Coded Aperture Pair for Depth from Defocus

Supplementary File

Online Submission ID: 1250

ICCV 2009

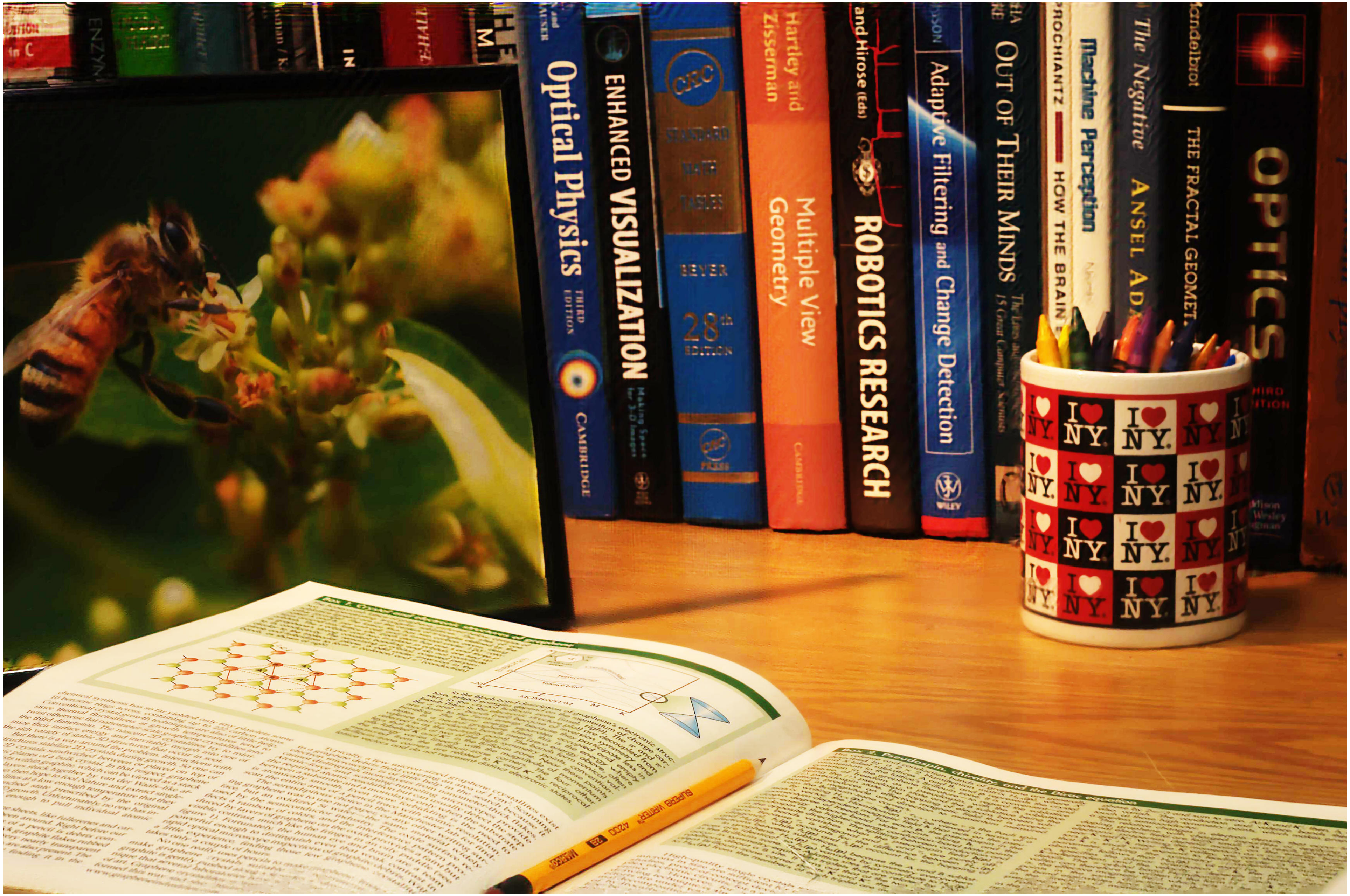
