Computer Graphics (Fall 2008)

COMS 4160, Lectures 16, 17:

Nuts and bolts of Ray Tracing

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Acknowledgements: Thomas Funkhouser and Greg Humphreys

Heckbert's Business Card Ray Tracer

```
typedef struct{double x,y,z}vec;vec U,black,amb={.02,.02,.02};struct sphere{ vec cen,color;
double rad,kd,ks,kt,kl,ir}*s,*best,sph[]={0.,6.,.5,1.,1.,1.,.9, .05,.2,.85,0.,1.7,-1.,8.,-.5,1.,.5,.2,1.,
.7,.3,0.,.05,1.2,1.,8.,-.5,.1,.8,.8, 1.,.3,.7,0.,0.,1.2,3.,-6.,15.,1.,.8,1.,7.,0.,0.,0.,.6,1.5,-3.,-3.,12.,
.8,1., 1.,5.,0.,0.,0.,.5,1.5,};yx;double u,b,tmin,sqrt(),tan();double vdot(A,B)vec A ,B;{return A.x
*B.x+A.y*B.y+A.z*B.z;}vec vcomb(a,A,B)double a;vec A,B;{B.x+=a* A.x;B.y+=a*A.y;B.z+=a*A.z;
return B;}vec vunit(A)vec A;{return vcomb(1./sqrt( vdot(A,A)),A,black);}struct sphere*intersect
(P,D)vec P,D;{best=0;tmin=1e30;s= sph+5;while(s-->sph)b=vdot(D,U=vcomb(-1.,P,s->cen)),
u=b*b-vdot(U,U)+s->rad*s ->rad,u=u>0?sqrt(u):1e31,u=b-u>1e-7?b-u:b+u,tmin=u>=1e-7&&
u<tmin?best=s,u: tmin;return best;}vec trace(level,P,D)vec P,D;{double d,eta,e;vec N,color;
struct sphere*s,*l;if(!level--)return black;if(s=intersect(P,D));else return amb;color=amb;eta=
s->ir;d= -vdot(D,N=vunit(vcomb(-1.,P=vcomb(tmin,D,P),s->cen )));if(d<0)N=vcomb(-1.,N,black),
eta=1/eta,d= -d;l=sph+5;while(l-->sph)if((e=l ->kl*vdot(N,U=vunit(vcomb(-1.,P,l->cen))))>0&&
intersect(P,U)==l)color=vcomb(e ,l->color,color);U=s->color;color.x*=U.x;color.y*=U.y;color.z
*=U.z;e=1-eta* eta*(1-d*d);return vcomb(s->kt,e>0?trace(level,P,vcomb(eta,D,vcomb(eta*d-
sqrt (e),N,black))):black,vcomb(s->ks,trace(level,P,vcomb(2*d,N,D)),vcomb(s->kd, color,vcomb
(s->kl,U,black))));}main(){printf("%d %d\n",32,32);while(yx<32*32) U.x=yx%32-32/2,U.z=32/2-
yx++/32,U.y=32/2/tan(25/114.5915590261),U=vcomb(255., trace(3,black,vunit(U)),black),printf
("%.0f %.0f %.0f\n",U);}/*minray!*/
```

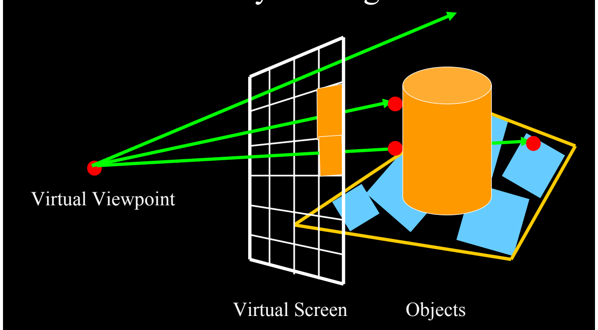
Outline

- *Camera Ray Casting (choosing ray directions) [2.3]*
- Ray-object intersections [2.4]
- Ray-tracing transformed objects [2.4]
- Lighting calculations [2.5]
- Recursive ray tracing [2.6]

