Computer Graphics (Fall 2008)

COMS 4160, Lecture 18: Illumination and Shading 1

http://www.cs.columbia.edu/~cs4160

Rendering: 1960s (visibility)

- Roberts (1963), Appel (1967) - hidden-line algorithms
- Warnock (1969), Watkins (1970) - hidden-surface
- Sutherland (1974) - visibility = sorting

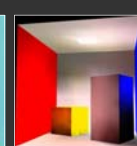
Images from FvDFH, Pixar's Shutterbug
Slide ideas for history of Rendering courtesy Marc Levoy

Rendering: 1970s (lighting)

1970s - raster graphics
- Gouraud (1971) - diffuse lighting, Phong (1974) - specular lighting
- Blinn (1974) - curved surfaces, texture
- Catmull (1974) - Z-buffer hidden-surface algorithm

Rendering (1980s, 90s: Global Illumination)

early 1980s - global illumination
- Whitted (1980) - ray tracing
- Goral, Torrance et al. (1984) radiosity
- Kajiya (1986) - the rendering equation


Outline

- *Preliminaries*
- *Basic diffuse and Phong shading*
- Gouraud, Phong interpolation, smooth shading
- Formal reflection equation

For today's lecture, slides and chapter 9 in textbook

Motivation

- Objects not flat color, perceive shape with appearance
- Materials interact with lighting
- Compute correct shading pattern based on lighting
  - This is not the same as shadows (separate topic)
- Some of today's lecture review of last OpenGL lec.
  - Idea is to discuss illumination, shading independ. OpenGL
- Today, initial hacks (1970-1980)
  - Next lecture: formal notation and physics

Linear Relationship of Light

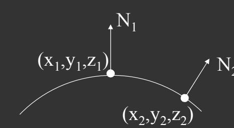
- Light energy is simply sum of all contributions

$$I = \sum_k I_k$$

- Terms can be calculated separately and later added:
  - multiple light sources
  - multiple interactions (diffuse, specular, more later)
  - multiple colors (R-G-B, or per wavelength)

General Considerations

Surfaces have a position, and a normal at every point.

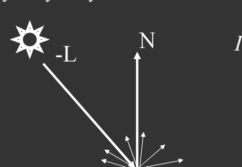


Other vectors used

- L = vector to the light source
  light position minus surface point position
- E = vector to the viewer (eye)
  viewer position minus surface point position

Diffuse Lambertian Term

- Rough matte (technically Lambertian) surfaces
  - Not shiny: matte paint, unfinished wood, paper, …
- Light reflects equally in all directions
- Obey Lambert's cosine law
  - Not exactly obeyed by real materials

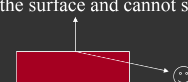


$$I \sim N \bullet L$$

Meaning of negative dot products

- If (N dot L) is negative, then the light is behind the surface, and cannot illuminate it.

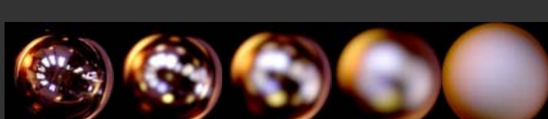
- If (N dot E) is negative, then the viewer is looking at the underside of the surface and cannot see it's front-face.



- In both cases, I is clamped to Zero.

Phong Illumination Model

- Specular or glossy materials: highlights
  - Polished floors, glossy paint, whiteboards
  - For plastics highlight is color of light source (not object)
  - For metals, highlight depends on surface color
- Really, (blurred) reflections of light source



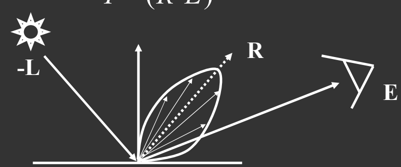
Roughness

Idea of Phong Illumination

- Find a simple way to create highlights that are view-dependent and happen at about the right place
- Not physically based
- Use dot product (cosine) of eye and reflection of light direction about surface normal
- Alternatively, dot product of half angle and normal
- Raise cosine lobe to some power to control sharpness

Phong Formula

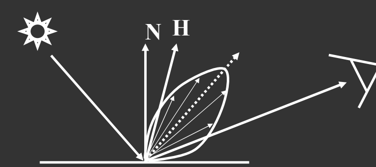
$$I \sim (R \bullet E)^p$$



$R = ?$ $\qquad$ $R = -L + 2(L \bullet N)N$

Alternative: Half-Angle (Blinn-Phong)

$$I \sim (N \bullet H)^p$$



- In practice, both diffuse and specular components

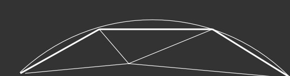
Outline

- Preliminaries
- Basic diffuse and Phong shading
- *Gouraud, Phong interpolation, smooth shading*
- Formal reflection equation

