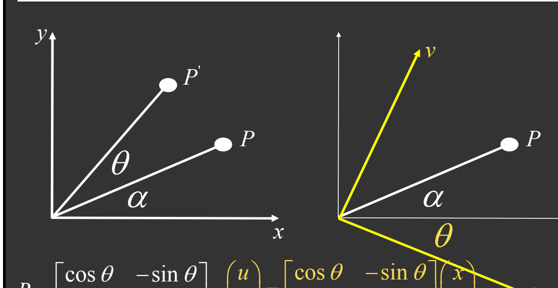


Coordinate Frames: In general

- Can differ both origin and orientation (e.g. 2 people)
- One good example: World, camera coord frames (H1)



Coordinate Frames: Rotations



$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \qquad \begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Geometric Interpretation 3D Rotations

- Rows of matrix are 3 unit vectors of new coord frame
- Can construct rotation matrix from 3 orthonormal vectors

$$R_{uvw} = \begin{pmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \\ x_w & y_w & z_w \end{pmatrix} \qquad u = x_u X + y_u Y + z_u Z$$

Axis-Angle formula (summary)

$$(b \backslash a)_{ROT} = (I_{3\times3}\cos\theta - aa^T\cos\theta)b + (A^*\sin\theta)b$$

$$(b \to a)_{ROT} = (aa^T)b$$

$$R(a,\theta) = I_{3\times3}\cos\theta + aa^T(1-\cos\theta) + A^*\sin\theta$$

$$R(a,\theta) = \cos\theta\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + (1-\cos\theta)\begin{pmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & yz & z^2 \end{pmatrix} + \sin\theta\begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}$$

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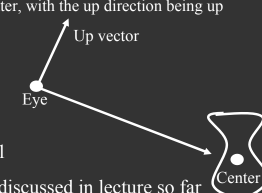
Not fully covered in textbooks. However, look at sections 6.5 and 7.2.1
We've already covered the key ideas, so we go over it quickly showing how things fit together

Case Study: Derive gluLookAt

Defines camera, fundamental to how we view images

- gluLookAt(eyex, eyey, eyez, centerx, centery, centerz, upx, upy, upz)
- Camera is at eye, looking at center, with the up direction being up



- May be important for HW1
- Combines many concepts discussed in lecture so far
- Core function in OpenGL for later assignments

Steps

- gluLookAt(eyex, eyey, eyez, centerx, centery, centerz, upx, upy, upz)
- Camera is at eye, looking at center, with the up direction being up

- *First, create a coordinate frame for the camera*
- Define a rotation matrix
- Apply appropriate translation for camera (eye) location

Constructing a coordinate frame?

We want to associate **w** with **a**, and **v** with **b**

- But **a** and **b** are neither orthogonal nor unit norm
- And we also need to find **u**

$$w = \frac{a}{\|a\|}$$

$$u = \frac{b \times w}{\|b \times w\|}$$

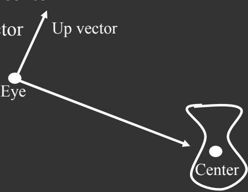
$$v = w \times u$$

from lecture 2

Constructing a coordinate frame

$$w = \frac{a}{\|a\|} \qquad u = \frac{b \times w}{\|b \times w\|} \qquad v = w \times u$$

- We want to position camera at origin, looking down –Z dirn
- Hence, vector **a** is given by **eye** – **center**
- The vector **b** is simply the **up** vector



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Steps

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- First, create a coordinate frame for the camera
- Define a rotation matrix
- *Apply appropriate translation for camera (eye) location*

Translation

- gluLookAt(eyex, eyey, eyez, centerx, centery, centerz, upx, upy, upz)
- Camera is at eye, looking at center, with the up direction being up

- *Cannot* apply translation after rotation
- The translation must come first (to bring camera to origin) before the rotation is applied

Combining Translations, Rotations

$$P' = (RT)P = MP = R(P+T) = RP + RT$$

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gluLookAt final form

$$\begin{pmatrix} x_u & y_u & z_u & 0 \\ x_v & y_v & z_v & 0 \\ x_w & y_w & z_w & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x_u & y_u & z_u & -x_u e_x - y_u e_y - z_u e_z \\ x_v & y_v & z_v & -x_v e_x - y_v e_y - z_v e_z \\ x_w & y_w & z_w & -x_w e_x - y_w e_y - z_w e_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$