Data Structures and Algorithms

Session 17. March 30, 2009

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Announcements

- Homework 4 due next Monday
- I have old hw’s and midterms in my office

Review

- Finished Huffman optimality proof sketch
- Hash Tables ADT
  - O(1) insert and search
  - Use hash function to determine table index
  - Collision resolution strategies:
    - Separate Chaining, Probing

Double Hashing Example

- N = 7

- h1(x) = x mod 7,   h2(x) = 5-x mod 5
- insert 9

		9				

- insert 16

		9				16

- insert 2

		7			2	16

Today's Plan

- Loose ends re: hashing
  - Rehashing
  - String hash function example
- Graphs
  - Definitions, implementation

Rehashing

- Like ArrayLists, we have to guess the number of elements we need to insert into a hash table
- Whatever our collision policy is, the hash table becomes inefficient when load factor is too high.
- To alleviate load, **rehash**:
  - create larger table, scan current table, insert items into new table using new hash function

When to Rehash

- For quadratic probing, insert may fail if load > 1/2
  - We can rehash as soon as load > 1/2
  - Or, we can rehash only when insert fails
- Heuristically choose a load factor threshold, rehash when threshold breached

Rehash Example

- Current Table:

0	8	7	17	25		
0	1	2	3	4	5	6

- quad. probing with h(x) = (x mod 7)
  8, 0, 25, 17, 7

- New table
  - h(x) = (x mod 17)

0	17						7	8	25							
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Rehash Cost

- No profound algorithm: re-insert each item
- Linear time
- If you rehash, inserting N items costs $O(1)*N + O(N) = O(N)$
- Insert still costs O(1) amortized

Hashing a String

- Simple but bad h(x)
  - add up all the character codes (ASCII/Unicode)
- ASCII 'a' is 97
- If keys are lowercase 5 character words, h(x) > 485

