Data Structures in Java

Session 23

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Announcements

- Homework 6 due Dec. 10, last day of class
- Final exam Thursday, Dec. 17th, 4-7 PM, Hamilton 602 (this room)
  - same format as midterm (open book/notes)
- Distinguished Lecture: Barbara Liskov, MIT.
  Turing Award Winner 2009.
  11 AM Monday. Davis Auditorium, CEPSR/Schapiro

Review

- External sorting: merge to alternating disks
- Complexity Classes
  - P, NP: Euler path
  - NP-Complete, NP-Hard: Hamiltonian path, Satisfiability, Graph Coloring
  - NP-Hard: Traveling Salesman
  - Undecidable: Halting Problem

Today's Plan

- Note about hw4
- Finish discussion of complexity
  - Polynomial Time Approximation Schemes
  - Graph Isomorphism
- k-d trees

Rehashing

- A hash table does not store the input order
- When rehashing, elements are inserted into the new table in the array order
- No penalty on homework, but make sure it's correct on the final

NP-Hardness

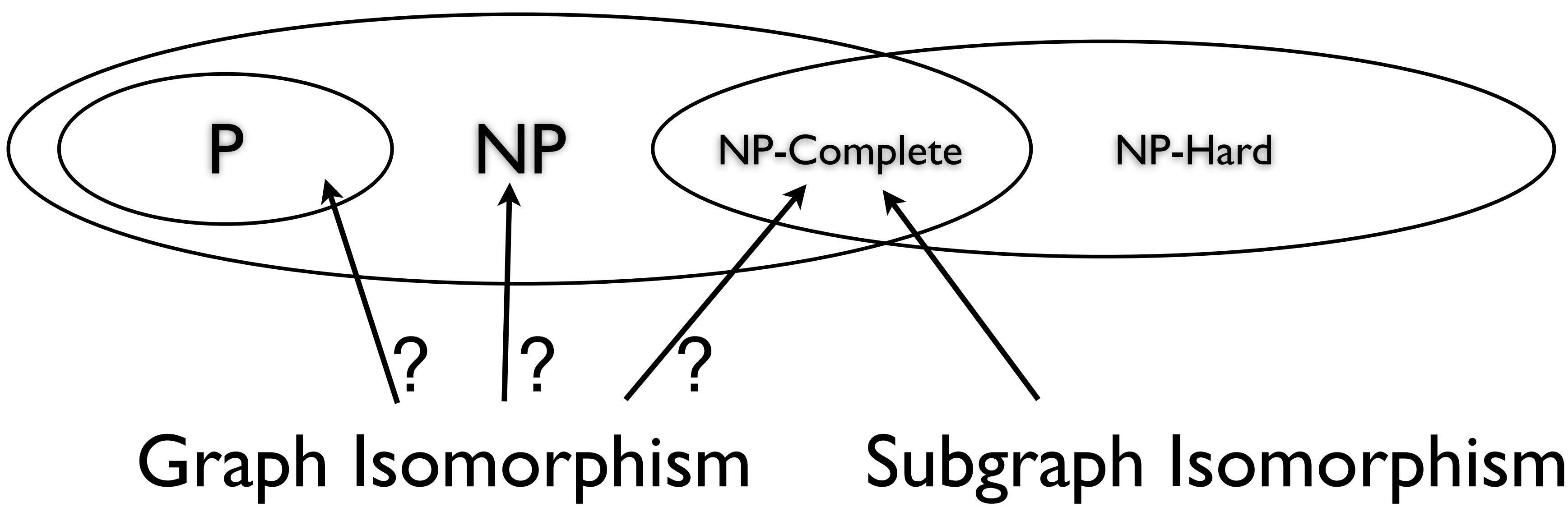
- An algorithm for an NP-Hard problem can be used to solve any NP problem via polynomial time conversion
- But we don't know algorithms to solve NP-Hard problems in poly. time
- If we did, P = NP, so most conjecture that NP-Hard problems must be intractable

