Announcements

• Homework 5 solutions posted
• Homework 6 to be posted this weekend
• Final exam Thursday, Dec. 17th, 4-7 PM, Hamilton 602 (this room)
  • same format as midterm (open book/notes)
• Distinguished Lecture: Barbara Liskov, MIT. Turing Award Winner 2009.
  11 AM Monday. Davis Auditorium, CEPSR/Schapiro
Review

• Sorting Algorithm Examples
• QuickSort space clarification
• External sorting*
Today’s Plan

• Correction on external sorting
• Complexity Classes
  • P, NP, NP-Complete, NP-Hard
External Sorting

Disk 3
Disk 2
Disk 1
Disk 0
External Sorting
External Sorting

Disk 0

Disk 1

Disk 2

Disk 3
External Sorting
External Sorting
Complexity of Problems

- We’ve been concerned with the complexity of algorithms
- It is important to also consider the complexity of problems
- Understanding complexity is important for theory, and also for practice
  - understanding the hardness of problems helps us build better algorithms
Three Graph Problems

• Euler Path: does a path exist that uses every edge exactly once?

• Hamiltonian Path: does a path exist that visits every node exactly once?

• Traveling Salesman: find the shortest path that visits every node exactly once?
Complexity Classes

- **P** - solvable in polynomial time
- **NP** - solvable in polynomial time by a nondeterministic computer
  - i.e., you can check a solution in polynomial time
- **NP-complete** - a problem in NP such that any problem in NP is polynomially reducible to it
- **Undecidable** - no algorithm can solve the problem
Probable Complexity Class Hierarchy

P

NP

NP-Complete

NP-Hard

Undecidable
Polynomial Time $\mathbb{P}$

- All the algorithms we cover in class are solvable in polynomial time
- An algorithm that runs in polynomial time is considered *efficient*
- A problem solvable in polynomial time is considered *tractable*
Nondeterministic Polynomial Time $\text{NP}$

- Consider a magical nondeterministic computer
  - infinitely parallel computer
- Equivalently, to solve any problem, check every possible solution in parallel
  - return one that passes the check
NP-Complete

- Special class of NP problems that can be used to solve any other NP problem
- Hamiltonian Path, Satisfiability, Graph Coloring etc.
- NP-Complete problems can be reduced to other NP-Complete problems:
  - polynomial time algorithm to convert the input and output of algorithms
NP-Hard

- A problem is NP-Hard if it is at least as complex as all NP-Complete problems
- NP-hard problems may not even be NP
Undecidable

- No algorithm can solve undecidable problems
- **halting problem** - given a computer program, determine if the computer program will finish

- Sketch of undecidability proof: Assume we have an algorithm that detects infinite loops
  - Write program LOOP(P) that runs P on itself
  - If P(P) halts, infinite loop, otherwise output
Halting Problem

LOOP(P):
If P(P) will halt: infinite loop
If P(P) runs forever: output

• What happens if we call LOOP(LOOP)?

LOOP(LOOP):
If LOOP(LOOP) will halt: infinite loop
If LOOP(LOOP) runs forever: output
P vs. NP

- So far, nobody has proven a super-polynomial lower bound on any NP-Complete problems,
- nor has anyone found a polynomial time algorithm for an NP-complete problem
- Therefore no problem is proven to be in one class but not the other;
- it is unknown if P and NP are the same
Complexity Class Hierarchy

- P?
- P?
- NP
- NP-Complete
- NP-Hard
- Undecidable
P Problems

- Most problems we’ve solved
- Proving a polynomial (or logarithmic) bound on any algorithm proves a problem is in P
- Euler paths - does a path exist that uses every edge?
- Return if there are zero or two nodes with odd degree.
Finding an Euler Circuit

- Run a partial DFS; search down a path until you need to backtrack (mark edges instead of nodes)
- At this point, you will have found a circuit
- Find first node along the circuit that has unvisited edges; run a DFS starting with that edge
- Splice the new circuit into the main circuit, repeat until all edges are visited
Euler Circuit Example
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NP-Complete Problems
Satisfiability

• Given Boolean expression of N variables, can we set variables to make expression true?

• First NP-Complete proof because Cook’s Theorem gave polynomial time procedure to convert any NP problem to a Boolean expression

• I.e., if we have efficient algorithm for Satisfiability, we can efficiently solve any NP problem
NP-Complete Problems
Graph Coloring

• Given a graph is it possible to color with \( k \) colors all nodes so no adjacent nodes are the same color?

• Coloring countries on a map

• Sudoku is a form of this problem. All squares in a row, column and blocks are connected. \( k = 9 \)
NP-Complete Problems

Hamiltonian Path

• Given a graph with N nodes, is there a path that visits each node exactly once?
NP-Hard Problems

Traveling Salesman

- Closely related to Hamiltonian Path problem
- Given complete graph $G$, find the shortest path that visits all nodes
- If we are able to solve TSP, we can find a Hamiltonian Path; set connected edge weight to constant, disconnected to infinity
- TSP is NP-hard
Reading

- Weiss 9.7
- http://www.tsp.gatech.edu/