Outline in Code

```
Image Raytrace (Camera cam, Scene scene, int width, int height)
{
    Image image = new Image (width, height) ;
    for (int i = 0 ; i < height ; i++)
        for (int j = 0 ; j < width ; j++) {
                Ray ray = RayThruPixel (cam, i, j) ;
                Intersection hit = Intersect (ray, scene) ;
                image[i][j] = FindColor (hit) ;
                }
    return image ;
}
```

Ray Casting



Ray misses all objects: Pixel colored black

Ray intersects object: shade using color, lights, materials

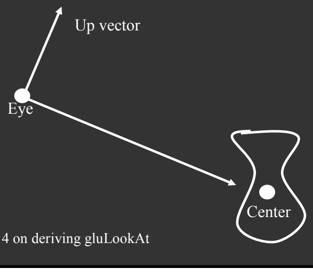
Multiple intersections: Use closest one (as does OpenGL)

Finding Ray Direction

- Goal is to find ray direction for given pixel i and j
- Many ways to approach problem
  - Objects in world coord, find dirn of each ray (we do this)
  - Camera in canonical frame, transform objects (OpenGL)
- Basic idea
  - Ray has origin (camera center) and direction
  - Find direction given camera params and i and j
- Camera params as in gluLookAt
  - Lookfrom[3], LookAt[3], up[3], fov

Similar to gluLookAt derivation

- gluLookAt(eyex, eyey, eyez, centerx, centery, centerz, upx, upy, upz)
- Camera at eye, looking at center, with up direction being up



From 4160 lecture 4 on deriving gluLookAt

Constructing a coordinate frame?

We want to associate **w** with **a**, and **v** with **b**

- But **a** and **b** are neither orthogonal nor unit norm
- And we also need to find **u**

$$w = \frac{a}{\|a\|}$$

$$u = \frac{b \times w}{\|b \times w\|}$$

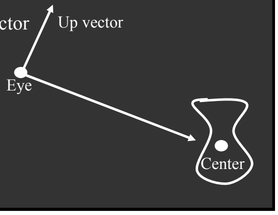
$$v = w \times u$$

Slide 20 from 4160 lecture 2

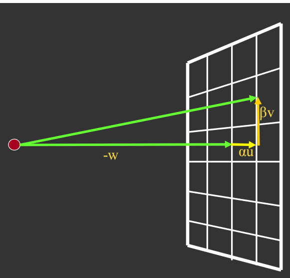
Camera coordinate frame

$$w = \frac{a}{\|a\|} \qquad u = \frac{b \times w}{\|b \times w\|} \qquad v = w \times u$$

- We want to position camera at origin, looking down –Z dirn
- Hence, vector **a** is given by **eye** – **center**
- The vector **b** is simply the **up** vector



Canonical viewing geometry



$$ray = eye + \frac{\alpha u + \beta v - w}{|\alpha u + \beta v - w|}$$

$$\alpha = \tan\left(\frac{fovx}{2}\right) \times \left(\frac{j - (width/2)}{width/2}\right) \qquad \beta = \tan\left(\frac{fovy}{2}\right) \times \left(\frac{(height/2) - i}{height/2}\right)$$