Poly. Time Approximation

- Certain optimization NP-Hard problems have **polynomial time approximation schemes** (PTAS)
  - An efficient method to find a solution within a constant of the true optimum
    - e.g., Optimal TSP path length = ℓ
      PTAS TSP path length $\leq \ell(1+\epsilon)$
  - For fixed constant, must be poly. time, but can scale poorly w.r.t. constant
  - E.g., $O(p(N)^{(\frac{1}{\epsilon}!)})$ is a valid PTAS time

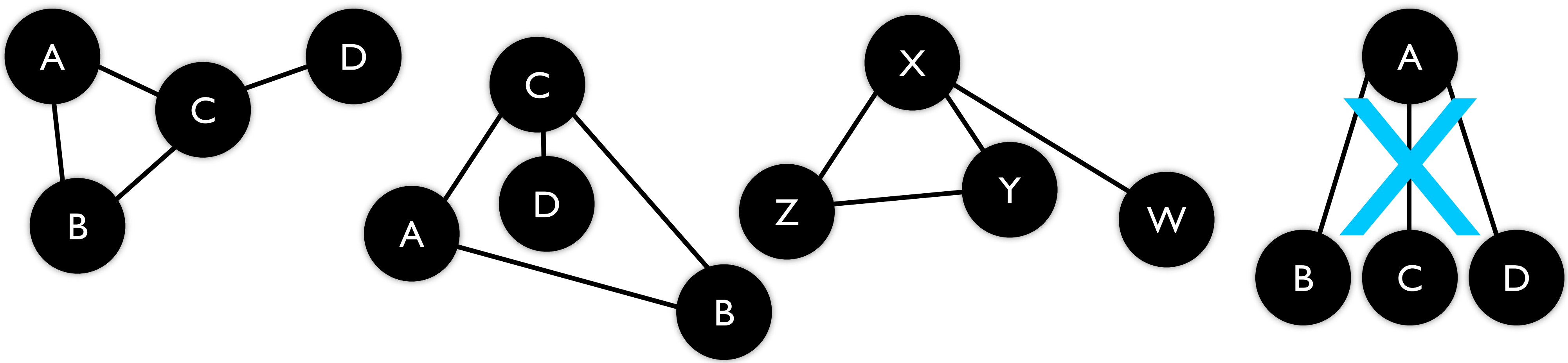
Graph Isomorphism Complexity

- The Graph Isomorphism problem is NP,
  - but is unknown if NP-Complete/Hard,
  - and no poly. time algorithm is known