Experiments

Scene 1

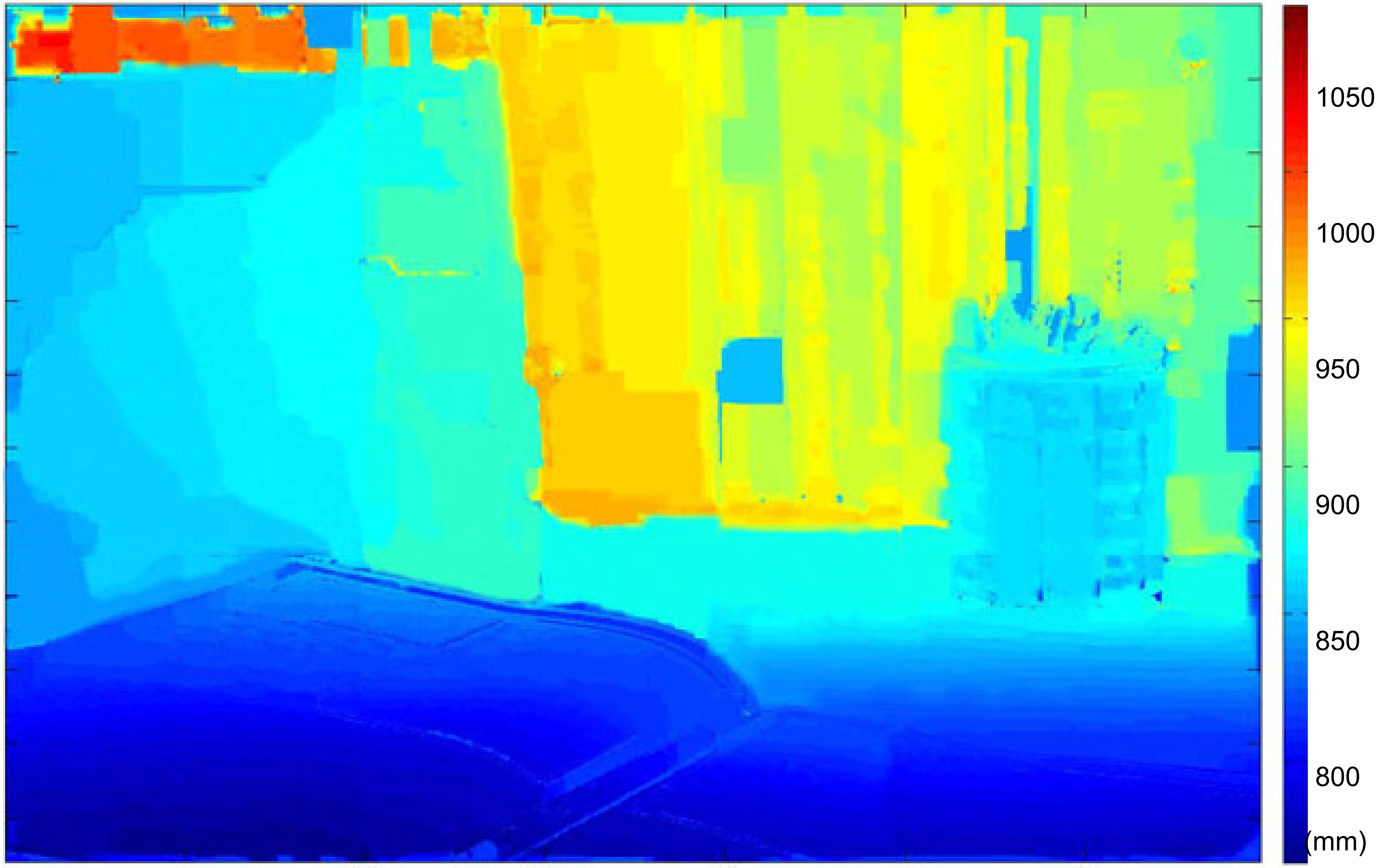
Captured Image 1

(The aperture pattern is shown in the left-top corner)