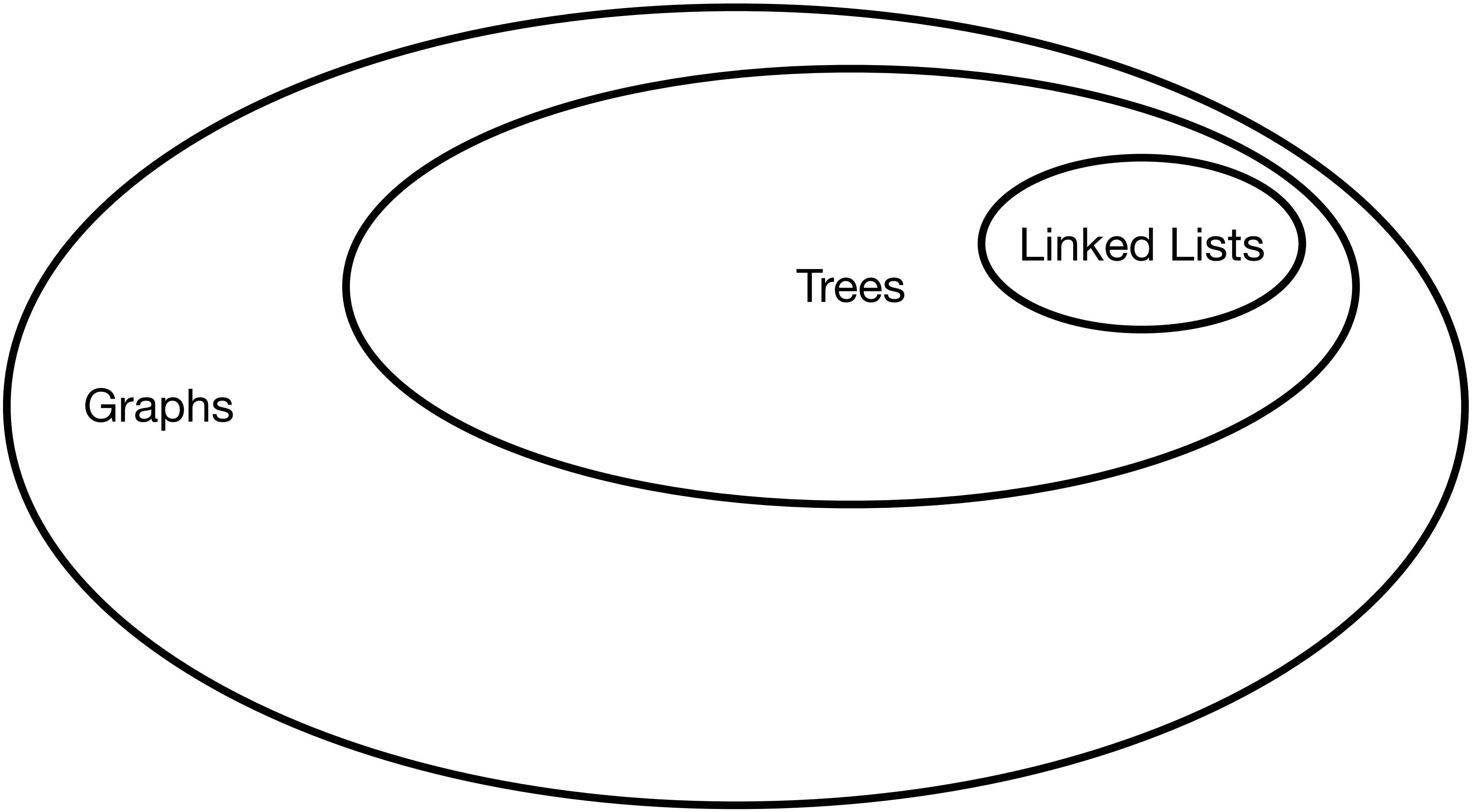
Hashing a String II

- Weiss: Treat first 3 characters of a string as a 3 digit, base 27 number
- Once again, ‘a’ is 97, ‘A’ is 65

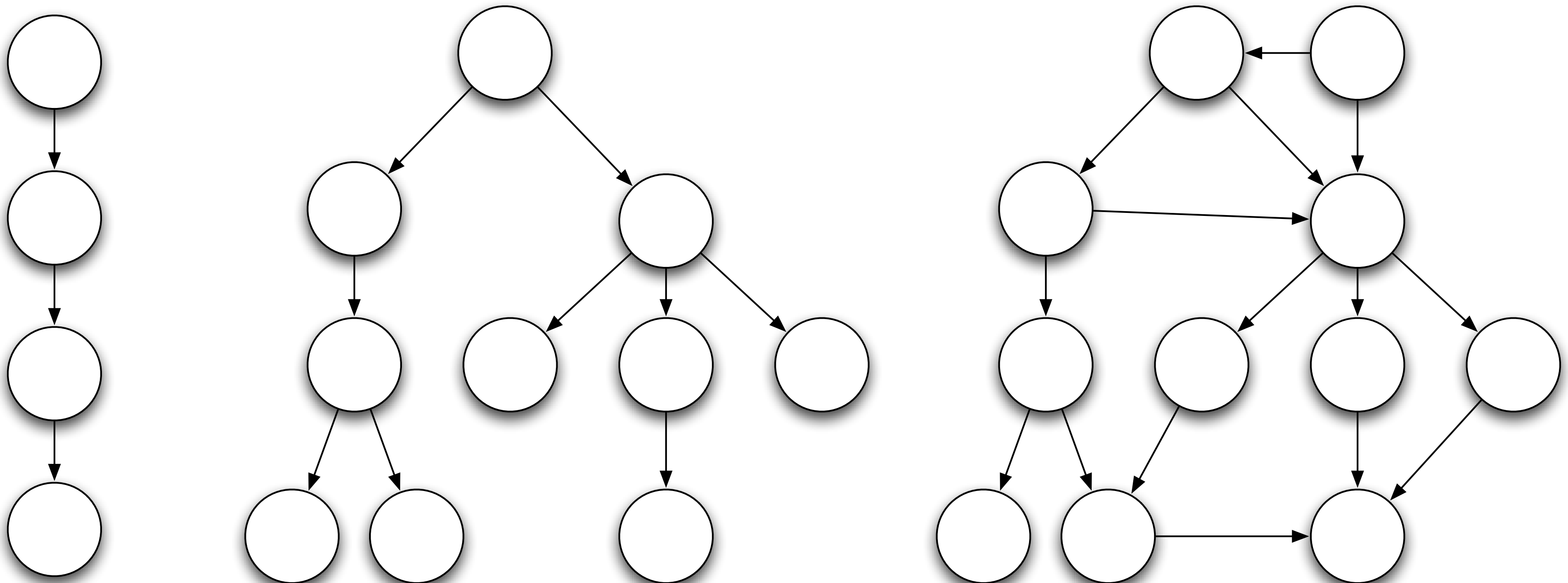
String.hashCode()

