Computer Graphics (Fall 2008)

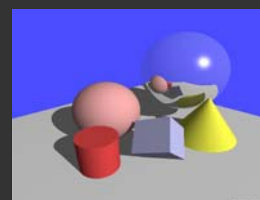
COMS 4160, Lecture 22: Global Illumination

http://www.cs.columbia.edu/~cs4160

Illumination Models

So far considered mainly local illumination
- Light directly from light sources to surface

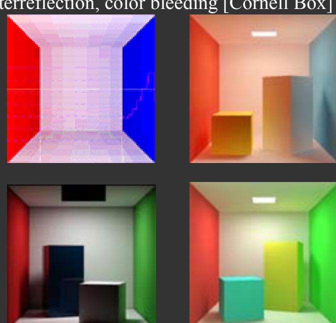
Global Illumination: multiple bounces
- Already ray tracing: reflections/refractions



Some images courtesy Henrik Wann Jensen

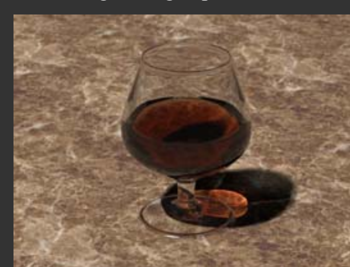
Global Illumination

Diffuse interreflection, color bleeding [Cornell Box]



Global Illumination

Caustics: Focusing through specular surface



Major research effort in 80s, 90s till today

Overview of lecture

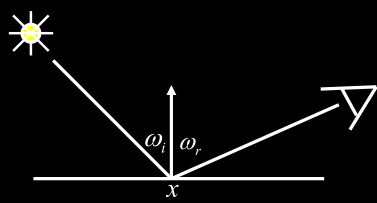
- ***Theory*** for all methods (ray trace, radiosity)
- We derive ***Rendering Equation*** [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases

Fairly theoretical lecture (but important). Not well covered in any of the textbooks. Closest are 2.6.2 in Cohen and Wallace handout (but uses slightly different notation, argument [swaps x, x' among other things]) and 19.2 in Shirley (different notation, omits emission, but has a reasonably good intuitive discussion that we somewhat follow).

Outline

- *Reflectance Equation (review)*
- *Global Illumination*
- *Rendering Equation*
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)

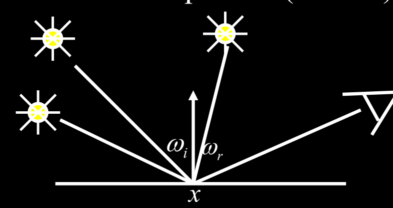
Reflectance Equation (review)



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i)\, f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflectance Equation (review)

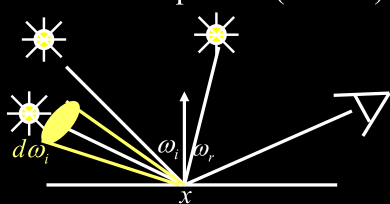


Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i)\, f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Reflectance Equation (review)



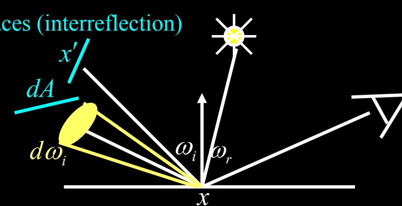
Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i)\, f(x,\omega_i,\omega_r) \cos\theta_i\, d\omega_i$$

Reflected Light (Output Image) | Emission | Incident Light (from light source) | BRDF | Cosine of Incident angle

Global Illumination

Surfaces (interreflection)



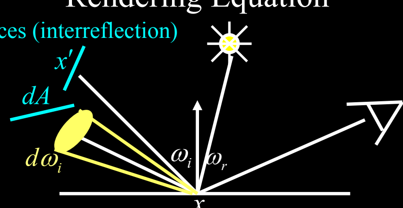
$\omega_i \sim x' - x$

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i)\, f(x,\omega_i,\omega_r) \cos\theta_i\, d\omega_i$$

Reflected Light (Output Image) | Emission | Reflected Light (from surface) | BRDF | Cosine of Incident angle

Rendering Equation

Surfaces (interreflection)



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i)\, f(x,\omega_i,\omega_r) \cos\theta_i\, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Rendering Equation (Kajiya 86)