Captured Image 2



Recovered All-focused Image



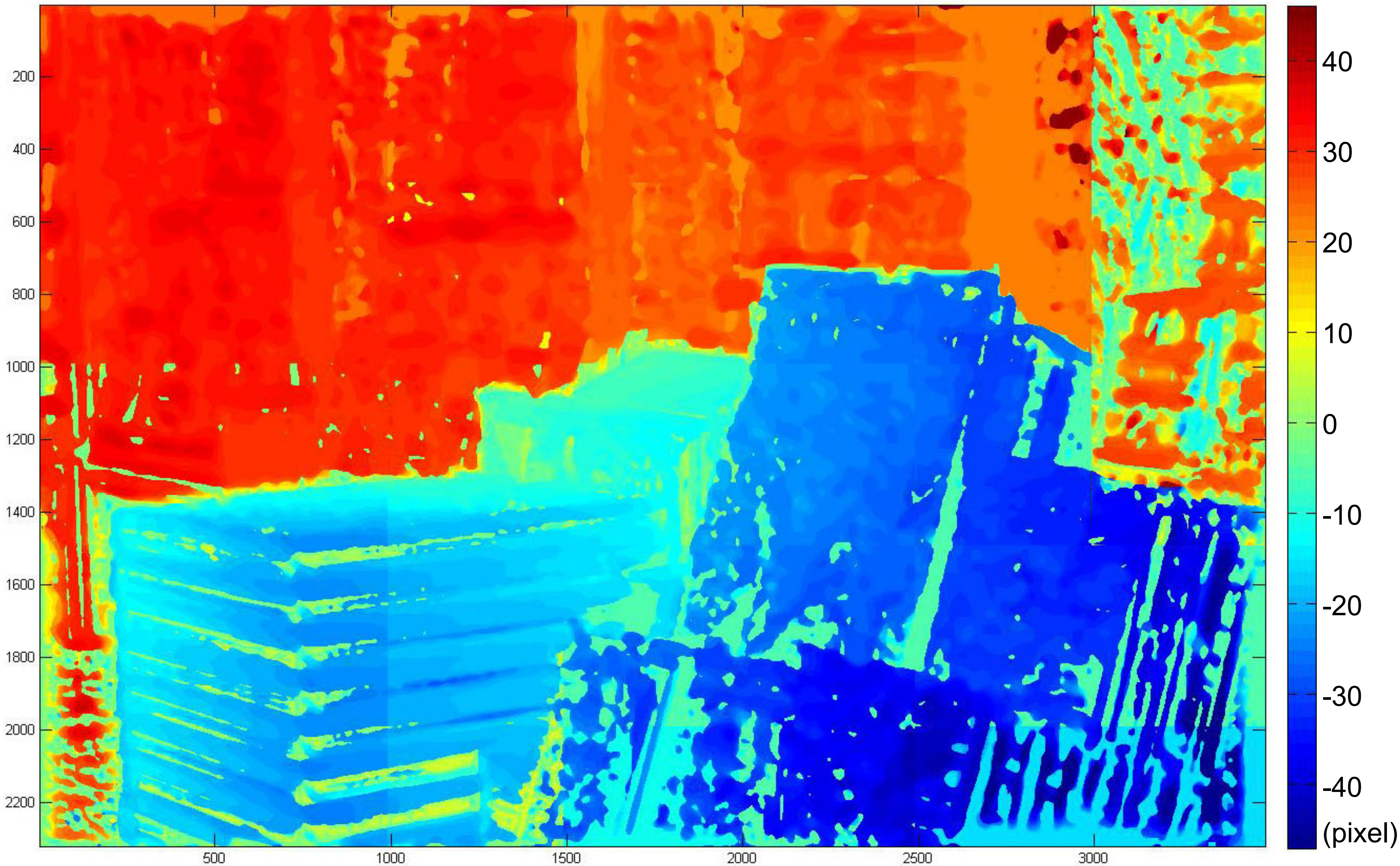
Estimated Depth Map

Scene 2

Captured Image 1

Captured Image 2

Recovered All-focused Image



Estimated Depth Map

Ground truth image taken with f/16
(The view point is slightly shifted)

Scene 3

Captured Image 1

(using a large circular aperture)

Captured Image 2
(using a small circular aperture)