- Java's built in String hashCode() method
  - `s[0]*31^(n-1) + s[1]*31^(n-2) + ... + s[n-1]`
- nth degree polynomial of base 31
- String characters are coefficients

Graphs



Graphs



LINKED LIST

TREE

GRAPH

Graph Terminology

- A **graph** is a set of **nodes** and **edges**
  - nodes aka vertices
  - edges aka arcs, links
- Edges exist between pairs of nodes
  - if nodes x and y share an edge, they are **adjacent**

Graph Terminology

- Edges may have **weights** associated with them
- Edges may be **directed** or **undirected**
- A **path** is a series of adjacent vertices
  - the **length** of a path is the sum of the edge weights along the path (1 if unweighted)
- A **cycle** is a path that starts and ends on a node

Graph Properties

- An undirected graph with no cycles is a tree
- A directed graph with no cycles is a special class called a **directed acyclic graph (DAG)**
- In a **connected** graph, a path exists between every pair of vertices
- A **complete** graph has an edge between every pair of vertices

Graph Applications: A few examples

- Computer networks
- The World Wide Web
- Social networks
- Public transportation
- Probabilistic Inference
- Flow Charts

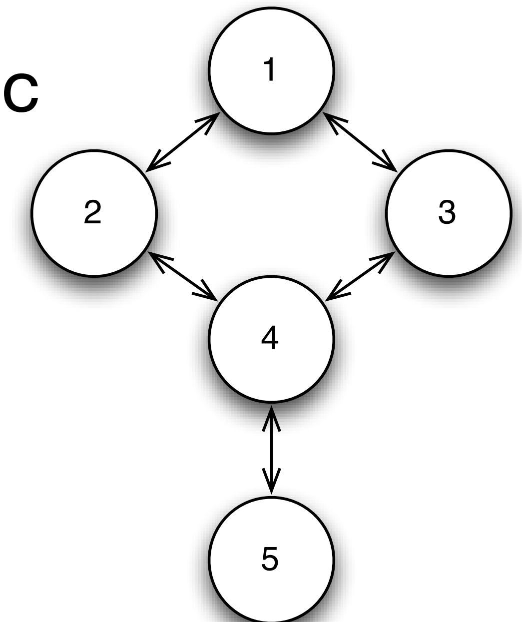
Implementation

- Option 1:
  - Store all nodes in an indexed list
  - Represent edges with **adjacency matrix**
- Option 2:
  - Explicitly store **adjacency lists**

Adjacency Matrices

- 2d-array **A** of boolean variables
- A[i][j] is true when node **i** is adjacent to node **j**
  - If graph is undirected, A is symmetric

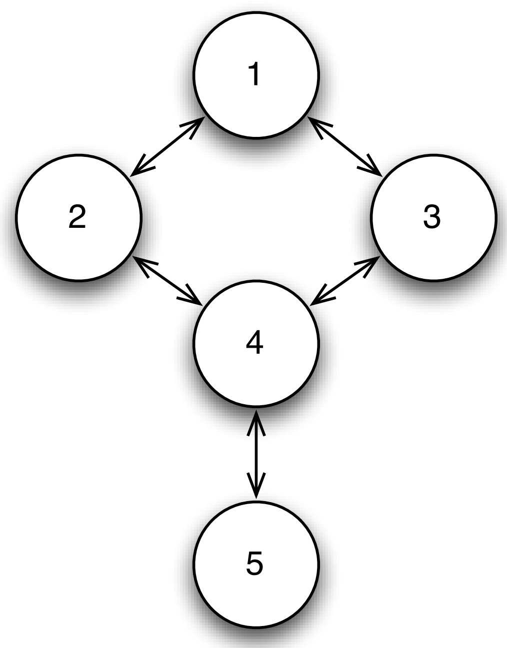
	1	2	3	4	5
1	0	**1**	**1**	0	0
2	**1**	0	0	**1**	0
3	**1**	0	0	**1**	0
4	0	**1**	**1**	0	**1**
5	0	0	0	**1**	0



Adjacency Lists

- Each node stores references to its neighbors

1	2	3	
2	1	4	
3	1	4	
4	2	3	5
5	4		



Reading

- Homework 4
  - Weiss Section 5 (Hashing)
- Weiss Section 9.1-9.2