Not in text. If interested, look at FvDFH pp 736-738

Triangle Meshes as Approximations

- Most geometric models large collections of triangles.
- Triangles have 3 vertices with position, color, normal
- Triangles are approximation to actual object surface



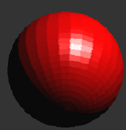
Vertex Shading

- We know how to calculate the light intensity given:
  - surface position
  - normal
  - viewer position
  - light source position (or direction)
- 2 ways for a vertex to get its normal:
  - given when the vertex is defined
  - take normals from faces that share vertex, and average

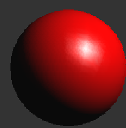
Coloring Inside the Polygon

- How do we shade a triangle between it's vertices, where we aren't given the normal?
- Inter-vertex interpolation can be done in object space (along the face), but it is simpler to do it in image space (along the screen).

Flat vs. Gouraud Shading



glShadeModel(GL_FLAT)



glShadeModel(GL_SMOOTH)

Flat - Determine that each face has a single normal, and color the entire face a single value, based on that normal.

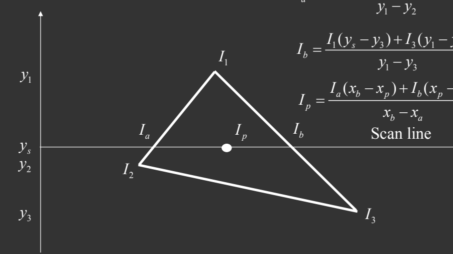
Gouraud – Determine the color at each vertex, using the normal at that vertex, and interpolate linearly for the pixels between the vertex locations.

Gouraud Shading – Details

$$I_a = \frac{I_1(y_s - y_2) + I_2(y_1 - y_s)}{y_1 - y_2}$$

$$I_b = \frac{I_1(y_s - y_3) + I_3(y_1 - y_s)}{y_1 - y_3}$$

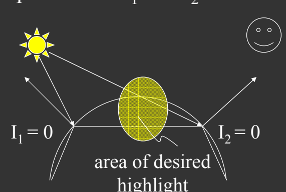
$$I_p = \frac{I_a(x_b - x_p) + I_b(x_p - x_a)}{x_b - x_a}$$



Actual implementation efficient: difference equations while scan converting

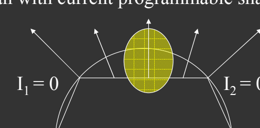
Gouraud and Errors

- $I_1 = 0$ because (N dot E) is negative.
- $I_2 = 0$ because (N dot L) is negative.
- Any interpolation of I_1 and I_2 will be 0.



2 Phongs make a Highlight

- Besides the Phong Reflectance model ($\cos^n$), there is a Phong Shading model.
- Phong Shading: Instead of interpolating the intensities between vertices, interpolate the ***normals***.
- The entire lighting calculation is performed for each pixel, based on the interpolated normal. (OpenGL doesn't do this, but you can with current programmable shaders)



Problems with Interpolated Shading

- Silhouettes are still polygonal
- Interpolation in screen, not object space: perspective distortion
- Not rotation or orientation-independent
- How to compute vertex normals for sharply curving surfaces?

- But at end of day, polygons are mostly preferred to explicitly representing curved objects like spline patches for rendering

Outline

- Preliminaries
- Basic diffuse and Phong shading
- Gouraud, Phong interpolation, smooth shading
- *Formal reflection equation*

Motivation

- Lots of ad-hoc tricks for shading
  - – Kind of looks right, but?
- Physics of light transport
  - – Will lead to formal reflection equation
- One of the more formal lectures
  - – But important to solidify theoretical framework

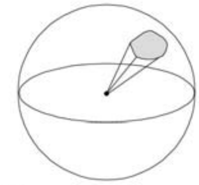
Angles and Solid Angles

- **Angle** $\theta = \frac{l}{r}$

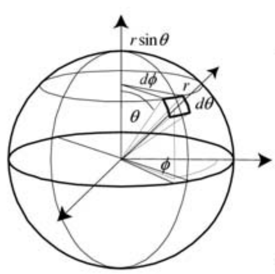
  $\Rightarrow$ **circle has 2π radians**

- **Solid angle** $\Omega = \frac{A}{R^2}$

  $\Rightarrow$ **sphere has 4π steradians**



Differential Solid Angles



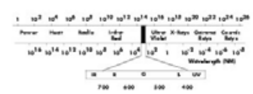
$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = 4\pi$$

The Light Field

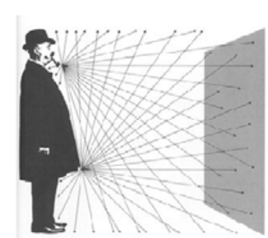
Electromagnetic waves and power spectrum