Recovered All-focused Image

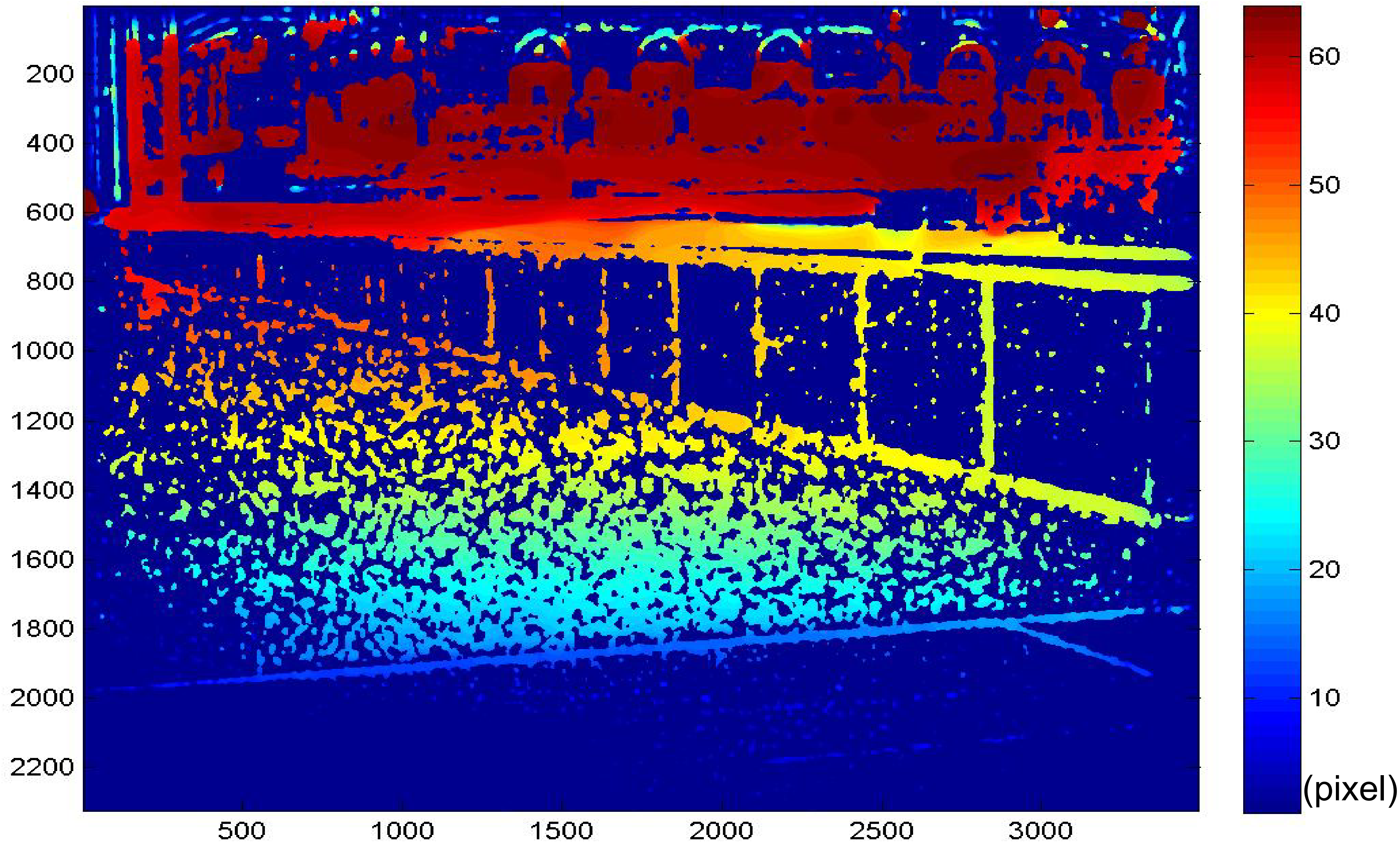
(using the conventional small/large circular aperture pair)

Captured Image 1

(using the optimized coded aperture 1)