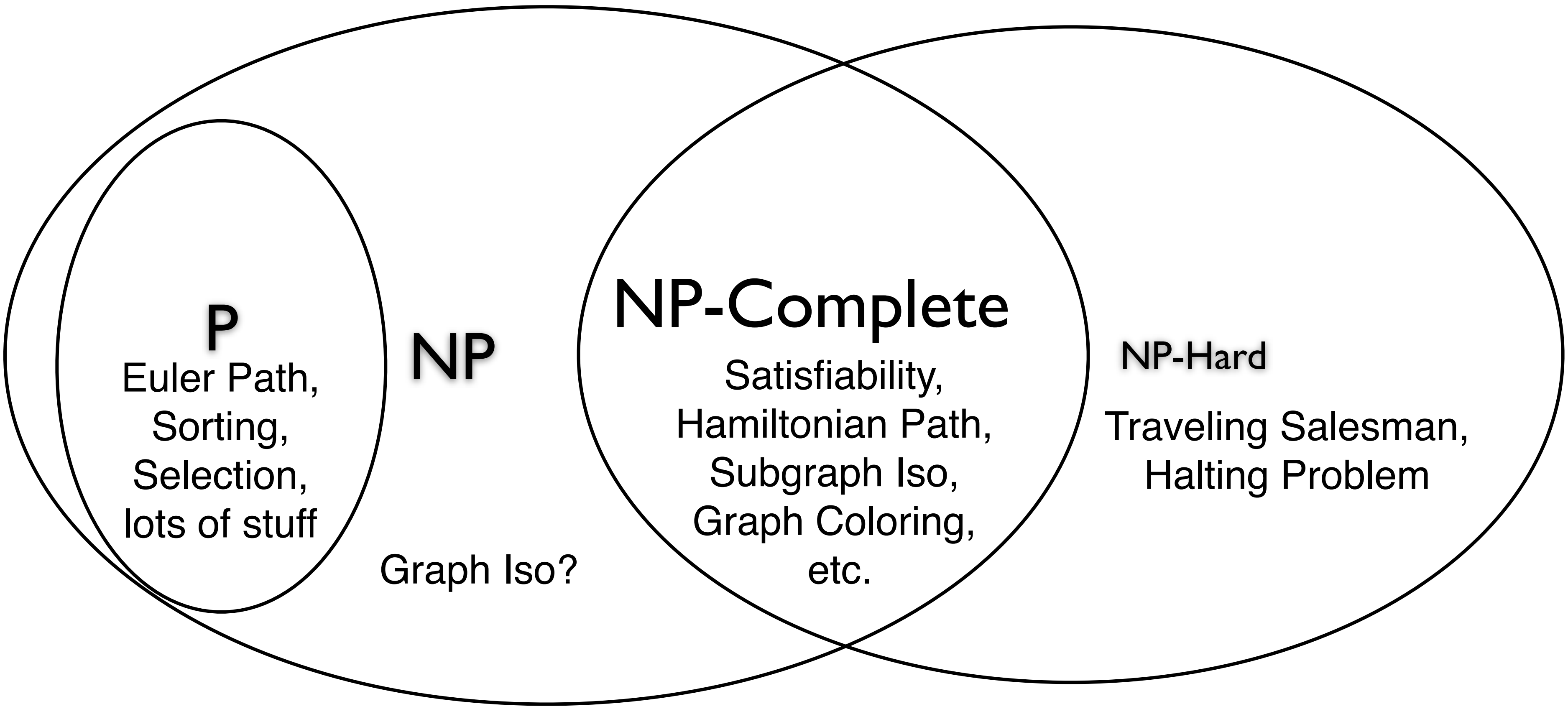
Graph Isomorphism Definition

- Given graphs G and H, is there a 1-to-1 mapping of vertices from G to vertices from H that preserves the edge structure?



- Subgraph Isomorphism: is a subgraph of G isomorphic to H?

Complexity


P
Euler Path,
Sorting,
Selection,
lots of stuff
NP
Graph Iso?
NP-Complete
Satisfiability,
Hamiltonian Path,
Subgraph Iso,
Graph Coloring,
etc.
NP-Hard
Traveling Salesman,
Halting Problem

Advanced Data Structures

- We've mostly studied fundamental data structures
  - wide application areas, very general
- In practice, you'll often have more specific goals, and thus need to design your own data structures

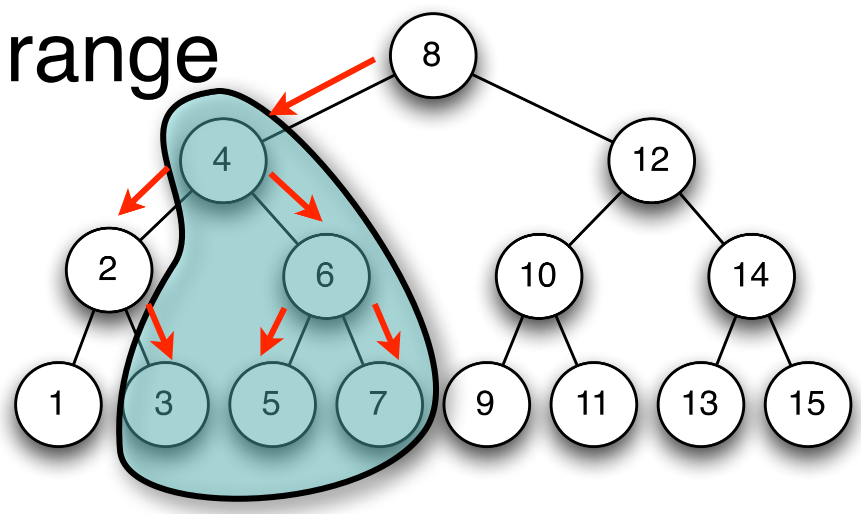
kd-Trees

- Useful data structure for data mining and machine learning applications
- Store elements by k-dimensional keys
  - e.g., age, height, weight
- Retrieve elements by ranges in the k dimensions
  - e.g., Searching for a new basketball center
    18-24 year olds, 6’6”-7’4”, 200+ lbs

