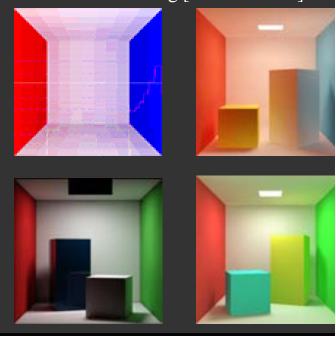
Computer Graphics (Fall 2008)

COMS 4160, Lecture 23: Radiosity

http://www.cs.columbia.edu/~cs4160

Radiosity

Cornell box with color bleeding [Goral et al 84]



Photograph of a sculpture.
The front faces are all diffuse white
The color is because of reflection from rear-facing colored faces

Raytracing makes all faces white. It can handle specular reflection and shadows, but not diffuse-diffuse interreflection or color bleeding

Radiosity correctly captures the color bleeding from the back of the boards to the front.

Advantages and Disadvantages

- Radiosity methods track rate at which energy (radiosity) leaves [diffuse] surfaces
- Determine equilibrium of light energy in a view-independent way
- Allows for diffuse interreflection, color bleeding, and walkthroughs
- Difficult to handle specular objects, mirrors

General Approach

- Assume diffuse surfaces discretized into a finite set of patches or finite elements
- Radiosity equation is a matrix equation or set of simultaneous linear equations derived by approximations to the rendering equation
- Solve iteratively using numerical methods

Earliest Radiosity pictures

Radiosity was first developed in other fields
- Heat transport, Lighting Design
- In graphics: Goral et al. 84

Parry Moon and Domina Spencer (MIT), Lighting Design, 1948

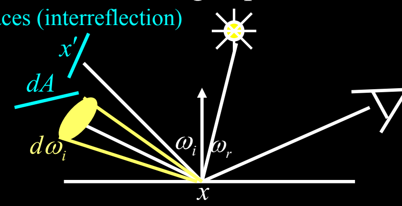
Outline

- *Rendering equation review*
- *Radiosity equation*
- Form factors
- Methods to compute form factors

High-level overview only. Best textual reference is probably Sections 16.3.1 and 16.3.2 in FvDFH. This will be handed out. If curious, read the rest of 16.3 and parts of Cohen and Wallace.

Rendering Equation

Surfaces (interreflection)



$\omega_i \sim x' - x$

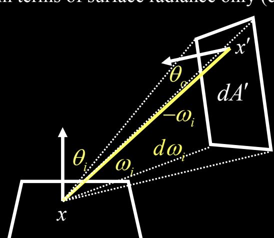
$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Change of Variables

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)



$$d\omega_i = \frac{dA' \cos\theta_o}{|x-x'|^2}$$

Change of Variables

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\frac{\cos\theta_i \cos\theta_o}{|x-x'|^2} dA'}$$

$$d\omega_i = \frac{dA' \cos\theta_o}{|x-x'|^2}$$

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

Rendering Equation: Standard Form

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\frac{\cos\theta_i \cos\theta_o}{|x-x'|^2} dA'}$$

Domain integral awkward. Introduce binary visibility fn V

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all surfaces } x'} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) G(x,x') V(x,x')\, dA'$$

$$d\omega_i = \frac{dA' \cos\theta_o}{|x-x'|^2}$$

Same as equation 2.52 Cohen Wallace. It swaps primed And unprimed, omits angular args of BRDF, - sign. Same as equation above 19.3 in Shirley, except he has no emission, slightly diff. notation

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

Radiosity Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all surfaces } x'} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) G(x,x') V(x,x')\, dA'$$

Drop angular dependence (diffuse Lambertian surfaces)

$$L_r(x) = L_e(x) + f(x) \int_S L_r(x') G(x,x') V(x,x')\, dA'$$

Change variables to radiosity (B) and albedo (ρ)

$$B(x) = E(x) + \rho(x) \int_S B(x') \frac{G(x,x') V(x,x')}{\pi} dA'$$

Expresses conservation of light energy at all points in space

Same as equation 2.54 in Cohen Wallace handout (read sec 2.6.3)
Ignore factors of π which can be absorbed.

