Diffusion Coded Photography: Supplementary Material

In this document we give derivations for several equations that were removed from the paper due to space constraints.

1 Diffuser with constant 2D Scatter Function

The first derivation we give is for the PSF of a Diffusion Coded camera with a constant 2D scatter function, as described in Section 3 of the paper. From Equation 4 in the paper, the kernel for this diffuser is

$$D(\bar{u},\bar{u}',\bar{x},\bar{x}') = \frac{1}{w^2}\delta(\bar{u}-\bar{u}')\sqcap\left(\frac{\bar{x}-\bar{x}'}{w}\right). \tag{1}$$

$$\hat{L}_\delta(\bar{u},\bar{x}) = \int_{\Omega_{\bar{u}}}\int_{\Omega_{\bar{x}}} \frac{1}{w^2}\delta(\bar{u}-\bar{u}')\sqcap\left(\frac{\bar{x}-\bar{x}'}{w}\right)\hat{L}_\delta(\bar{u}',\bar{x}')d\bar{u}'d\bar{x}' \tag{2}$$

$$= \frac{1}{w^2}\int_{\Omega_{\bar{x}}}\sqcap\left(\frac{\bar{x}-\bar{x}'}{w}\right)L_\delta(\bar{u},\bar{x}')d\bar{x}' \tag{3}$$

$$\hat{P}(\bar{x}) = \frac{1}{w^2}\int_{\Omega_{\bar{u}}}\int_{\Omega_{\bar{x}}}\sqcap\left(\frac{\bar{x}-\bar{x}'}{w}\right)L_\delta(\bar{u},\bar{x}')d\bar{x}'d\bar{u} \tag{4}$$

$$= \frac{1}{w^2}\int_{\Omega_{\bar{x}}}\sqcap\left(\frac{\bar{x}-\bar{x}'}{w}\right)\left[\int_{\Omega_{\bar{u}}}L_\delta(\bar{u},\bar{x}')d\bar{u}\right]d\bar{x}' \tag{5}$$

$$= \frac{1}{w^2}\sqcap\left(\frac{\bar{x}}{w}\right)\otimes P(\bar{x}) \tag{6}$$

Which is the same result as Equation 8 in the paper, the result being that the effect of the diffuser is to blur the image E that would be captured were it not present.

2 Radially-Symmetric Light Fields

In this section we verify mathematically that Equation 10 in Section 4 of the paper represents the light field of a unit energy point source. The equation for the light field is

$$L_\delta(\rho,r) = \frac{4}{\pi A^2}\sqcap\left(\frac{\rho}{A}\right)\frac{\delta(r-s_0\rho)}{\pi|r|}. \tag{7}$$

In polar coordinates, the energy e of a light field is calculated by integrating over all variables

$$e = \pi^2\int_{\Omega_\rho}\int_{\Omega_r}L_\delta(\rho,r)|\rho|d\rho|r|dr \tag{8}$$

or equivalently

$$e = \pi\int_{\Omega_r}P(r)|r|dr, \tag{9}$$

where $P(r)$ is the PSF resulting from the image of the point source L_δ. The PSF for the point source is

$$P(r) = \pi\int_{\Omega_\rho}\frac{4}{\pi A^2}\sqcap\left(\frac{\rho}{A}\right)\frac{\delta(r-s_0\rho)}{\pi|r|}|\rho|d\rho \tag{10}$$

$$= \frac{4}{\pi s_0^2A^2}\frac{1}{|r|}\int_{\Omega_\rho}\delta(r-\rho)\sqcap\left(\frac{\rho}{s_0A}\right)|\rho|d\rho. \tag{11}$$

The integral in Equation 11 is just a convolution between $\sqcap\left(\frac{r}{s_0A}\right)|r|$ and a delta function. Thus, the resulting PSF takes the familiar shape of a pillbox with diameter s_0A

$$P(r) = \frac{4}{\pi s_0^2A^2}\sqcap\left(\frac{r}{s_0A}\right). \tag{12}$$

The energy for the point source light field is then

$$e = \pi\int_{\Omega_r}\frac{4}{\pi s_0^2A^2}\sqcap\left(\frac{r}{s_0A}\right)|r|dr \tag{13}$$

$$= \frac{4}{s_0^2A^2}\int_{s_0A/2}^{-s_0A/2}|r|dr \tag{14}$$

$$= 1, \tag{15}$$

which verifies that the point source has unit energy.

3 Radially-Symmetric Diffuser

We now give a derivation for the PSF of a Diffusion Coded camera using the diffuser kernel from Equation 14 in Section 4 of the paper. The light field of a point source filtered by the radially symmetric kernel is

$$\hat{L}_\delta(\rho,r) = \pi^2\int_{\Omega_{\rho'}}\int_{\Omega_{r'}}D(\rho,\rho',r,r')L_\delta(\rho',r)|\rho'|d\rho'|r'|dr' \tag{16}$$

$$= \frac{4\pi}{A^2}\int_{\Omega_{\rho'}}\int_{\Omega_{r'}}\frac{\delta(\rho-\rho')}{\pi|\rho'|}\frac{\sqcap(\frac{r-r'}{w})}{\pi w|r|}\sqcap\left(\frac{\rho'}{A}\right)\frac{\delta(r'-s_0\rho')}{\pi|r'|}|\rho'|d\rho'|r'|dr' \tag{17}$$

$$= \frac{4}{\pi A^2}\sqcap\left(\frac{\rho}{A}\right)\frac{1}{w|r|}\int_{\Omega_r}\delta(r'-s_0\rho)\sqcap\left(\frac{r-r'}{w}\right)dr' \tag{18}$$

$$= \frac{4}{\pi A^2}\sqcap\left(\frac{\rho}{A}\right)\frac{\sqcap(\frac{r-s_0\rho}{w})}{\pi w|r|}. \tag{19}$$

The PSF for the light field filtered by this diffuser is

$$\hat{P}(r) = \pi\int_{\Omega_\rho}\hat{L}_\delta(\rho,r)|\rho|d\rho \tag{20}$$

$$= \pi\int_{\Omega_\rho}\frac{4}{\pi A^2}\frac{\sqcap(\frac{r-s_0\rho}{w})}{\pi w|r|}\sqcap\left(\frac{\rho}{A}\right)|\rho|d\rho \tag{21}$$

$$= \frac{4}{\pi A^2w|r|}\int_{\Omega_\rho}\sqcap(\frac{r-s_0\rho}{w})\sqcap\left(\frac{\rho}{A}\right)|\rho|d\rho \tag{22}$$

$$= \frac{4}{\pi s_0^2A^2w|r|}\int_{\Omega_\rho}\sqcap\left(\frac{r-\rho}{w}\right)\sqcap\left(\frac{\rho}{s_0A}\right)|\rho|d\rho \tag{23}$$

$$= \frac{4}{\pi s_0^2A^2}\frac{1}{w|r|}\left[\sqcap\left(\frac{r}{w}\right)\otimes\left(\sqcap\left(\frac{r}{s_0A}\right)\cdot|r|\right)\right], \tag{24}$$

which is the same result as the PSF given by Equation 16 in Section 4 of the paper.