1-d Range Search

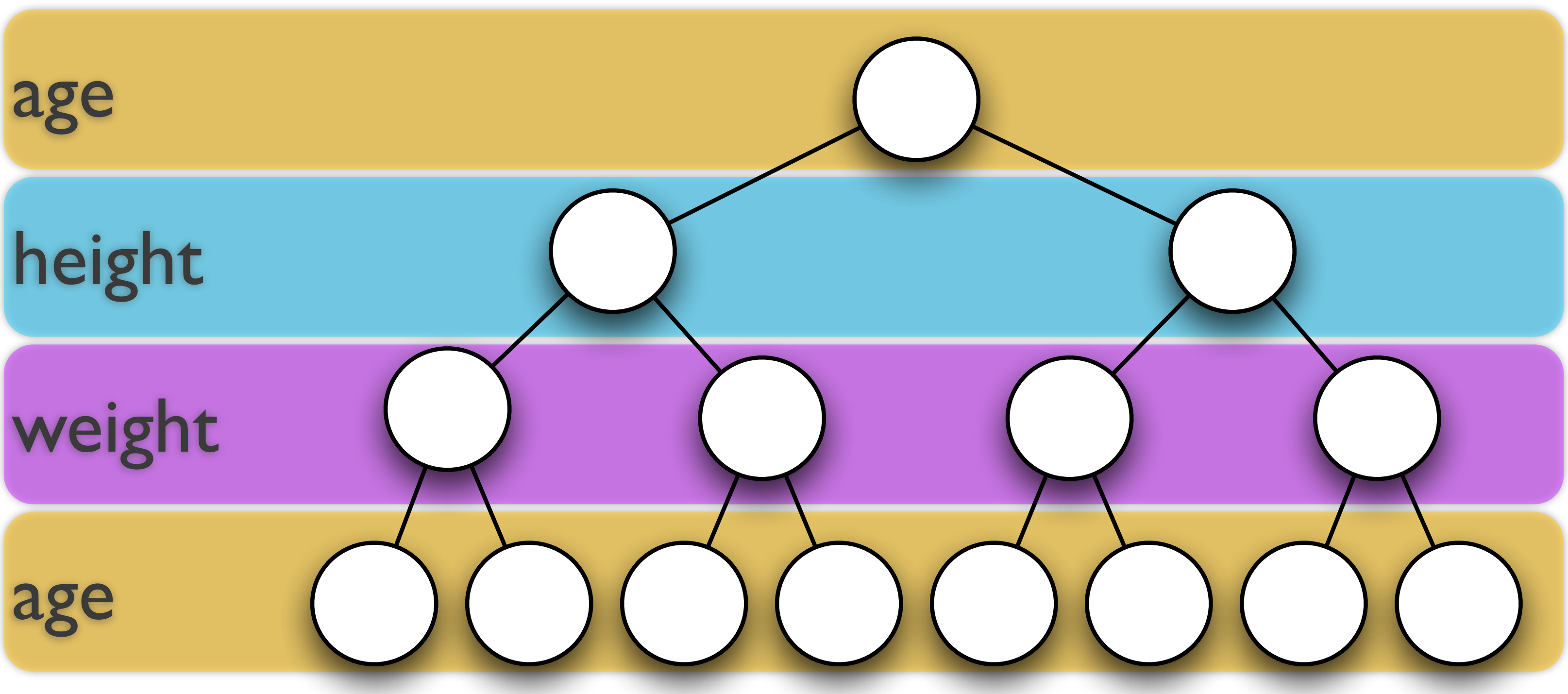
- BST recursive search:
  (1) if **key** is in range, print node
  (2) if **key** > **lower bound**, search left
  (3) if **key** < **upper bound**, search right
- **O(M+log N)** for **M** items returned
  - **O(log N)** to find nodes in range
  - **O(1)** at each node