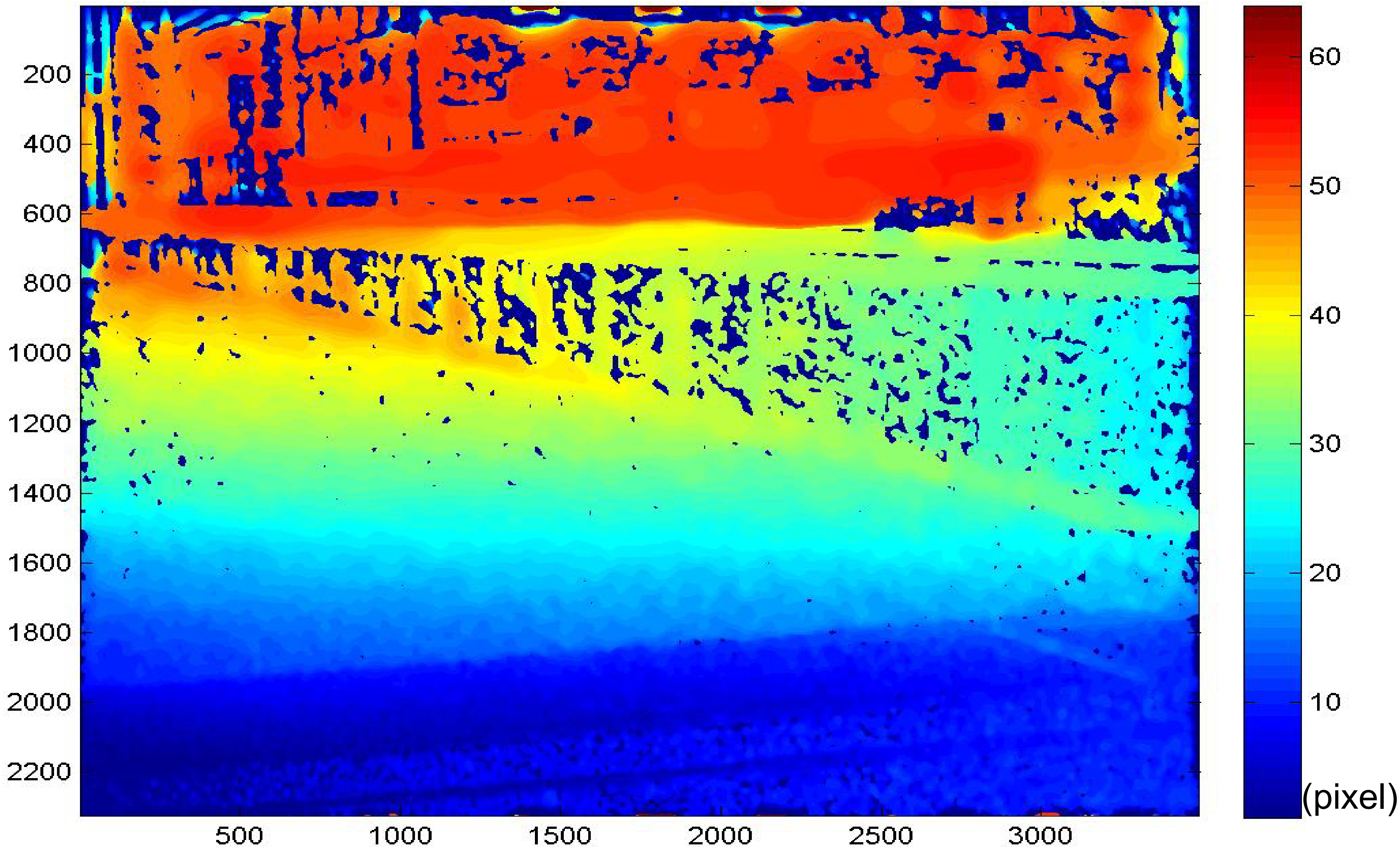
Captured Image 2
(using the optimized coded aperture 2)

Recovered All-focused Image
(using the optimized coded aperture pair)