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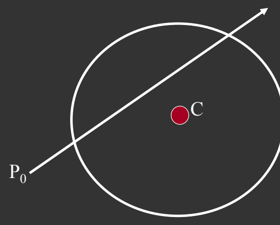
            }
    return image ;
}
```

Ray-Sphere Intersection

$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$sphere \equiv (\vec{P} - \vec{C}) \bullet (\vec{P} - \vec{C}) - r^2 = 0$$



Ray-Sphere Intersection

$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$sphere \equiv (\vec{P} - \vec{C}) \bullet (\vec{P} - \vec{C}) - r^2 = 0$$

Substitute

$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$sphere \equiv (\vec{P}_0 + \vec{P}_1 t - \vec{C}) \bullet (\vec{P}_0 + \vec{P}_1 t - \vec{C}) - r^2 = 0$$

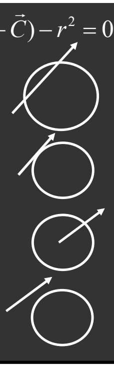
Simplify

$$t^2 (\vec{P}_1 \bullet \vec{P}_1) + 2t\, \vec{P}_1 \bullet (\vec{P}_0 - \vec{C}) + (\vec{P}_0 - \vec{C}) \bullet (\vec{P}_0 - \vec{C}) - r^2 = 0$$

Ray-Sphere Intersection

$$t^2 (\vec{P}_1 \bullet \vec{P}_1) + 2t\, \vec{P}_1 \bullet (\vec{P}_0 - \vec{C}) + (\vec{P}_0 - \vec{C}) \bullet (\vec{P}_0 - \vec{C}) - r^2 = 0$$

Solve quadratic equations for t



- 2 real positive roots: pick smaller root
- Both roots same: tangent to sphere
- One positive, one negative root: ray origin inside sphere (pick + root)
- Complex roots: no intersection (check discriminant of equation first)

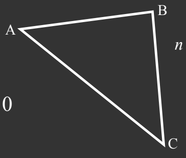
Ray-Sphere Intersection

- Intersection point: $ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$
- Normal (for sphere, this is same as coordinates in sphere frame of reference, useful other tasks)

$$normal = \frac{\vec{P} - \vec{C}}{|\vec{P} - \vec{C}|}$$

Ray-Triangle Intersection

- One approach: Ray-Plane intersection, then check if inside triangle
- Plane equation:

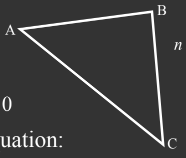


$$n = \frac{(C-A) \times (B-A)}{|(C-A) \times (B-A)|}$$

$$plane \equiv \vec{P} \bullet \vec{n} - \vec{A} \bullet \vec{n} = 0$$

Ray-Triangle Intersection

- One approach: Ray-Plane intersection, then check if inside triangle
- Plane equation:



$$n = \frac{(C-A) \times (B-A)}{|(C-A) \times (B-A)|}$$

$$plane \equiv \vec{P} \bullet \vec{n} - \vec{A} \bullet \vec{n} = 0$$

- Combine with ray equation:

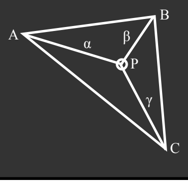
$$ray \equiv \vec{P} = \vec{P}_0 + \vec{P}_1 t$$

$$(\vec{P}_0 + \vec{P}_1 t) \bullet \vec{n} = \vec{A} \bullet \vec{n}$$

$$t = \frac{\vec{A} \bullet \vec{n} - \vec{P}_0 \bullet \vec{n}}{\vec{P}_1 \bullet \vec{n}}$$

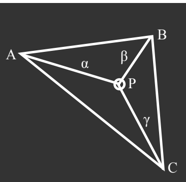
Ray inside Triangle

- Once intersect with plane, still need to find if in triangle
- Many possibilities for triangles, general polygons (point in polygon tests)
- We find parametrically [barycentric coordinates]. Also useful for other applications (texture mapping)



$$P = \alpha A + \beta B + \gamma C$$
$$\alpha \geq 0, \beta \geq 0, \gamma \geq 0$$
$$\alpha + \beta + \gamma = 1$$

Ray inside Triangle



$$P = \alpha A + \beta B + \gamma C$$
$$\alpha \geq 0, \beta \geq 0, \gamma \geq 0$$
$$\alpha + \beta + \gamma = 1$$

$$P - A = \beta(B - A) + \gamma(C - A)$$
$$0 \leq \beta \leq 1 \;,\; 0 \leq \gamma \leq 1$$
$$\beta + \gamma \leq 1$$

Other primitives

- Much early work in ray tracing focused on ray-primitive intersection tests
- Cones, cylinders, ellipsoides
- Boxes (especially useful for bounding boxes)
- General planar polygons
- Many more
- Consult chapter in Glassner (handed out) for more details and possible extra credit

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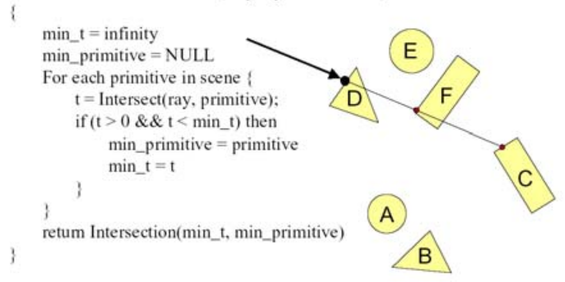
Ray Scene Intersection