Search for 3-7



kd-Tree Structure

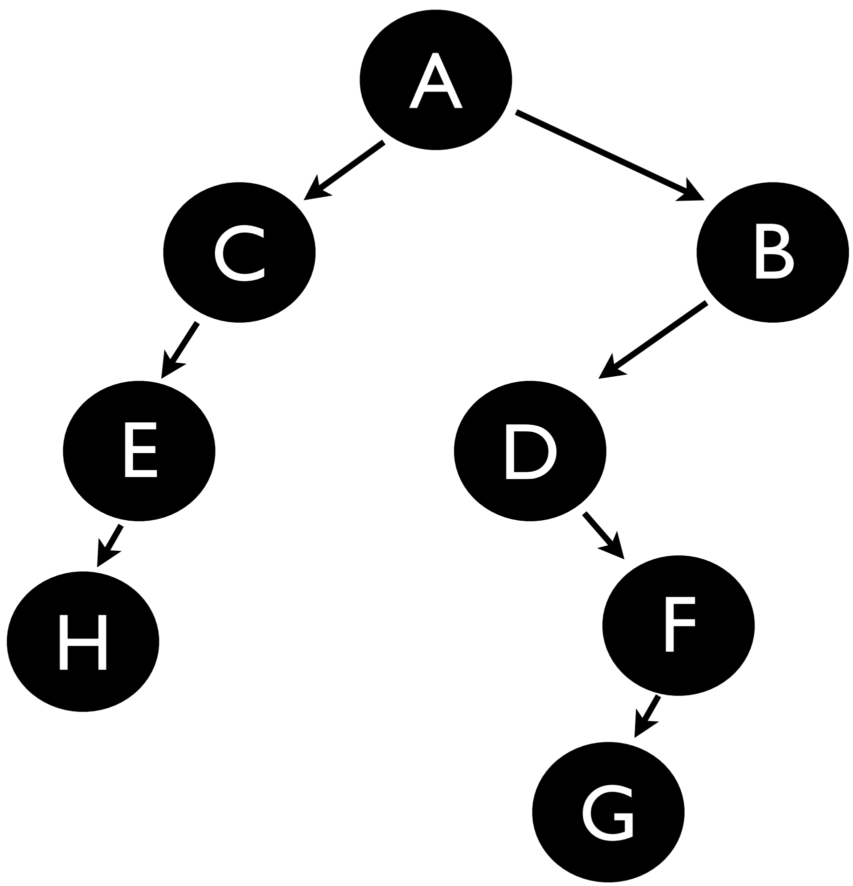
- Binary search tree
  - each level splits on alternating keys