Estimated Depth Map

(using the conventional small/large circular aperture pair)



Estimated Depth Map

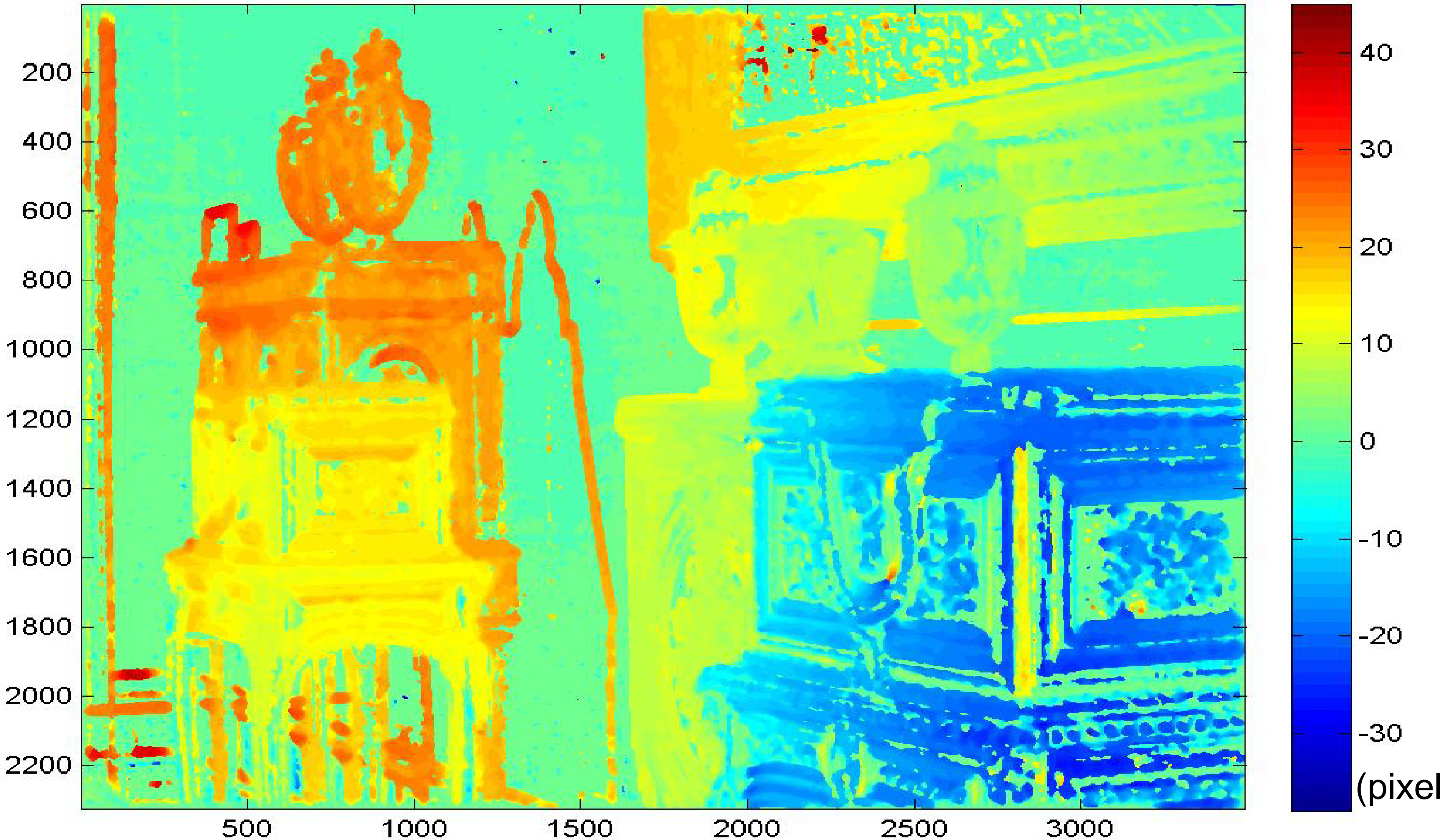
(using the optimized coded aperture pair)

Scene 4

Captured Image 1
(using the optimized coded aperture 1)

Captured Image 2
(using the optimized coded aperture 2)

Recovered All-focused Image
(using the optimized coded aperture pair)



Estimated Depth Map

(using the optimized coded aperture pair)

(The blur size of a surface without any texture will be estimated as 0)

Scene 5

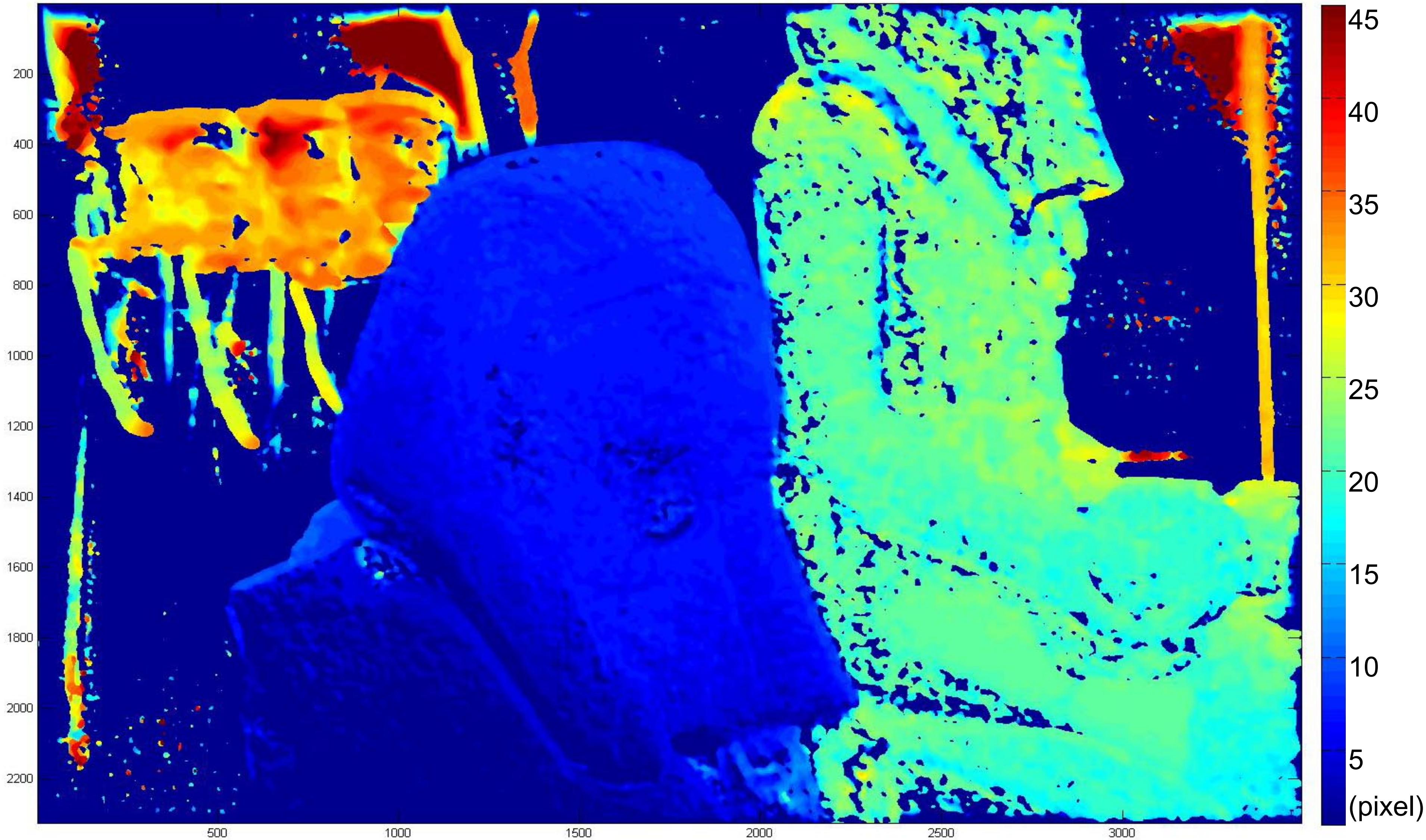
Captured Image 1

(using the optimized coded aperture 1)

Captured Image 2
(using the optimized coded aperture 2)

Recovered All-focused Image

(using the optimized coded aperture pair)



Estimated Depth Map
(using the optimized coded aperture pair)
(The blur size of a surface without any texture will be estimated as 0)

Coded Aperture Pairs for Depth from Defocus

Appendix A: Proof of Equation (6)

$$E(d|K_1^{d^*}, K_2^{d^*}, \sigma) = \sum_{\xi} \frac{A \cdot |K_1^d \cdot K_2^{d^*} - K_2^d \cdot K_1^{d^*}|^2}{\sum_i |K_i^d|^2 + C} + \sigma^2 \cdot \sum_{\xi} \frac{C^2}{\sum_i |K_i^d|^2 + C} + n \cdot \sigma^2.$$

Proof:

$$\begin{aligned} E(d|K_1^{d^*}, K_2^{d^*}, \sigma) &= \mathop{\mathbb{E}}_{F_0} E(d|K_1^{d^*}, K_2^{d^*}, \sigma, F_0) && \text{(A-1)} \\ &= \mathop{\mathbb{E}}_{F_0, F_1, F_2} E(d|K_1^{d^*}, K_2^{d^*}, F_1, F_2, F_0) && \text{(A-2)} \\ &= \mathop{\mathbb{E}}_{F_0, F_1, F_2} \left[\sum_{i=1,2} \|\hat{F}_0 \cdot K_i - F_i\|^2 + \|C \cdot \hat{F}_0\|^2 \right], && \text{(A-3)} \end{aligned}$$

where $\mathbb{E}(x)$ is the expectation of x, and F_i is the i^{th} captured image. Substituting $\hat{F}_0$ with Equation (4), we get:

$$\begin{aligned} E(d|K_1^{d^*}, K_2^{d^*}, \sigma) &= \mathop{\mathbb{E}}_{F_0, F_1, F_2} \Bigg[\sum_{i=1,2} \| \tfrac{F_1 \cdot \bar{K}_1^d + F_2 \cdot \bar{K}_2^d}{|K_1^d|^2 + |K_2^d|^2 + |C|^2} \cdot K_i - F_i \|^2 \\ &\quad + \|C \cdot \tfrac{F_1 \cdot \bar{K}_1^d + F_2 \cdot \bar{K}_2^d}{|K_1^d|^2 + |K_2^d|^2 + |C|^2} \|^2 \Bigg]. && \text{(A-4)} \end{aligned}$$

Then, by substituting F_i with Equation (2), we have:

$$\begin{aligned} & E(d|K_1^{d^*}, K_2^{d^*}, \sigma) \\ &= \mathop{\mathbb{E}}_{F_0, \zeta_1, \zeta_2} \Bigg[\sum_{i=1,2} \| \tfrac{(F_0 \cdot K_1^{d^*} + \zeta_1) \cdot \bar{K}_1^d + (F_0 \cdot K_2^{d^*} + \zeta_2) \cdot \bar{K}_2^d}{|K_1^d|^2 + |K_2^d|^2 + |C|^2} \cdot K_i - (F_0 \cdot K_i^{d^*} + \zeta_i) \|^2 \\ & \quad + \|C \cdot \tfrac{(F_0 \cdot K_1^{d^*} + \zeta_1) \cdot \bar{K}_1^d + (F_0 \cdot K_2^{d^*} + \zeta_2) \cdot \bar{K}_2^d}{|K_1^d|^2 + |K_2^d|^2 + |C|^2} \|^2 \Bigg]. && \text{(A-5)} \end{aligned}$$

Since ζ_1 and ζ_2 are independent Gaussian white noise $N(0, \sigma)$, we have $\mathbb{E}\,\zeta_i^2 = \sigma^2$, $\mathbb{E}\,\zeta_i = 0$, and $\mathbb{E}\,\zeta_1 \zeta_2 = 0$. Let $B = K_1^2 + K_2^2 + C^2$. Then, Equation (A-5) can be rearranged to be:

$$\begin{aligned} & E(d|K_1^{d^*}, K_2^{d^*}, \sigma) \\ &= \mathop{\mathbb{E}}_{F_0, \zeta_1, \zeta_2} \sum_{i=1,2} \left[\| \tfrac{F_0[(K_1^{d^*} \bar{K}_1 + K_2^{d^*} \bar{K}_2^d) \cdot K_i^d - K_i^{d^*} B]}{B} \|^2 + \| \tfrac{(\zeta_1 \bar{K}_1^d + \zeta_2 \bar{K}_2^d) K_i^d}{B} - \zeta_i \|^2 \right] \\ & \quad + \|C \cdot \tfrac{F_0 \cdot (K_1^{d^*} \bar{K}_1^d + K_2^{d^*} \bar{K}_2^d)}{B} + \tfrac{\zeta_1 \cdot \bar{K}_1^d + \zeta_2 \cdot \bar{K}_2^d}{B} \|^2 \\ &= \mathop{\mathbb{E}}_{F_0} \sum_{i=1,2} \left[\| \tfrac{F_0[(K_1^{d^*} \bar{K}_1^d + K_2^{d^*} \bar{K}_2^d) \cdot K_i^d - K_i^{d^*} B]}{B} \|^2 + \sigma^2 \cdot (\| \tfrac{{K_i^d}^2 + C}{B} \|^2 + \| \tfrac{K_1^d K_2^d}{B} \|^2) \right] \\ & \quad + \|C \cdot \tfrac{F_0 \cdot (K_1^{d^*} \bar{K}_1^d + K_2^* \bar{K}_2^d)}{B} \|^2 + \sigma^2 \cdot (\|C \cdot \tfrac{K_1^d}{B}\|^2 + \|C \cdot \tfrac{K_2^d}{B}\|^2). && \text{(A-6)} \end{aligned}$$

According to the $1/f$ law, we define the expectation of the power spectrum of F_0 as $A(\xi) = \int_{F_0} |F_0(\xi)|^2 \mu(F_0)$. In addition, it is known that $C = \sigma^2/A$. Then, Equation (A-6) can be further re-arranged and simplified as:

$$
\begin{aligned}
& E(d|K_1^{d^*}, K_2^{d^*}, \sigma) \\
=& \sum_{\xi} \Bigg[\frac{A \cdot |K_1^d \cdot K_2^{d^*} - K_2^d \cdot K_1^{d^*}|^2}{B} \\
& + \sigma^2 \cdot (\| \frac{K_1^{d^2} + C^2}{B} \|^2 + \| \frac{K_2^{d^2} + C^2}{B} \|^2 + 2 \cdot \| \frac{K_1^d K_2^d}{B} \|^2 + \| C \cdot \frac{K_1^d}{B} \|^2 + \| C \cdot \frac{K_2^d}{B} \|^2) \Bigg] \\
=& \sum_{\xi} \Bigg[\frac{A \cdot |K_1^d \cdot K_2^{d^*} - K_2^d \cdot K_1^{d^*}|^2 + \sigma^2 \cdot C^2}{B} + \sigma^2) \Bigg] \\
=& \sum_{\xi} \frac{A \cdot |K_1^d \cdot K_2^{d^*} - K_2^d \cdot K_1^{d^*}|^2}{|K_1^d|^2 + |K_2^d|^2 + C^2} + \sigma^2 \cdot \sum_{\xi} \frac{C^2}{|K_1^d|^2 + |K_2^d|^2 + C^2} + n \cdot \sigma^2 . \square
\end{aligned}
\quad \text{(A-7)}
$$