```
Intersection FindIntersection(Ray ray, Scene scene)
{
    min_t = infinity
    min_primitive = NULL
    For each primitive in scene {
        t = Intersect(ray, primitive);
        if (t > 0 && t < min_t) then
            min_primitive = primitive
            min_t = t
        }
    }
    return Intersection(min_t, min_primitive)
}
```

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Transformed Objects

- E.g. transform sphere into ellipsoid
- Could develop routine to trace ellipsoid (compute parameters after transformation)
- May be useful for triangles, since triangle after transformation is still a triangle in any case
- But can also use original optimized routines

Transformed Objects

- Consider a general 4x4 transform M
  - Will need to implement matrix stacks like in OpenGL
- Apply inverse transform M^{-1} to ray
  - Locations stored and transform in homogeneous coordinates
  - Vectors (ray directions) have homogeneous coordinate set to 0 [so there is no action because of translations]
- Do standard ray-surface intersection as modified
- Transform intersection back to actual coordinates
  - Intersection point p transforms as Mp
  - Distance to intersection if used may need recalculation
  - Normals n transform as $M^{-t}n$. Do all this before lighting

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                    }
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}
```
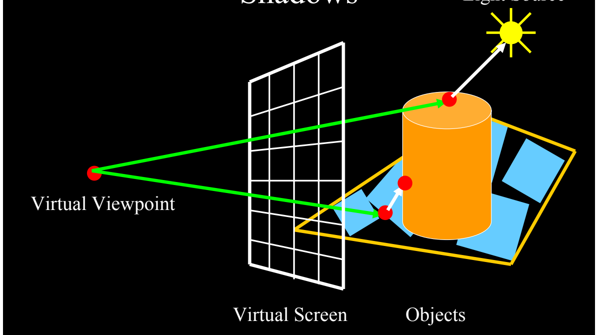

Shadows

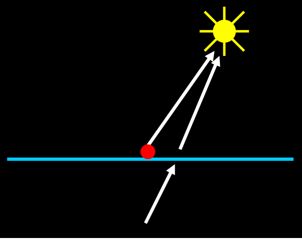


Shadow ray to light is unblocked: object visible

Shadow ray to light is blocked: object in shadow

Shadows: Numerical Issues

- Numerical inaccuracy may cause intersection to be below surface (effect exaggerated in figure)
- Causing surface to incorrectly shadow itself
- Move a little towards light before shooting shadow ray



Lighting Model

- Similar to OpenGL
- Lighting model parameters (global)
  - Ambient r g b (no per-light ambient as in OpenGL)
  - Attenuation const linear quadratic (like in OpenGL)

$$L = \frac{L_0}{const + lin * d + quad * d^2}$$

- Per light model parameters
  - Directional light (direction, RGB parameters)
  - Point light (location, RGB parameters)

Material Model

- Diffuse reflectance (r g b)
- Specular reflectance (r g b)
- Shininess s
- Emission (r g b)
- All as in OpenGL

Shading Model

$$I = K_a + K_e + \sum_{i=1}^{n} V_i L_i (K_d \max(l_i \bullet n, 0) + K_s (\max(h_i \bullet n, 0))^s)$$

- Global ambient term, emission from material
- For each light, diffuse specular terms
- Note visibility/shadowing for each light (not in OpenGL)
- Evaluated per pixel per light (not per vertex)