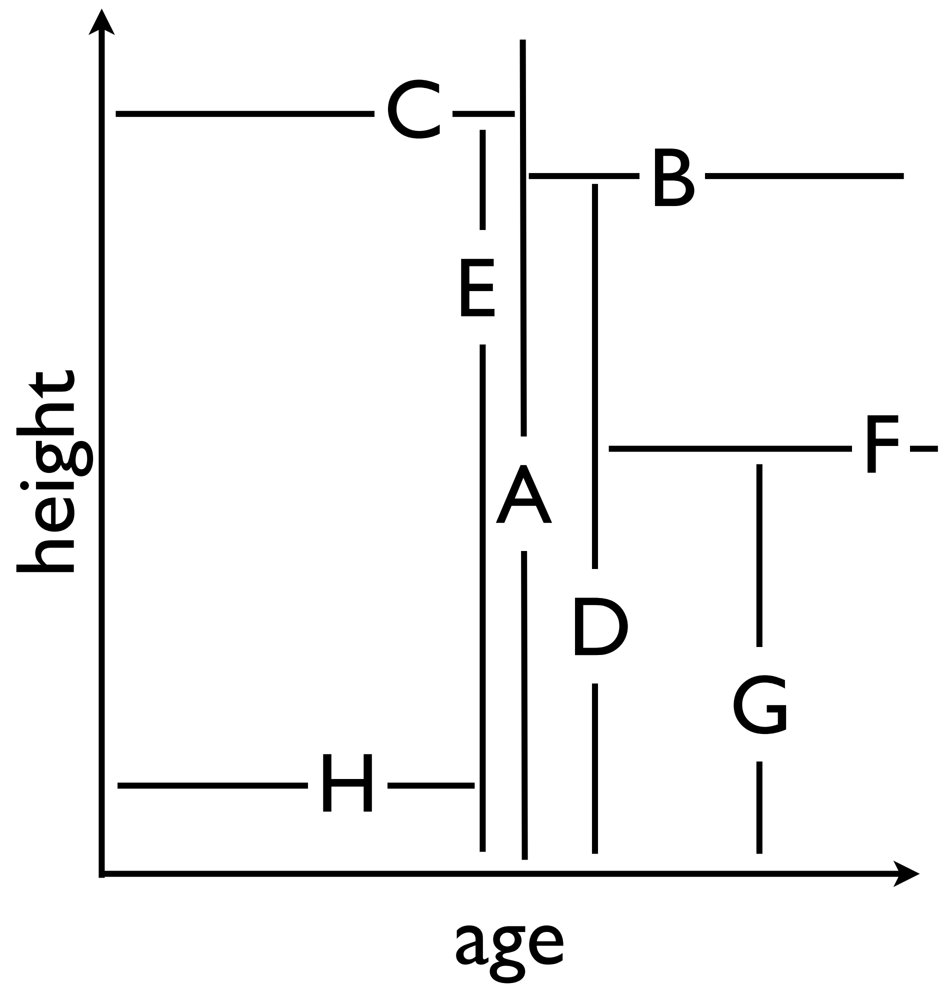


Search Algorithm

- Given lower and upper bounds for each dimension
- If key is in range, print
  If key > current dimension's lower bound
      search left child
  If key < current dimension's upper bound
      search right child
- Insert recursion is just like BST

