Computer Graphics (Fall 2006)

COMS 4160, Lecture 17: Illumination and Shading 2

http://www.cs.columbia.edu/~cs4160

Lecture includes number of slides from other sources: Hence different color scheme to be compatible with these other sources

To Do

- Submit HW 3, do well
- Start early on HW 4

Outline

- Preliminaries
- Basic diffuse and Phong shading
- Gouraud, Phong interpolation, smooth shading
- *Formal reflection equation*
- Texture mapping (next week)
- Global illumination (next unit)

See handout (chapter 2 of Cohen and Wallace)

Motivation

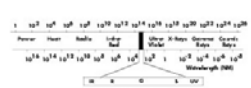
- Lots of ad-hoc tricks for shading
  – Kind of looks right, but?
- Study this more formally
- Physics of light transport
  – Will lead to formal reflection equation
- One of the more technical/theoretical lectures
  – But important to solidify theoretical framework

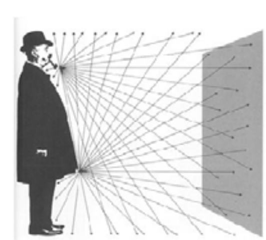
Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing light energy
- We have already seen special cases: Lambertian, Phong
- In this lecture, we study all this abstractly

The Light Field

Electromagnetic waves and power spectrum





Ignore polarization

Ignore photons

Spatial distribution

From London and Upton

CS348B Lecture 4 — Pat Hanrahan, Spring 2002

Radiometry

- Physical measurement of electromagnetic energy
- We consider light field
  - *Radiance, Irradiance*
  - *Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF*
  - Reflection Equation
  - Simple BRDF models

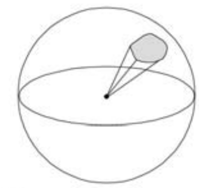
Angles and Solid Angles

- **Angle** $\theta = \frac{l}{r}$

  $\Rightarrow$ **circle has 2π radians**

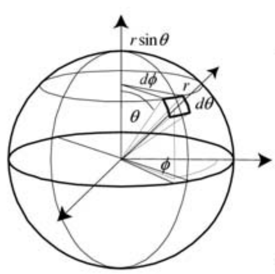
- **Solid angle** $\Omega = \frac{A}{R^2}$

  $\Rightarrow$ **sphere has 4π steradians**



CS348B Lecture 4 — Pat Hanrahan, Spring 2002

Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2 \sin\theta\,d\theta\,d\phi$$

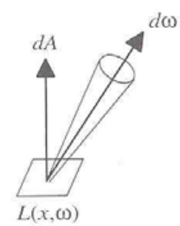
$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = 4\pi$$

CS348B Lecture 4 — Pat Hanrahan, Spring 2002

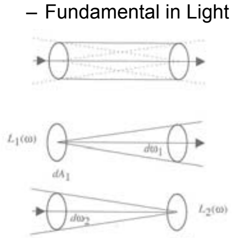
Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray
- Symbol: $L(x,\omega)$ (W/m^2 sr)
- Flux given by
  $d\Phi = L(x,\omega) \cos\theta \, d\omega \, dA$



Radiance properties

- Radiance is constant as it propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.



$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

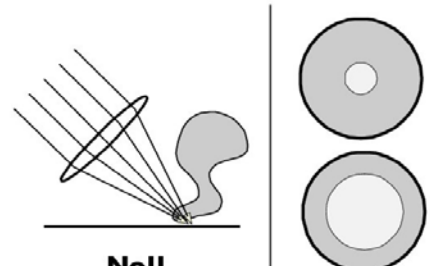
$$d\omega_1 = dA_2 / r^2 \qquad d\omega_2 = dA_1 / r^2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

Quiz

Does radiance increase under a magnifying glass?

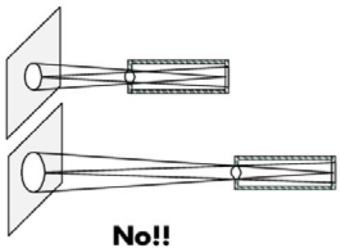


No!!

CS348B Lecture 4 — Pat Hanrahan, Spring 2002

Quiz

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?



No!!