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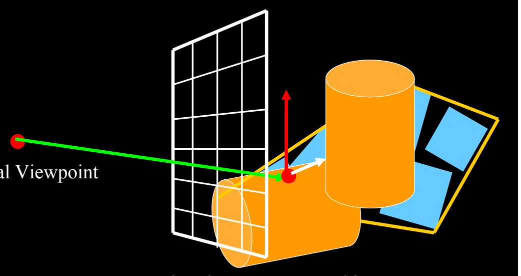
Mirror Reflections/Refractions

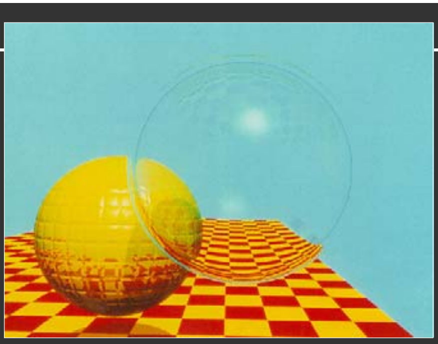


Virtual Viewpoint

Virtual Screen

Objects

Generate reflected ray in mirror direction,
Get reflections and refractions of objects



Turner Whitted 1980

Basic idea

For each pixel

- Trace Primary Eye Ray, find intersection
- Trace Secondary Shadow Ray(s) to all light(s)
  - Color = Visible ? Illumination Model : 0 ;
- Trace Reflected Ray
  - Color += reflectivity * Color of reflected ray

Recursive Shading Model

$$I = K_a + K_e + \sum_{i=1}^{n} V_i L_i (K_d \max(l_i \bullet n, 0) + K_s (\max(h_i \bullet n, 0))^s) + K_s I_R + K_T I_T$$

- Highlighted terms are recursive specularities [mirror reflections] and transmission (latter is extra credit)
- Trace secondary rays for mirror reflections and refractions, include contribution in lighting model
- GetColor calls RayTrace recursively (the I values in equation above of secondary rays are obtained by recursive calls)

Problems with Recursion

- Reflection rays may be traced forever
- Generally, set maximum recursion depth
- Same for transmitted rays (take refraction into account)

Effects needed for Realism

- (Soft) Shadows
- Reflections (Mirrors and Glossy)
- Transparency (Water, Glass)
- Interreflections (Color Bleeding)
- Complex Illumination (Natural, Area Light)
- Realistic Materials (Velvet, Paints, Glass)

Discussed in this lecture so far
Not discussed but possible with distribution ray tracing
Hard (but not impossible) with ray tracing; radiosity methods

Some basic add ons

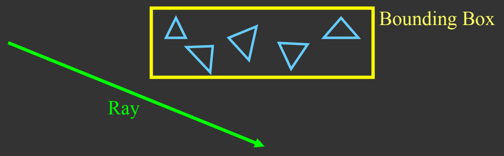
- Area light sources and soft shadows: break into grid of n x n point lights
  - Use jittering: Randomize direction of shadow ray within small box for given light source direction
  - Jittering also useful for antialiasing shadows when shooting primary rays
- More complex reflectance models
  - Simply update shading model
  - But at present, we can handle only mirror global illumination calculations

Acceleration

Testing each object for each ray is slow
- Fewer Rays
  Adaptive sampling, depth control
- Generalized Rays
  Beam tracing, cone tracing, pencil tracing etc.
- Faster Intersections
  - Optimized Ray-Object Intersections
  - ***Fewer Intersections***

Acceleration Structures

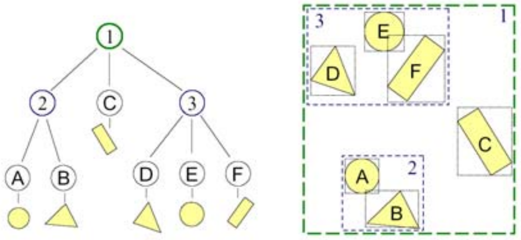
Bounding boxes (possibly hierarchical)
If no intersection bounding box, needn't check objects



Spatial Hierarchies (Oct-trees, kd trees, BSP trees)

