

# Computer Graphics (Spring 2008)

COMS 4160, Lecture 20: Illumination and Shading 2  
<http://www.cs.columbia.edu/~cs4160>

## BRDF

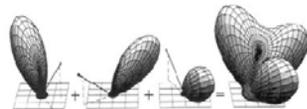
- Reflected Radiance proportional to Irradiance
- Constant proportionality: BRDF [CW pp 28,29]
  - Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr

$$f(\omega_i, \omega_r) = \frac{L_r(\omega_r)}{L_i(\omega_i) \cos \theta_i d\omega_i}$$

$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i, \omega_r) \cos \theta_i d\omega_i$$

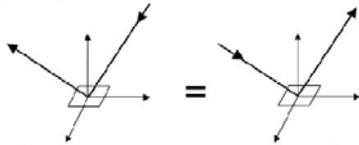
### Properties of BRDF's

#### 1. Linear



From Sillion, Arvo, Westin, Greenberg

#### 2. Reciprocity principle $f_r(\omega_i \rightarrow \omega_r) = f_r(\omega_r \rightarrow \omega_i)$



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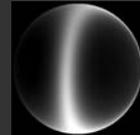
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### Isotropic vs Anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction



Isotropic

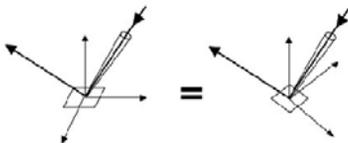


Anisotropic

### Properties of BRDF's

#### 3. Isotropic vs. anisotropic

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_r, \theta_i, \phi_r - \phi_i)$$



Reciprocity and isotropy

$$f_r(\theta_i, \theta_r, \phi_r - \phi_i) = f_r(\theta_r, \theta_i, \phi_i - \phi_r) = f_r(\theta_i, \theta_r, |\phi_r - \phi_i|)$$

#### 4. Energy conservation

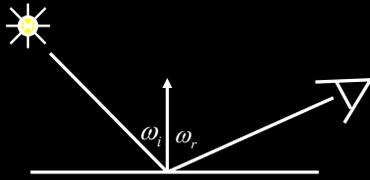
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## Radiometry

- Physical measurement of electromagnetic energy
- We consider light field
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models

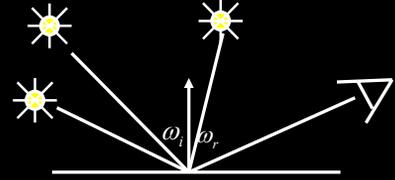
## Reflection Equation



$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Radiance (Output Image)    Incident radiance (from light source)    BRDF    Cosine of Incident angle

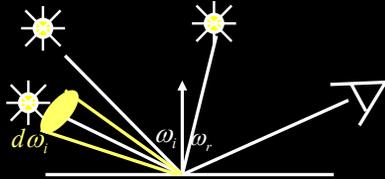
## Reflection Equation



$$L_r(\omega_r) = \sum_i L_i(\omega_i) f(\omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Radiance (Output Image)    <sup>Sum over all light sources</sup> Incident radiance (from light source)    BRDF    Cosine of Incident angle

## Reflection Equation



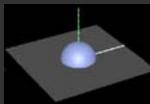
$$L_r(\omega_r) = \int_{\Omega} L_i(\omega_i) f(\omega_i, \omega_r) (\omega_i \cdot n) d\omega_i$$

Reflected Radiance (Output Image)    <sup>Replace sum with integral</sup> Incident radiance (from light source)    BRDF    Cosine of Incident angle

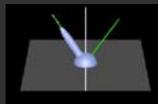
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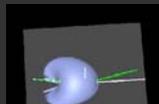
## Brdf Viewer plots



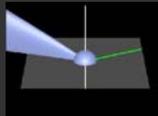
Diffuse



Torrance-Sparrow



Anisotropic



bv written by Szymon Rusinkiewicz

## Demo

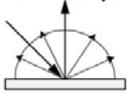


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## Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).



$$\begin{aligned} L_{r,d}(\omega_r) &= \int f_{r,d} L_i(\omega_i) \cos \theta_i d\omega_i \\ &= f_{r,d} \int L_i(\omega_i) \cos \theta_i d\omega_i \\ &= f_{r,d} E \end{aligned}$$

$$M = \int L_r(\omega_r) \cos \theta_r d\omega_r = L_r \int \cos \theta_r d\omega_r = \pi L_r$$