2-d Tree

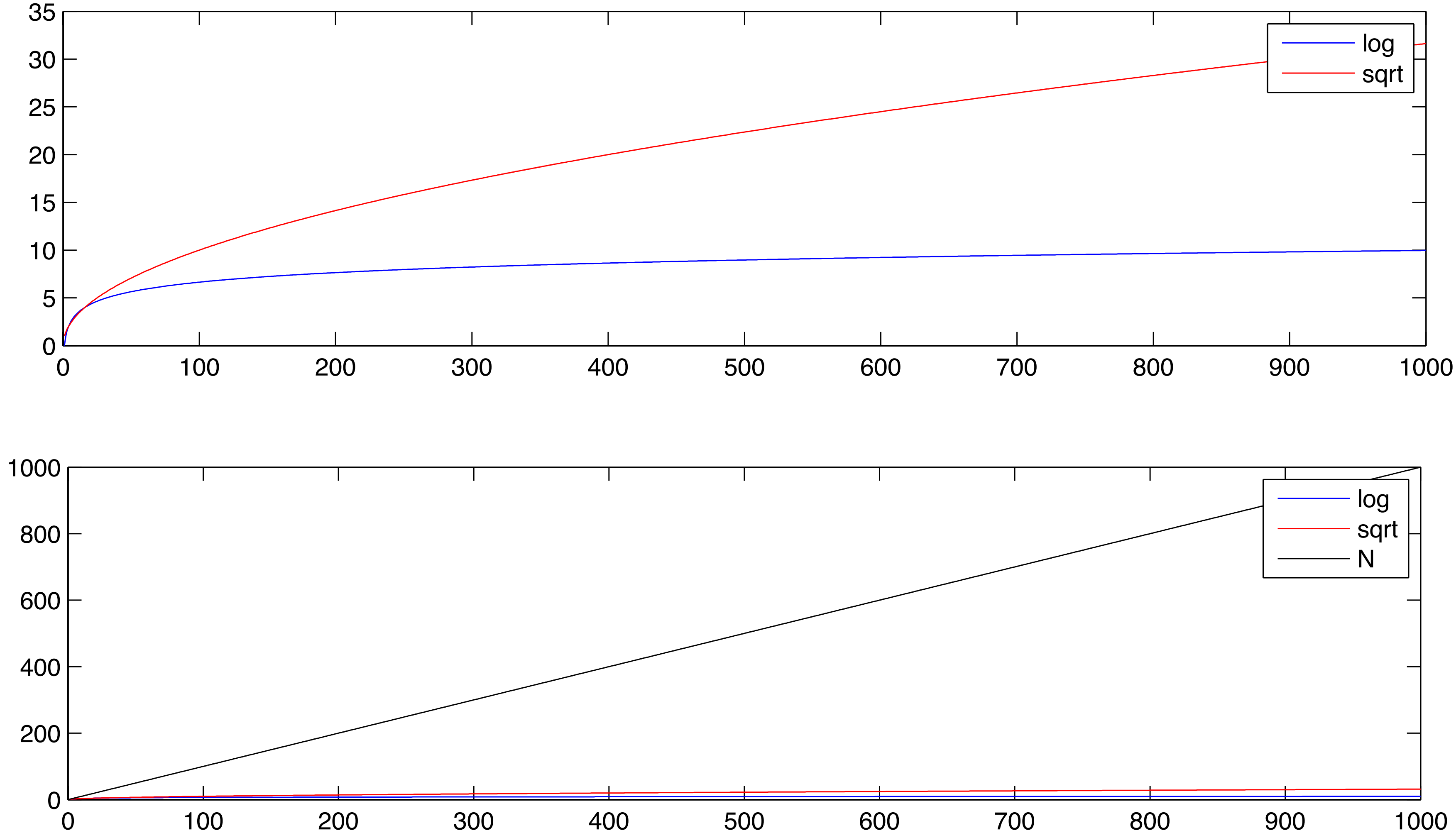




kd-Tree Analysis

- Since each level represents a different keys, balancing is not possible
- If we have all the points, we can build a perfectly balanced tree. How?
- Then worst case $O(M + kN^{1-1/k})$

Square Root vs. Log



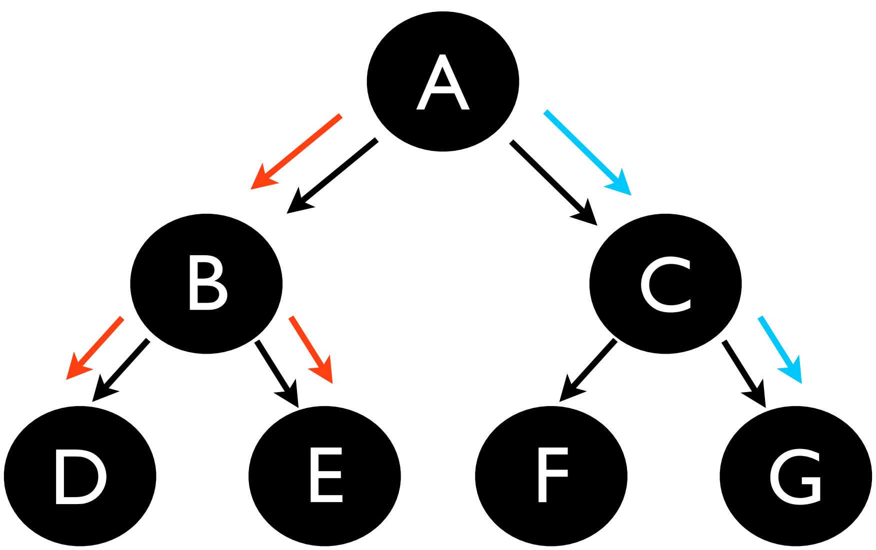
Nearest Neighbor Search

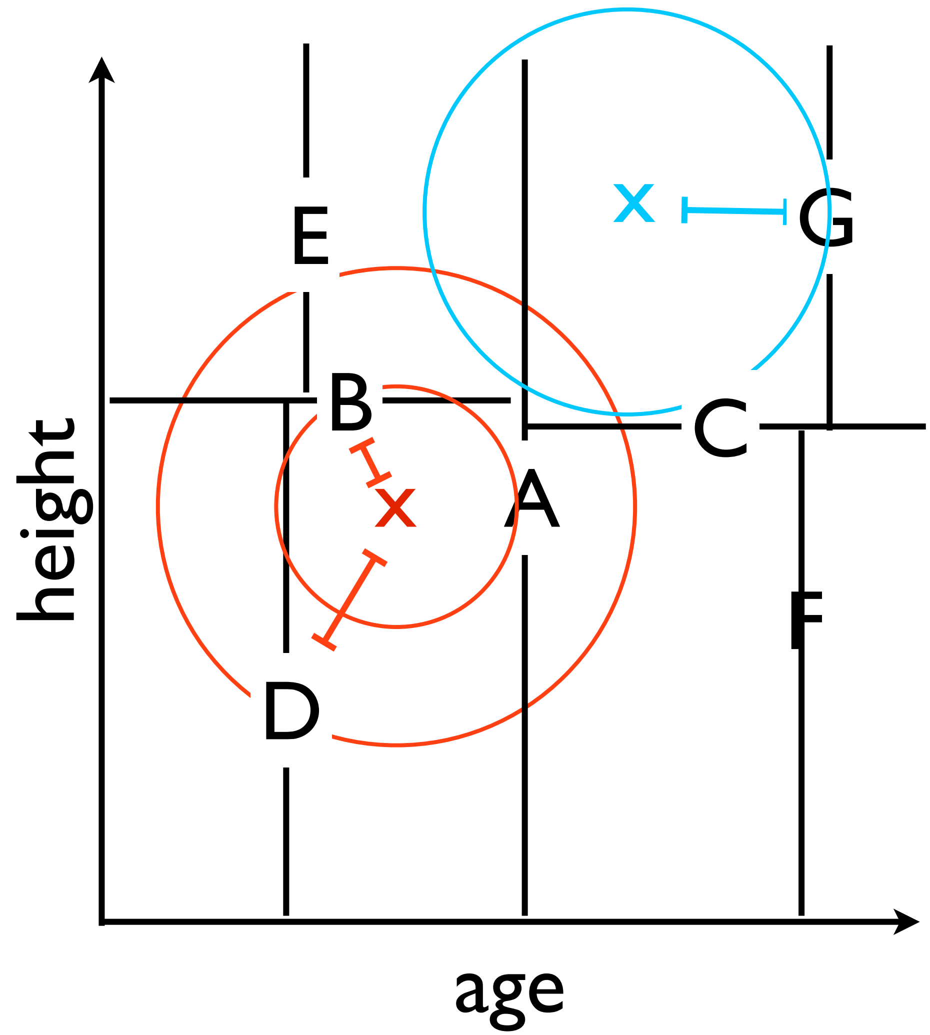
- kd-trees are especially helpful for finding nearest neighbors
- Given a data set, find the nearest point to any element **x**
- Naive O(N) approach is to compute distances everywhere
- Instead, kd-tree offers $O(kN^{1-1/k})$

Nearest Neighbor Algorithm

- Search for **x** in the kd-tree until you reach a leaf
    - Consider leaf point *current-best*
- Backtrack along search path, and at each node:
    - If current point is better, redefine *current-best*
    - If best can be in the unexplored child*, recurse down the unexplored child

Algorithm Illustration





kd-Tree Summary

- Smart data structure for storing and searching multi-dimensional keys
  - Branch BST cycling dimensions at each level
- Running time is sub-linear, but grows to linear when k is infinite
- Efficient range search, search, and nearest-neighbor search

Reading

- Complexity - Weiss 9.7
- kd-trees - Weiss 12.6