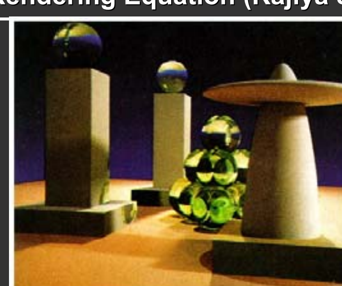


Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Outline

- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- *As a general Integral Equation and Operator*
- *Approximations (Ray Tracing, Radiosity)*
- Surface Parameterization (Standard Form)

The material in this part of the lecture is fairly advanced and not covered in any of the texts. The slides should be fairly complete. This section is fairly short, and I hope some of you will get some insight into solutions for general global illumination

Rendering Equation as Integral Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i)\, f(x,\omega_i,\omega_r)\cos\theta_i\, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v)\, K(u,v)\, dv$$

Kernel of equation

Linear Operator Equation

$$l(u) = e(u) + \int l(v)\, K(u,v)\, dv$$

Kernel of equation
Light Transport Operator

$$L = E + KL$$

Can be discretized to a simple matrix equation [or system of simultaneous linear equations] (L, E are vectors, K is the light transport matrix)

Solution Techniques

All global illumination methods try to solve (approximations of) the rendering equation

- Too hard for analytic solution: numerical General theory of solving integral equations

Radiosity (next lecture; usually diffuse surfaces)

- General class numerical ***finite element*** methods (divide surfaces in scene into a finite set elements or patches)
- Set up linear system (matrix) of simultaneous equations
- Solve iteratively

Ray Tracing and extensions

- General class numerical ***Monte Carlo*** methods
- Approximate set of all paths of light in scene

$$L = E + KL$$
$$IL - KL = E$$
$$(I-K)L = E$$
$$L = (I-K)^{-1}E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + \ldots)E$$
$$L = E + KE + K^2E + K^3E + \ldots$$

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

- E: Emission directly From light sources
- KE: Direct Illumination on surfaces
- K^2E: Global Illumination (One bounce indirect) [Mirrors, Refraction]
- K^3E: (Two bounce indirect) [Caustics etc]

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly From light sources

Direct Illumination on surfaces

OpenGL Shading

Global Illumination (One bounce indirect) [Mirrors, Refraction]

(Two bounce indirect) [Caustics etc]

Outline

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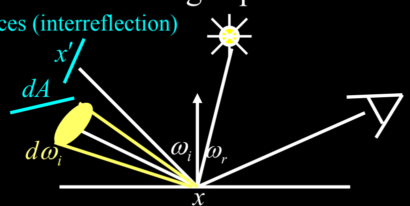
- Approximations (Ray Tracing, Radiosity)
- *Surface Parameterization (Standard Form)*

Page 461 of Shirley is reasonably close to this part of lecture, although it uses different notation. See also pages 38 and 39 in handout, which may have a clearer explanation of the ideas.

Rendering Equation



$$\omega_i \sim x' - x$$

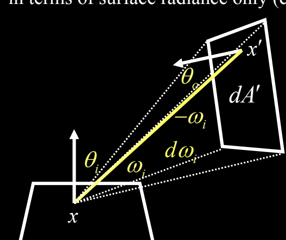
$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Change of Variables

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)



$$d\omega_i = \frac{dA'\cos\theta_o}{|x - x'|^2}$$

Change of Variables

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\frac{\cos\theta_i\cos\theta_o}{|x - x'|^2} dA'}$$

$$d\omega_i = \frac{dA'\cos\theta_o}{|x - x'|^2}$$

$$G(x,x') = G(x',x) = \frac{\cos\theta_i\cos\theta_o}{|x - x'|^2}$$

Rendering Equation: Standard Form

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\cos\theta_i d\omega_i}$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \boxed{\frac{\cos\theta_i\cos\theta_o}{|x - x'|^2} dA'}$$

Domain integral awkward. Introduce binary visibility fn V

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\text{all surfaces } x'} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) G(x,x') V(x,x')\, dA'$$

$$d\omega_i = \frac{dA'\cos\theta_o}{|x - x'|^2}$$

Same as equation 2.52 Cohen Wallace. It swaps primed And unprimed, omits angular args of BRDF, - sign. Same as equation above 19.3 in Shirley, except he has no emission, slightly diff. notation

$$G(x,x') = G(x',x) = \frac{\cos\theta_i\cos\theta_o}{|x - x'|^2}$$

Overview

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- We derive ***Rendering Equation*** [Kajiya 86]
  - Major theoretical development in field
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