COMS 6998-006 Network Theory **Columbia University, Spring 2008**

Lecture 18 - Title: Social Networks. Date: March 27, 2008

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1 Models of Social Networks

1.1 Classical Model

For a closed population, if S, I and R denote the fraction of the population that is susceptible, infected and recovering from a disease, then we have

$$\frac{dS}{dt} = -\beta SI \quad (1)$$
$$\frac{dI}{dt} = \beta SI - \delta I \quad (2)$$
$$\frac{dR}{dt} = \delta I \quad (3)$$
$$R_o = \frac{\beta}{\delta} \quad (4)$$
$$(5)$$

These equations can be simultaneously solved to give

$$S(t) = S(0)e^{R_o(R(t)-R(0))} \quad (6)$$
$$R_\infty = 1 - S(0)e^{-R_o(R_\infty - R(0))} \quad (7)$$
$$(8)$$

1.2 Alternative Models

The above classical model can be further refined using the following modifications:

1. Newborn Immunity - This accounts for added immunity for newborns in the population.
2. Persistent infectious state - This accounts for a small percent of the population that can persistently infect the whole population, even if they're not themselves diseased. (For. eg: Typhoid-Mary)
3. Short term immunity (S I S Model) - In this model, the population can revert from state S to state I and then back to S after recovery; useful for diseases like the common-cold.

1.3 Models of Diffusion

1.3.1 'Spread' of Obesity

Based on a study by Christakis and Fowler [**?**], the following observations were made:

- If your friends are obese, how likely are you obese?
- Homophily - People with similar tastes/interests/problems tend to cluster.
- Other factors that affect obesity - genes, geographic location, social customs, social influence and peer effects.

1.3.2 Adoption of a Technology

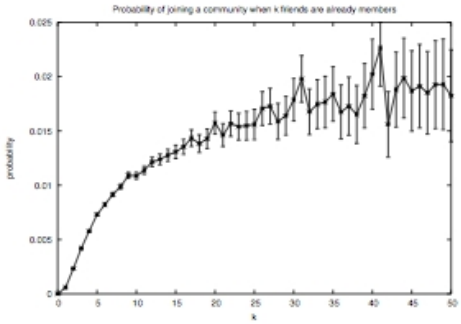


Figure 1: The probability p of joining a LiveJournal community as a function of the number of friends k already in the community. Error bars represent two standard errors.

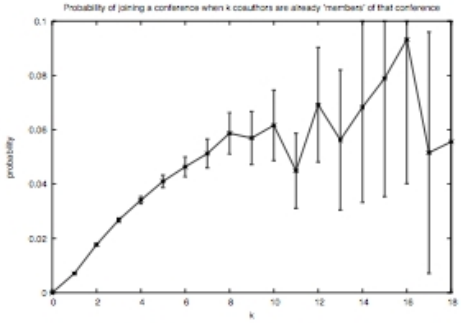


Figure 2: The probability p of joining a DBLP community as a function of the number of friends k already in the community. Error bars represent two standard errors.

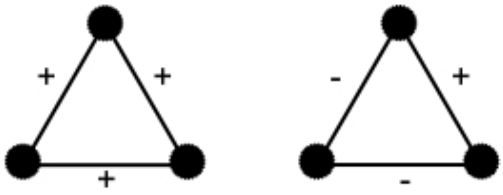
Figure 1: Plot of probability of adopting a certain technology vs the number of acquaintances that have adopted them[**?**].

1.4 Signed Networks

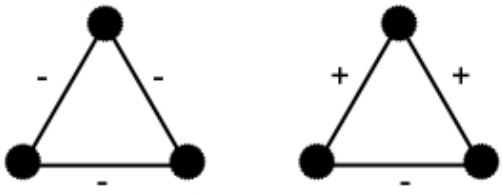
These networks have their interaction edges annotated with weights, usually positive, negative and neutral. It was observed [**?**] that triangles in any network tend to have an overall + sign - overall + indicates the triad is stable. A study of how friendship networks between countries, between the 1^{st} and 2^{nd} World Wars, evolve to achieve stable triads [**?**] is shown in Fig. **??**.

1.5 Evolution of Social Networks

Based primarily on the work by Kossinetts and Watts [**?**], where they studied the dynamics of email activity between students, faculty and staff at a large university, we can deduce that probability of a 'new' email between two people is proportional to the number of mutual acquaintances and the number of common classes they share.



(b) Stable signed triangles



(c) Unstable signed triangles

Figure 2: Stable and Unstable Triads

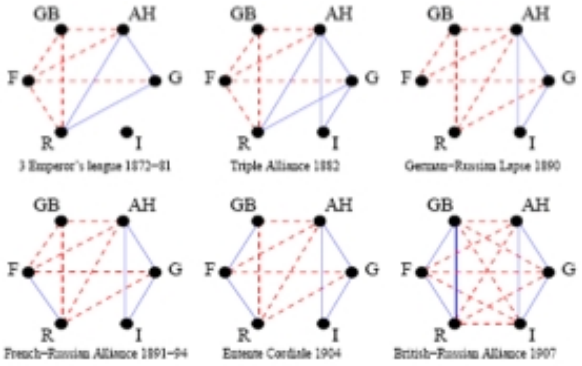


Fig. 10. Evolution of the major relationship changes between the protagonists of World War I from 1872–1907. Here GB = Great Britain, AH = Austria-Hungary, G = Germany, I = Italy, R = Russia, and F = France.

Figure 3: A historical example showing evolution of triads in friendship networks.

1.6 Weak Ties

A study by Mark Granovetter [**?**] showed how most people find their jobs through referrals by mere acquaintances than by close friends. He defined **strong ties** to be edges in a nearly-fully connected subgraph, **weak**

ties or 'bridges' to be edges between two modules or clusters and **missing ties** to be edges required to make a subgraph fully-connected. Missing ties are also created by *triadic closure* as the network clustering coefficient increases with time. He also found that a node's linkage to local bridges gives it structural advantages. For eg: a bilingual employee in a primarily monolingual company would be crucial when the company wants to globalize or collaborate with other partnerships.

1.7 Homophily

"The principle that contact between similar people occur at a higher rate than among dissimilar people"

There are two types of homophily according to Lazarsfeld and Merton: based on "status" (formal/informal) and based on "value" (attitude/beliefs).

For eg: AddHealth (http://www.cpc.unc.edu/projects/addhealth/

1.7.1 Race and ethnicity

Other studies observed a strong racial segregation in American schools [**?**], individuals who arrived more recently tend to have more homophilous networks [**?**] and most adults discuss "important matters" only with other adults of the same race.

1.7.2 Gender

Studies on gender based homophily showed that girls tend to resolve transitivity by deleting friendships while boys would add friendships. Also, gender based homophily was more prevalent among Blacks and Hispanics.

Other criteria that governed homophily also included age, religion, education, occupation, social class, sports interests etc.

1.8 Interesting Websites for Social Networks

- http://www.dyads.net/
- http://www.insna.org/
- http://netwiki.amath.unc.edu/

1.9 Readings

Miller McPherson, Lynn Smith-Lovin and James M Cook, *BIRDS OF A FEATHER: Homophily in Social Networks*, Annual Review of Sociology, **Vol. 27: 415-444 (Volume publication date August 2001)**