CS348B Lecture 4 Pat Hanrahan, Spring 2002

Radiance properties

- Sensor response proportional to surface radiance (constant of proportionality is throughput)
  - Far away surface: See more, but subtends smaller angle
  - Wall is equally bright across range of viewing distances

Consequences
  - Radiance associated with rays in a ray tracer
  - All other radiometric quantities derived from radiance

Irradiance, Radiosity

- Irradiance E is the radiant power per unit area
- Integrate incoming radiance over hemisphere
  - Projected solid angle (cos θ dω)
  - Uniform illumination: Irradiance = π [CW 24,25]
  - Units: W/m^2
- Radiosity
  - Power per unit area leaving surface (like irradiance)

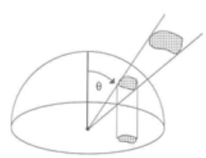
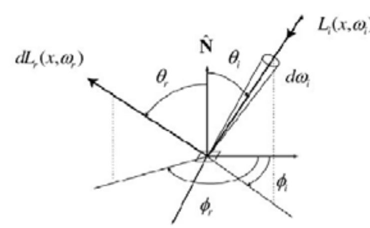


Figure 2.8: *Projection of differential area.*

The BRDF

Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i} \quad \left[\frac{1}{sr}\right]$$

CS348B Lecture 10 Pat Hanrahan, Spring 2002

BRDF

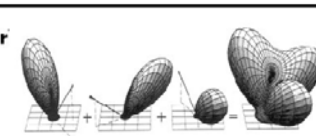
- Reflected Radiance proportional to Irradiance
- Constant proportionality: BRDF [CW pp 28,29]
  - Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr

$$f(\omega_i, \omega_r) = \frac{L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i d\omega_i}$$

$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i, \omega_r)\cos\theta_i d\omega_i$$

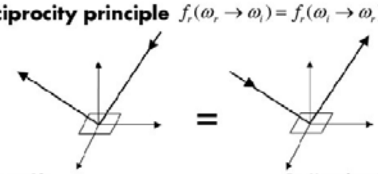
Properties of BRDF's

1. Linear



From Sillion, Arvo, Westin, Greenberg

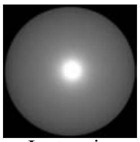
2. Reciprocity principle $f_r(\omega_r \to \omega_i) = f_r(\omega_i \to \omega_r)$



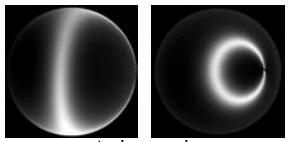
CS348B Lecture 10 Pat Hanrahan, Spring 2002

Isotropic vs Anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction



Isotropic

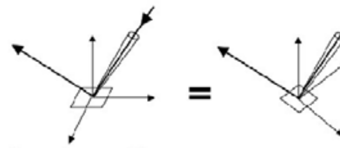


Anisotropic

Properties of BRDF's

3. Isotropic vs. anisotropic

$$f_r(\theta_i,\varphi_i;\theta_r,\varphi_r) = f_r(\theta_i,\theta_r,\varphi_r-\varphi_i)$$



Reciprocity *and* isotropy

$$f_r(\theta_i,\theta_r,\varphi_r-\varphi_i) = f_r(\theta_r,\theta_i,\varphi_i-\varphi_r) = f_r(\theta_i,\theta_r,|\varphi_r-\varphi_i|)$$

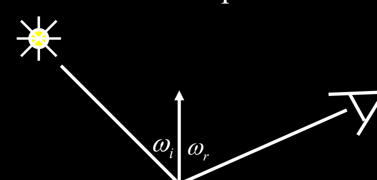
4. Energy conservation

CS348B Lecture 10 Pat Hanrahan, Spring 2002

Radiometry

- Physical measurement of electromagnetic energy
- We consider light field
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - *Reflection Equation*
  - Simple BRDF models

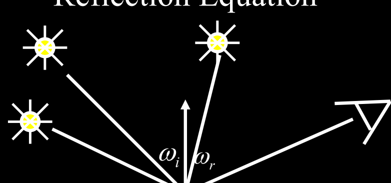
Reflection Equation



$$L_r(\omega_r) = L_i(\omega_i)\, f(\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle

Reflection Equation

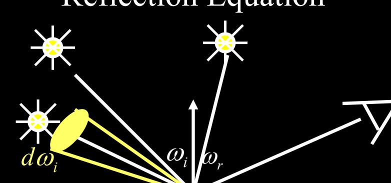


Sum over all light sources

$$L_r(\omega_r) = \sum_i L_i(\omega_i)\, f(\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle

Reflection Equation



Replace sum with integral

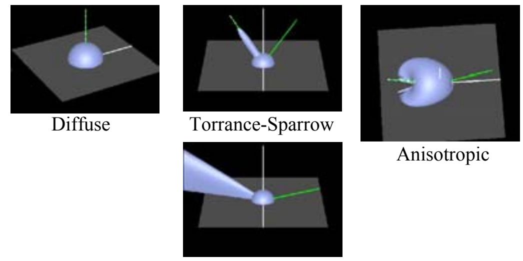
$$L_r(\omega_r) = \int_\Omega L_i(\omega_i)\, f(\omega_i,\omega_r)(\omega_i \bullet n)\, d\omega_i$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle

Radiometry

- Physical measurement of electromagnetic energy
- We consider light field
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - *Simple BRDF models*

Brdf Viewer plots



bv written by Szymon Rusinkiewicz

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).

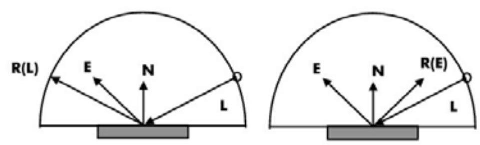
$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos\theta_i \, d\omega_i = f_{r,d} \int L_i(\omega_i) \cos\theta_i \, d\omega_i = f_{r,d} E$$

$$M = \int L_r(\omega_r) \cos\theta_r \, d\omega_r = L_r \int \cos\theta_r \, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \quad \Rightarrow \quad f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law $M = \rho_d E = \rho_d E_s \cos\theta_s$

Phong Model



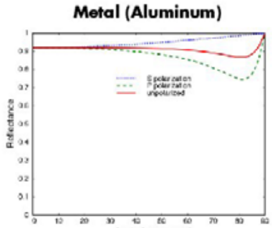
$(\hat{E} \bullet R(\hat{L}))^s$ $(\hat{L} \bullet R(\hat{E}))^s$

Reciprocity: $(\hat{E} \bullet R(\hat{L}))^s = (\hat{L} \bullet R(\hat{E}))^s$

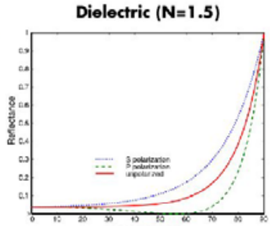
Distributed light source!

Fresnel Reflectance

Metal (Aluminum)



Dielectric (N=1.5)



Gold F(0)=0.82
Silver F(0)=0.95

Glass n=1.5 F(0)=0.04
Diamond n=2.4 F(0)=0.15

Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

Experiment

Reflections from a shiny floor

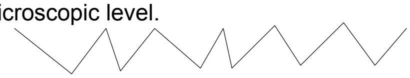
From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Analytical BRDF: TS example

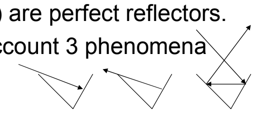
- One famous analytically derived BRDF is the Torrance-Sparrow model.
- T-S is used to model specular surface, like the Phong model.
  - more accurate than Phong
  - has more parameters that can be set to match different materials
  - derived based on assumptions of underlying geometry. (instead of 'because it works well')

Torrance-Sparrow

- Assume the surface is made up grooves at the microscopic level.



- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena



Shadowing Masking Interreflection

Torrance-Sparrow Result

Fresnel term: allows for wavelength dependency

Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.

$$f = \frac{F(\theta_i)G(\omega_i,\omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

Distribution: distribution function determines what percentage of microfacets are oriented to reflect in the viewer direction.

How much of the macroscopic surface is visible to the light source

How much of the macroscopic surface is visible to the viewer

Other BRDF models

- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research

Complex Lighting

- So far we've looked at simple, discrete light sources.
- Real environments contribute many colors of light from many directions.
- The complex lighting of a scene can be captured in an Environment map.
  - Just paint the environment on a sphere.

Environment Maps

- Instead of determining the lighting direction by knowing what lights exist, determine what light exists by knowing the lighting direction.

Blinn and Newell 1976, Miller and Hoffman, 1984
Later, Greene 86, Cabral et al. 87

Conclusion

- All this (OpenGL, physically based) are local illumination and shading models
- Good lighting, BRDFs produce convincing results
  - Matrix movies, modern realistic computer graphics
- Do not consider global effects like shadows, interreflections (from one surface on another)
  - Subject of next unit (global illumination)