Data Structures and Algorithms

Session 11. February 25, 2009

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Announcements

- Homework 3 is out. Due 3/9
- Midterm review March 9th
- Midterm Exam March 11th
- Manu is stepping down as a TA
- New office hour Priyamvad Tuesday 3-5 PM

Review

- HW1 solutions
- Splay Trees
  - Move accessed node to root
    - Zig-Zag: double rotate (a la AVL)
    - Zig-Zig: reverse order
- Prefix Trees (tries)

Today’s Plan

- Note about HW1 Problem 5
- Visualization of Splay Trees
- Cover Tries at normal pace
- Introduction to Priority Queues

Amortized Running Time

- In classical analysis, we try to prove:

$$MT(N) = O(M \log N)$$

- If this is impossible, we can guarantee that **M** operations take:

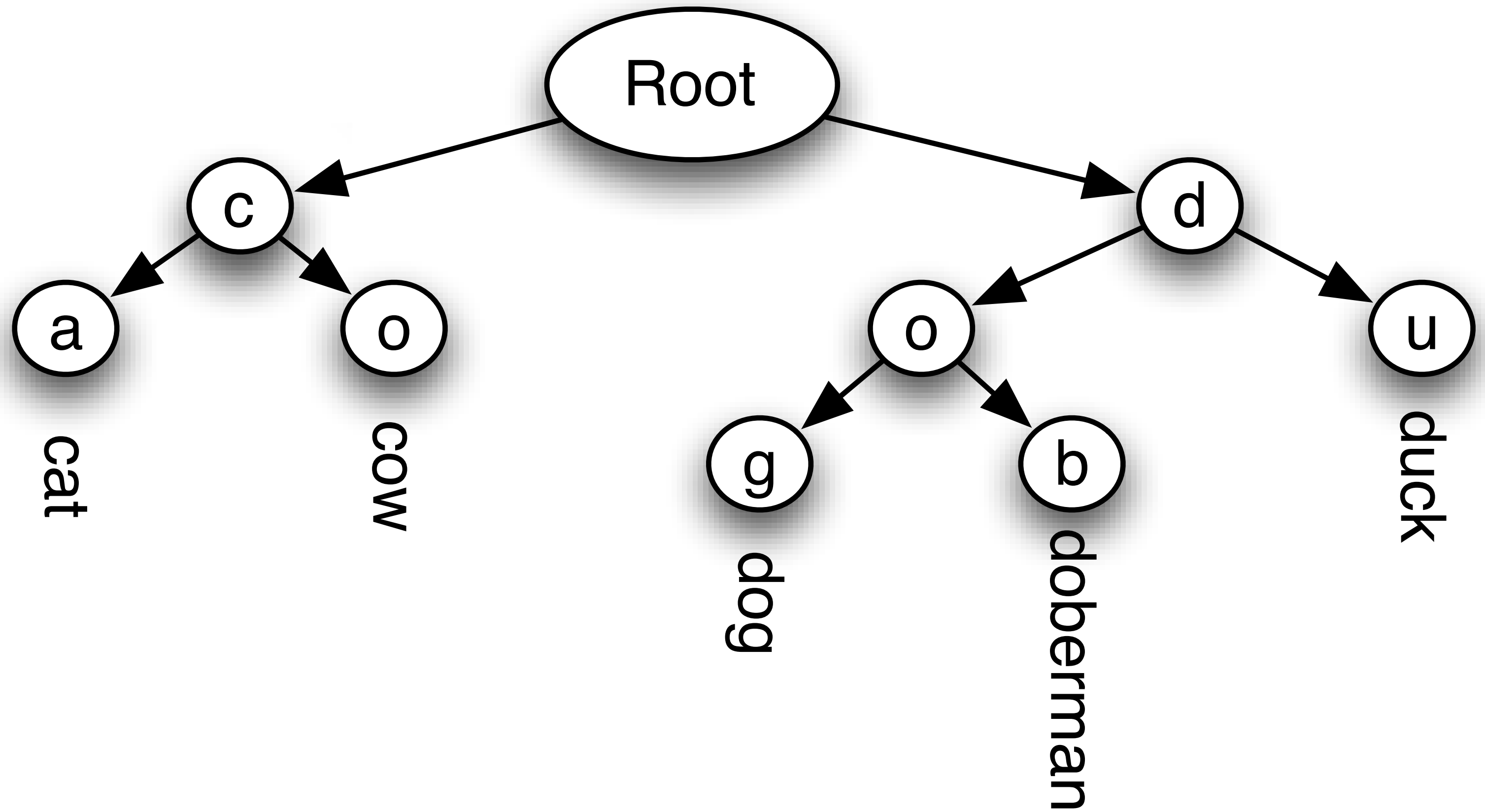
$$\sum_{i=1}^{M} T_i(N) = O(M \log N)$$

Prefix Trees (Tries)

- Nicknamed “Trie”, short for re**trie**val
- Efficiently store objects for fast retrieval via keys
  - Usually key is a String
- Basic strategy:
  - split into sub-tries based on current letter

Trie Example

- “cat”, “cow”, “dog”, “doberman”, “duck”



Trie Details

- Not all words are at leaves
  - cat, cataclysm, cataclysmic
- Initially, one letter is enough to uniquely identify
- When a new word is inserted that conflicts, need to branch
  - Originally-unique word must be moved to lower level

Trie Analysis

- In the worst case, inserting a key of length **k** or (looking up) is **O(k)**
- This is not dependent on **N**! (surprise, not factorial)
- Much better than **log(N)** for huge data like dictionaries
- Sometimes we can access words even faster.
  - E.g., we can find qwerty uniquely with just “qw”

Priority Queues

- New abstract data type Priority Queue:
  - Insert: add node with key
  - deleteMin: delete the node with smallest key
  - (increase/decrease priority)

Heap Implementation

- Priority queues are most commonly implemented using Binary Heaps
  - Binary tree with special properties
- Heap Structure Property: all nodes are full, (except possibly one at the bottom level)
- Heap Order Property: any node is smaller than its children

Array Implementation

- A full tree is regular: we can easily store in an array
  - Root at **A[1]**
  - Root's children at **A[2]**, **A[3]**
  - Node **i** has children at **2i** and **(2i+1)**
  - Parent at **floor(i/2)**
- No links necessary, so faster (in most languages)

Insert

- To insert key **X**, create a hole in bottom level
- **Percolate up**
  - Is hole's parent is less than **X**
    - If so, put **X** in hole, heap order satisfied
    - If not, swap hole and parent and repeat

DeleteMin

- Save root node, and delete, creating a hole
- Take the last element in the heap **X**
- **Percolate down**:
    - Check if X is less than hole's children
        - if so, we're done
        - if not, swap hole and smallest child and repeat

Assignments

- Start/continue HW3
- Read Weiss Section 6.1-6.3