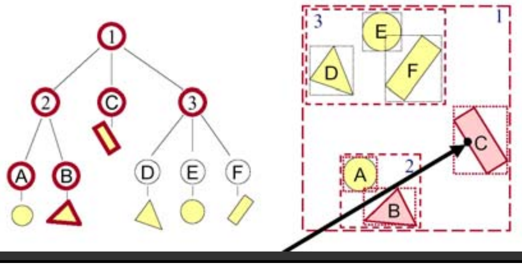
Bounding Volume Hierarchies 1

- Build hierarchy of bounding volumes
  - Bounding volume of interior node contains all children



Bounding Volume Hierarchies 2

- Use hierarchy to accelerate ray intersections
  - Intersect node contents only if hit bounding volume

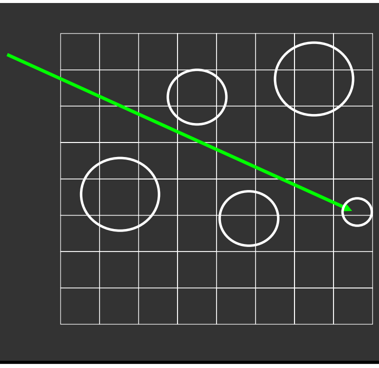


Bounding Volume Hierarchies 3

- Sort hits & detect early termination

```
FindIntersection(Ray ray, Node node)
{
    // Find intersections with child node bounding volumes
    ...
    // Sort intersections front to back
    ...
    // Process intersections (checking for early termination)
    min_t = infinity;
    for each intersected child i {
        if (min_t < bv_t[i]) break;
        shape_t = FindIntersection(ray, child);
        if (shape_t < min_t) { min_t = shape_t;}
    }
    return min_t;
}
```

Acceleration Structures: Grids

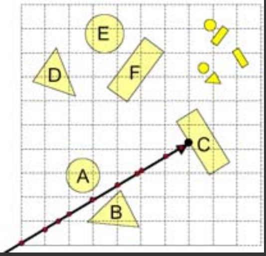


Uniform Grid: Problems

- Potential problem:
  - How choose suitable grid resolution?

Too little benefit
if grid is too coarse

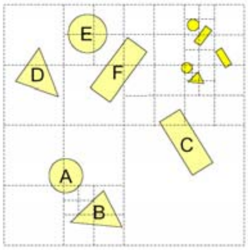
Too much cost
if grid is too fine



Octree

- Construct adaptive grid over scene
  - Recursively subdivide box-shaped cells into 8 octants
  - Index primitives by overlaps with cells

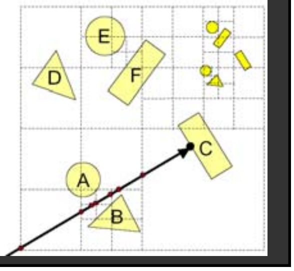
Generally fewer cells



Octree traversal

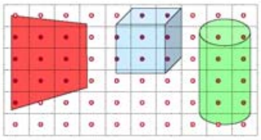
- Trace rays through neighbor cells
  - Fewer cells
  - More complex neighbor finding

Trade-off fewer cells for more expensive traversal



Other Accelerations

- Screen space coherence
  - Check last hit first
  - Beam tracing
  - Pencil tracing
  - Cone tracing
- Memory coherence
  - Large scenes
- Parallelism
  - Ray casting is “embarassingly parallelizable”
- etc.



Interactive Raytracing

- Ray tracing historically slow
- Now viable alternative for complex scenes
  - Key is sublinear complexity with acceleration; need not process all triangles in scene
- Allows many effects hard in hardware
- OpenRT project real-time ray tracing (http://www.openrt.de)

Raytracing on Graphics Hardware

- Modern Programmable Hardware general streaming architecture
- Can map various elements of ray tracing
- Kernels like eye rays, intersect etc.
- In vertex or fragment programs
- Convergence between hardware, ray tracing

[Purcell et al. 2002, 2003]

http://graphics.stanford.edu/papers/photongfx

Ring - Stencil Routing

8s @ 512x384, 16K photons

Cornell Box - Bitonic Sort

64s @ 512x512, 65K photons

Glass Ball - Stencil Routing

11s @ 512x384, 5K photons

Cornell Box - Increased Search Radius