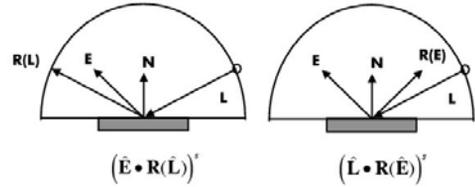
$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \Rightarrow f_{r,d} = \frac{\rho_d}{\pi}$$

**Lambert's Cosine Law**  $M = \rho_d E = \rho_d E \cos \theta_r$

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## Phong Model



$$\text{Reciprocity: } (\hat{E} \cdot \mathbf{R}(\hat{L}))^2 = (\hat{L} \cdot \mathbf{R}(\hat{E}))^2$$

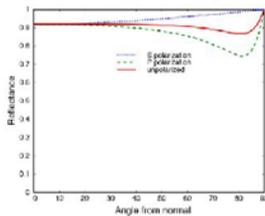
Distributed light source!

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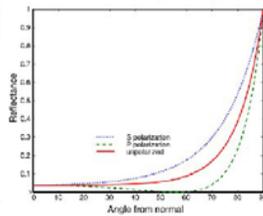
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## Fresnel Reflectance

Metal (Aluminum)



Dielectric (N=1.5)



Gold  $F(0)=0.82$   
Silver  $F(0)=0.95$

Glass  $n=1.5$   $F(0)=0.04$   
Diamond  $n=2.4$   $F(0)=0.15$

**Schlick Approximation**  $F(\theta) = F(0) + (1 - F(0))(1 - \cos \theta)^5$

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## Experiment

Reflections from a shiny floor



From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

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## Analytical BRDF: TS example

- One famous analytically derived BRDF is the Torrance-Sparrow model.
- T-S is used to model specular surface, like the Phong model.
  - more accurate than Phong
  - has more parameters that can be set to match different materials
  - derived based on assumptions of underlying geometry. (instead of 'because it works well')

## Torrance-Sparrow

- Assume the surface is made up of grooves at the microscopic level.



- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena



Shadowing Masking Interreflection

## Torrance-Sparrow Result

Fresnel term:  
allows for wavelength  
dependency

Geometric Attenuation:  
reduces the output based on the  
amount of shadowing or masking  
that occurs.

$$f = \frac{F(\theta_i)G(\omega_i, \omega_r)D(\theta_v)}{4 \cos(\theta_i) \cos(\theta_r)}$$

Distribution:  
distribution function  
determines what  
percentage of  
microfacets are  
oriented to reflect in  
the viewer direction.

How much of the  
macroscopic surface  
is visible to the light  
source

How much of the  
macroscopic  
surface is visible  
to the viewer

## Other BRDF models

- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research

## Complex Lighting

- So far we've looked at simple, discrete light sources.
- Real environments contribute many colors of light from many directions.
- The complex lighting of a scene can be captured in an Environment map.
  - Just paint the environment on a sphere.

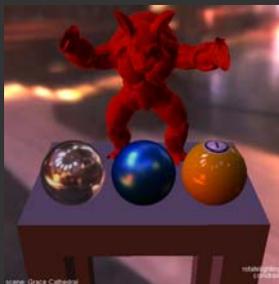
## Environment Maps

- Instead of determining the lighting direction by knowing what lights exist, determine what light exists by knowing the lighting direction.



Blinn and Newell 1976, Miller and Hoffman, 1984  
Later, Greene 86, Cabral et al. 87

## Demo



## Conclusion

- All this (OpenGL, physically based) are local illumination and shading models
- Good lighting, BRDFs produce convincing results
  - Matrix movies, modern realistic computer graphics
- Do not consider global effects like shadows, interreflections (from one surface on another)
  - Subject of next unit (global illumination)

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## What's Next

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- Have finished basic material for class
- Review of illumination and Shading
- Remaining topics are global illumination (written assignment 2): Lectures on rendering eq, radiosity
- Historical movie: Story of Computer Graphics
- Likely to finish these by April 21: No class April 28,
- Work instead on HW 4, written assignments
- May 5 will be demo session for HW 4