Outline

- Rendering equation review
- Radiosity equation
- *Form factors*
- Methods to compute form factors

Section 16.3.1,2 (eqs 16.63-65) in FvDFH

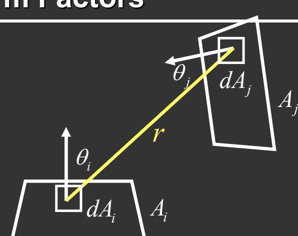
Discretization and Form Factors

$$B(x) = E(x) + \rho(x) \int_S B(x') \frac{G(x,x')V(x,x')}{\pi} dA'$$

$$B_i = E_i + \rho_i \sum_j B_j F_{j\to i} \frac{A_j}{A_i}$$

F is the ***form factor.*** It is dimensionless and is the fraction of energy leaving the entirety of patch j (*multiply by area of* j to get total energy) that arrives anywhere in the entirety of patch i (*divide by area of i* to get energy per unit area or radiosity).

Form Factors



$$A_i F_{i\to j} = A_j F_{j\to i} = \iint \frac{G(x,x')V(x,x')}{\pi} dA_i dA_j$$

$$G(x,x') = G(x',x) = \frac{\cos\theta_i \cos\theta_o}{|x-x'|^2}$$

Matrix Equation

$$B_i = E_i + \rho_i \sum_j B_j F_{j\to i} \frac{A_j}{A_i}$$

$$A_i F_{i\to j} = A_j F_{j\to i} = \iint \frac{G(x,x')V(x,x')}{\pi} dA_i dA_j$$

$$B_i = E_i + \rho_i \sum_j B_j F_{i\to j}$$

$$B_i - \rho_i \sum_j B_j F_{i\to j} = E_i$$

$$\sum_j M_{ij} B_j = E_i \qquad MB = E \qquad M_{ij} = I_{ij} - \rho_i F_{i\to j}$$

Outline

- Rendering equation review
- Radiosity equation
- Form factors
- *Methods to compute form factors*

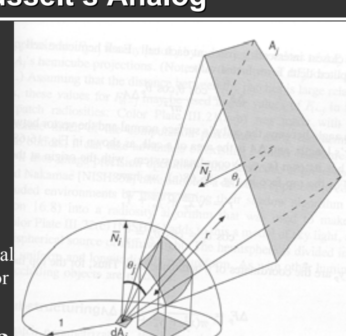
Section 16.3.2 in FvDFH

Nusselt's Analog

Analytically project into hemisphere above point. Then project onto hemisphere base

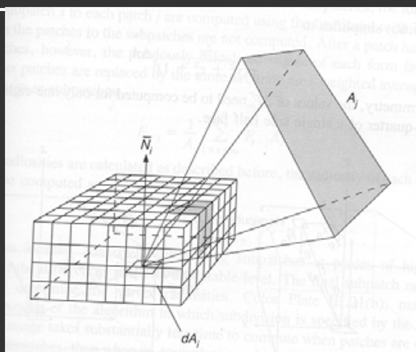
Form factor is ratio of area on base to area of entire base

This computes differential point to patch form factor



Why does it work?

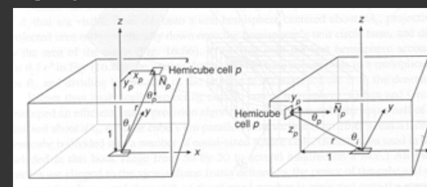
Hemicube



Hemicubes

$$\Delta F_p = \frac{\cos\theta_i \cos\theta_p}{\pi r^2}\Delta A$$

- Each small hemicube cell has a precomputed delta form factor: add up to get final value



- We can render the scene using normal Z-buffer scan conversion onto the faces of the hemicube!

Monte Carlo Ray Tracing

- Can be used to find form factors (slow)
- Can be used directly to shoot energy