Ignore polarization

Ignore photons

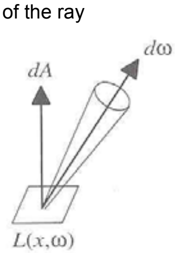
Spatial distribution



From London and Upton

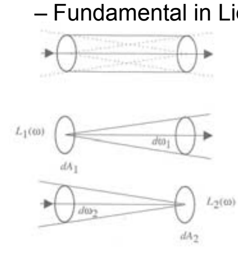
Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray
- Symbol: $L(x,\omega)$ (W/m^2 sr)
- Flux given by
  $d\Phi = L(x,\omega) \cos\theta \, d\omega \, dA$



Radiance properties

- Radiance is constant as it propagates along ray
  - – Derived from conservation of flux
  - – Fundamental in Light Transport.



$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

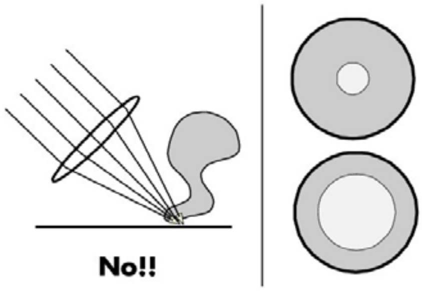
$$d\omega_1 = dA_2 / r^2 \qquad d\omega_2 = dA_1 / r^2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

Quiz

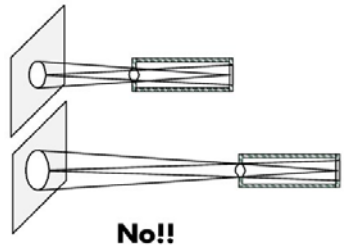
Does radiance increase under a magnifying glass?



No!!

Quiz

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?



No!!

Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)
  - Far away surface: See more, but subtends smaller angle
  - Wall equally bright across viewing distances

Consequences
  - Radiance associated with rays in a ray tracer
  - Other radiometric quants derived from radiance

Irradiance, Radiosity

- Irradiance E is radiant power per unit area
- Integrate incoming radiance over hemisphere
  - Projected solid angle ($\cos\theta\, d\omega$)
  - Uniform illumination: Irradiance = π [CW 24,25]
  - Units: W/m^2
- Radiosity
  - Power per unit area leaving surface (like irradiance)

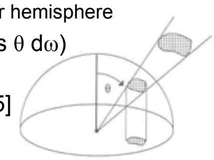


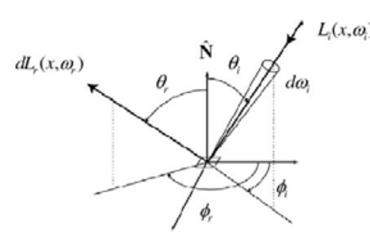
Figure 2.8: *Projection of differential area.*

Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing light energy
- We have already seen special cases: Lambertian, Phong
- In this lecture, we study all this abstractly

The BRDF

Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i} \quad \left[\frac{1}{sr}\right]$$

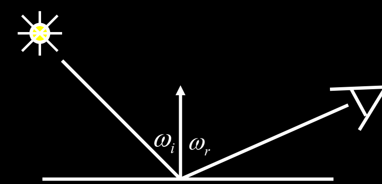
BRDF

- Reflected Radiance proportional to Irradiance
- Constant proportionality: BRDF [CW pp 28,29]
  - Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr

$$f(\omega_i,\omega_r)=\frac{L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i d\omega_i}$$

$$L_r(\omega_r)=L_i(\omega_i)f(\omega_i,\omega_r)\cos\theta_i d\omega_i$$

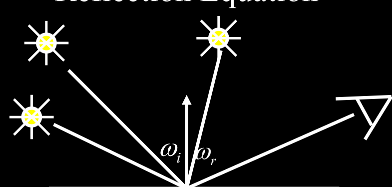
Reflection Equation



$$L_r(\omega_r)=L_i(\omega_i)f(\omega_i,\omega_r)(\omega_i\bullet n)$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle

Reflection Equation

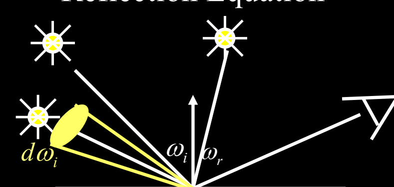


Sum over all light sources

$$L_r(\omega_r)=\sum_i L_i(\omega_i)f(\omega_i,\omega_r)(\omega_i\bullet n)$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle

Reflection Equation



Replace sum with integral

$$L_r(\omega_r)=\int_\Omega L_i(\omega_i)f(\omega_i,\omega_r)(\omega_i\bullet n)\,d\omega_i$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle