

# **Data Structures and Algorithms**

**Session 5. February 2, 2009**

**Instructor: Bert Huang**

**<http://www.cs.columbia.edu/~bert/courses/3137>**

# Announcements

- \* Homework 1 up on website
- \* Due at beginning of next class

# Review

- \* Dissected textbook's Java implementation of MyLinkedList and MyArrayList
  - \* add(), remove(), get(), set()
  - \* void vs. boolean add() – boolean is for Collections interface, which requires that add() returns whether the Collection changed

# Today

- \* New Abstract Data Type: Stack
- \* Stack example applications
- \* Stack Implementations (easy)

# Stack Definition

- \* Essentially a very restricted List
- \* Two (main) operations:
  - \* Push(AnyType x)
  - \* Pop(AnyType x)
- \* Analogy – Cafeteria Trays, PEZ

# Stack Applications

- \* Recursion
- \* Parsing text: infix vs. postfix
- \* Syntax checking ( ), { }, “”

# Evaluating Recursion

- \* Push recursive calls onto a Stack, evaluate top
- \* Consider computing factorials:
  - \*  $N! = N * (N-1)!$
  - \*  $1! = 1$
- \* (Note:  $O(N!)$  is REALLY bad)

# Stack Animation



# Stack Animation



6!

# Stack Animation

$$6! = 6 * 5!$$

# Stack Animation

$$5! = 5 * 4!$$

$$6! = 6 * 5!$$

# Stack Animation

$$4! = 4 * 3!$$

$$5! = 5 * 4!$$

$$6! = 6 * 5!$$

# Stack Animation

$$3! = 3 * 2!$$

$$4! = 4 * 3!$$

$$5! = 5 * 4!$$

$$6! = 6 * 5!$$

# Stack Animation

$$2! = 2 * 1!$$

$$3! = 3 * 2!$$

$$4! = 4 * 3!$$

$$5! = 5 * 4!$$

$$6! = 6 * 5!$$

# Stack Animation

$$1! = 1$$

$$2! = 2 * 1!$$

$$3! = 3 * 2!$$

$$4! = 4 * 3!$$

$$5! = 5 * 4!$$

$$6! = 6 * 5!$$

# Stack Animation

$$2! = 2 * 1 = 2$$

$$3! = 3 * 2!$$

$$4! = 4 * 3!$$

$$5! = 5 * 4!$$

$$6! = 6 * 5!$$



# Stack Animation

$$3! = 3 * 2 = 6$$

$$4! = 4 * 3!$$

$$5! = 5 * 4!$$

$$6! = 6 * 5!$$

# Stack Animation

$$4! = 4 * 3 = 24$$

$$5! = 5 * 4!$$

$$6! = 6 * 5!$$

# Stack Animation

$$5! = 5 * 24 = 120$$

$$6! = 6 * 5!$$

# Stack Animation

$$6! = 6 * 120 = 720$$

# Evaluating Postfix

\* Postfix notation places operator after operands

\* Ambiguous Infix:  $(3 + 2)^* 10$        $((3+2)^* 10)$

\* Postfix:  $3 2 + 10 *$        $((3 2 +) 10 *)$

(As opposed to)

$3 2 10 * +$

$(3 (2 10 *) +)$

# Evaluating Postfix

\* Postfix notation places operator after operands

\* Ambiguous Infix:  $3 + 2 * 10$   $((3+2) * 10)$

\* Postfix:  $3 2 + 10 *$   $((3 2 +) 10 *)$

(As opposed to)

$3 2 10 * +$

$(3 (2 10 *) +)$

# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
  - \* Evaluate operation, push result
- \* E.g., 3 2 + 10 \*



# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
- \* Evaluate operation, push result

\* E.g., 3 2 + 10 \*

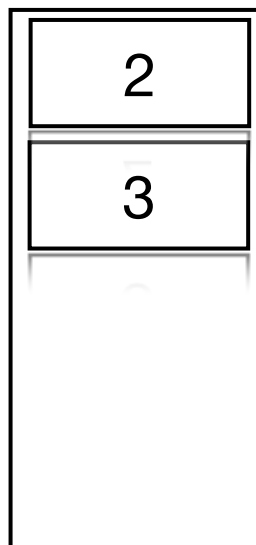




# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
- \* Evaluate operation, push result

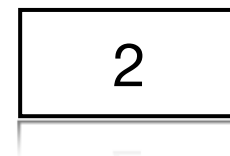
\* E.g., 3 2 + 10 \*



# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
- \* Evaluate operation, push result

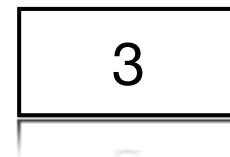
\* E.g., 3 2 + 10 \*



# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
- \* Evaluate operation, push result

\* E.g., 3 2 + 10 \*



# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
- \* Evaluate operation, push result

\* E.g., 3 2 + 10 \*



# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
- \* Evaluate operation, push result

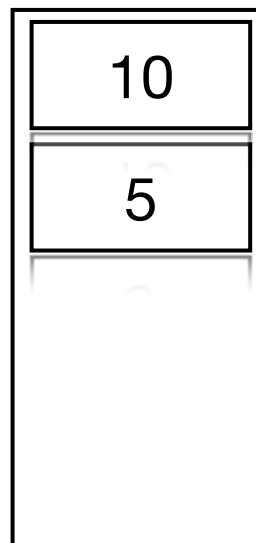
\* E.g., 3 2 + 10 \*



# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
- \* Evaluate operation, push result

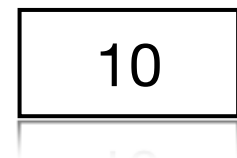
\* E.g., 3 2 + 10 \*



# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
- \* Evaluate operation, push result

\* E.g., 3 2 + 10 \*



# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
- \* Evaluate operation, push result

\* E.g., 3 2 + 10 \*





# Postfix Stack

- \* Push symbols as they appear
- \* Whenever we read an operator, pop two operands
- \* Evaluate operation, push result

\* E.g., 3 2 + 10 \*



# Syntax Checking

- \* Check for matching parenthesis ( ), braces { }, brackets [ ], etc.
- \* Sweep through code
  - \* If we see an opening symbol, push onto stack
  - \* If we see a closing symbol, pop from stack and compare

# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

# Syntax Checking

```
* public void add((int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

(

# Syntax Checking

```
* public void add((int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

(

# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

(

# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

(



# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

{

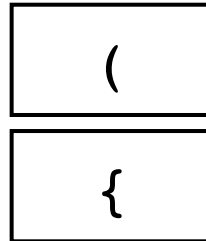
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

{

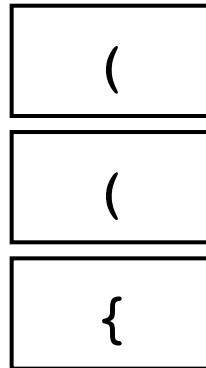
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```



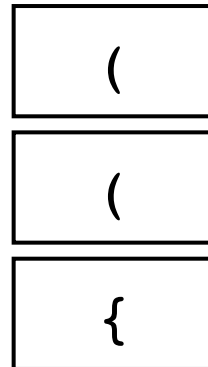
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```



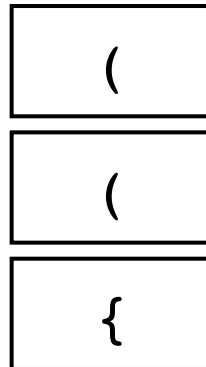
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```



# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```



# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```

(

(

{



# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

(

{

# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

{

# Syntax Checking

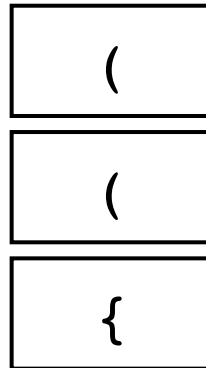
```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```

(

{

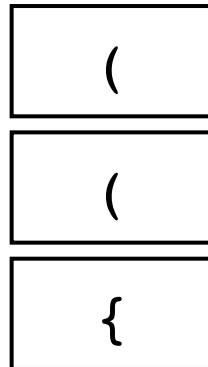
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```



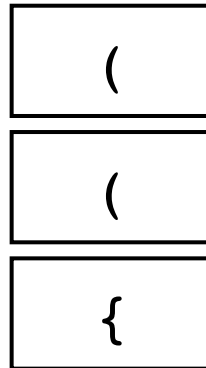
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```



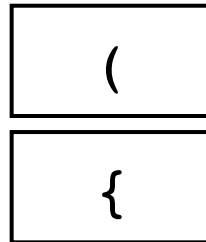
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```



# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1) for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```



# Syntax Checking

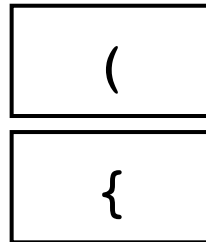
```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1) for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```

{



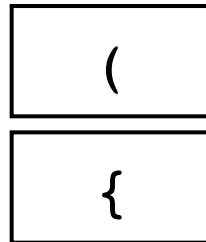
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```



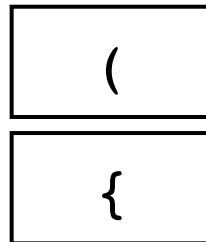
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```



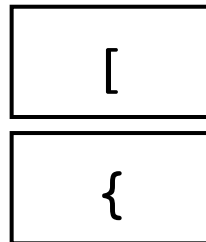
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```



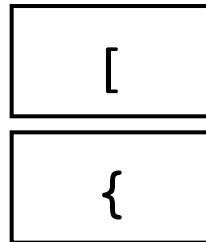
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```



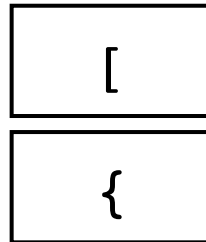
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```



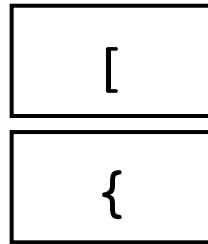
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
    theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
    theSize++; }
```



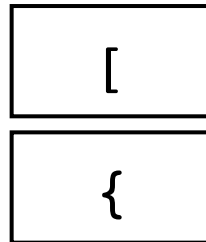
# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```



# Syntax Checking

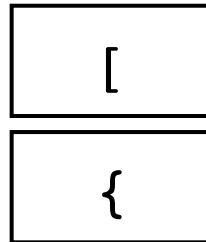
```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```





# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```



# Syntax Checking

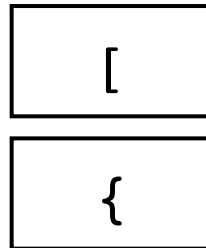
```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[idx] = x;
  theSize++; }
```

[

{

# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[idx] = x;
  theSize++; }
```



# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```

[

{

# Syntax Checking

```
* public void add( int idx, AnyType x)
  { if( theItems.length == size( ) ) ensureCapacity( size
    ( ) * 2 + 1); for( int i=theSize; i > idx; i-- )
  theItems[ i ] = theItems[ i - 1 ]; theItems[ idx ] = x;
  theSize++; }
```

{

# Stack Implementations

- \* Linked List:

- \*  $\text{Push}(x) \leftrightarrow \text{add}(x) \quad \leftrightarrow \quad \text{add}(x,0)$

- \*  $\text{Pop}(x) \leftrightarrow \text{remove}(0)$

- \* Array:

- \*  $\text{Push}(x) \leftrightarrow \text{Array}[k++] = x$

- \*  $\text{Pop}(x) \leftrightarrow \text{return Array[--k]}$

# Assignments

- \* Homework 1 due next class
- \* Homework 2 will be assigned next class, will cover Lists, Stacks, Queues
- \* Weiss Section 3.7 – Queues