Data Structures in Java

Session 22

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Announcements

- Homework 5 solutions posted
- Homework 6 to be posted this weekend
- Final exam Thursday, Dec. 17th, 4-7 PM, Hamilton 602 (this room)
  - same format as midterm (open book/notes)
- Distinguished Lecture: Barbara Liskov, MIT.
  Turing Award Winner 2009.
  11 AM Monday. Davis Auditorium, CEPSR/Schapiro

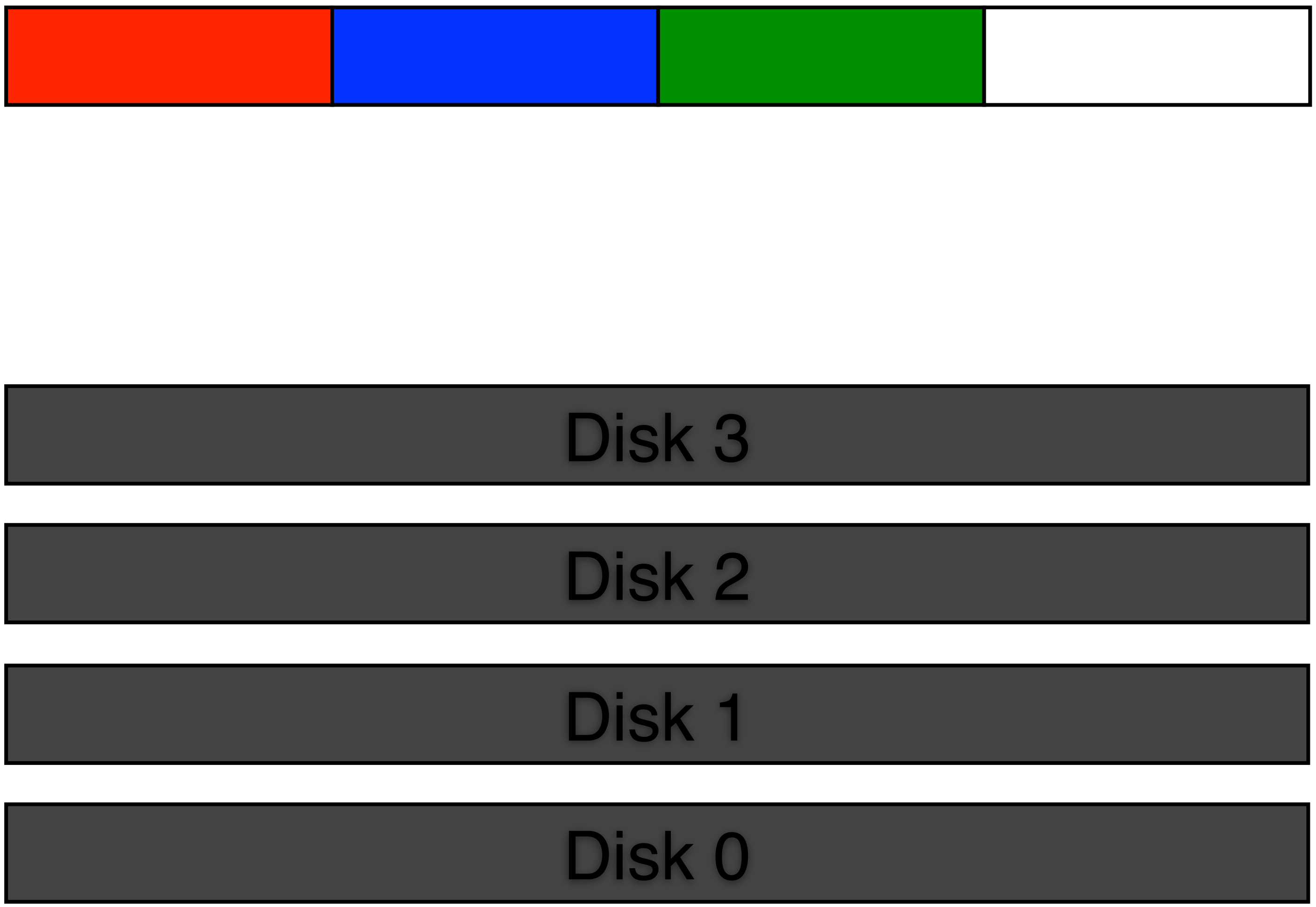
Review

- Sorting Algorithm Examples
- QuickSort space clarification
- External sorting*

Today's Plan

- Correction on external sorting
- Complexity Classes
  - P, NP, NP-Complete, NP-Hard

External Sorting



External Sorting

External Sorting

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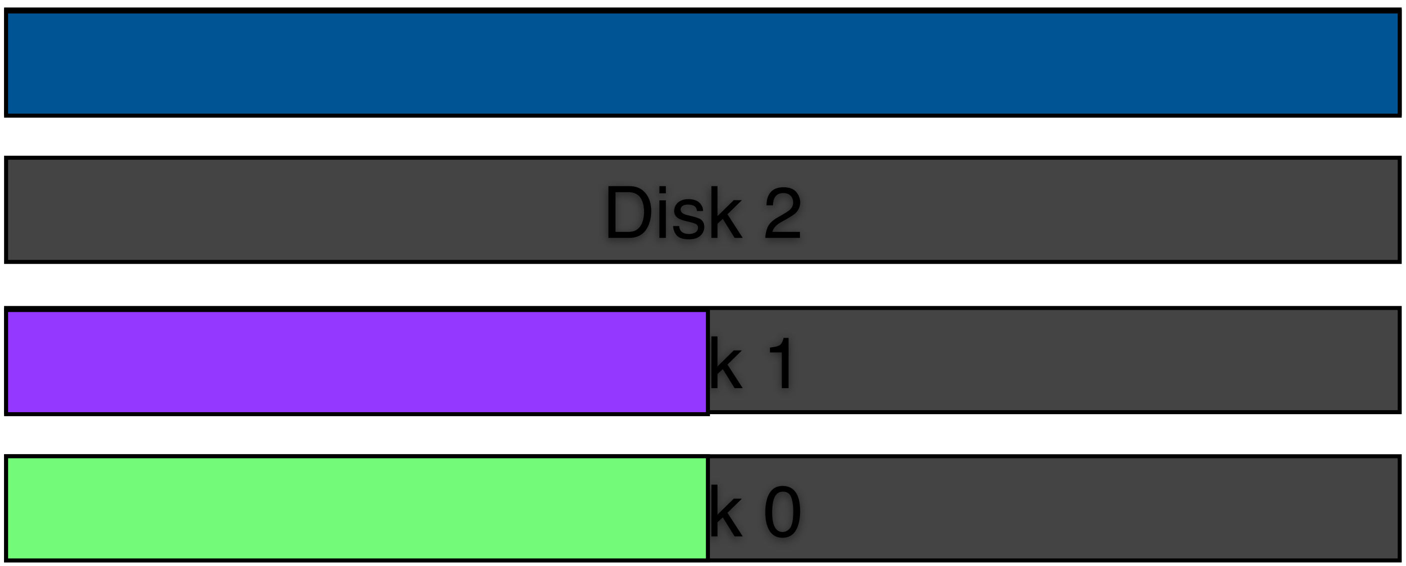
External Sorting



Complexity of Problems

- We’ve been concerned with the complexity of *algorithms*
- It is important to also consider the complexity of *problems*
- Understanding complexity is important for theory, and also for practice
    - understanding the hardness of problems helps us build better algorithms

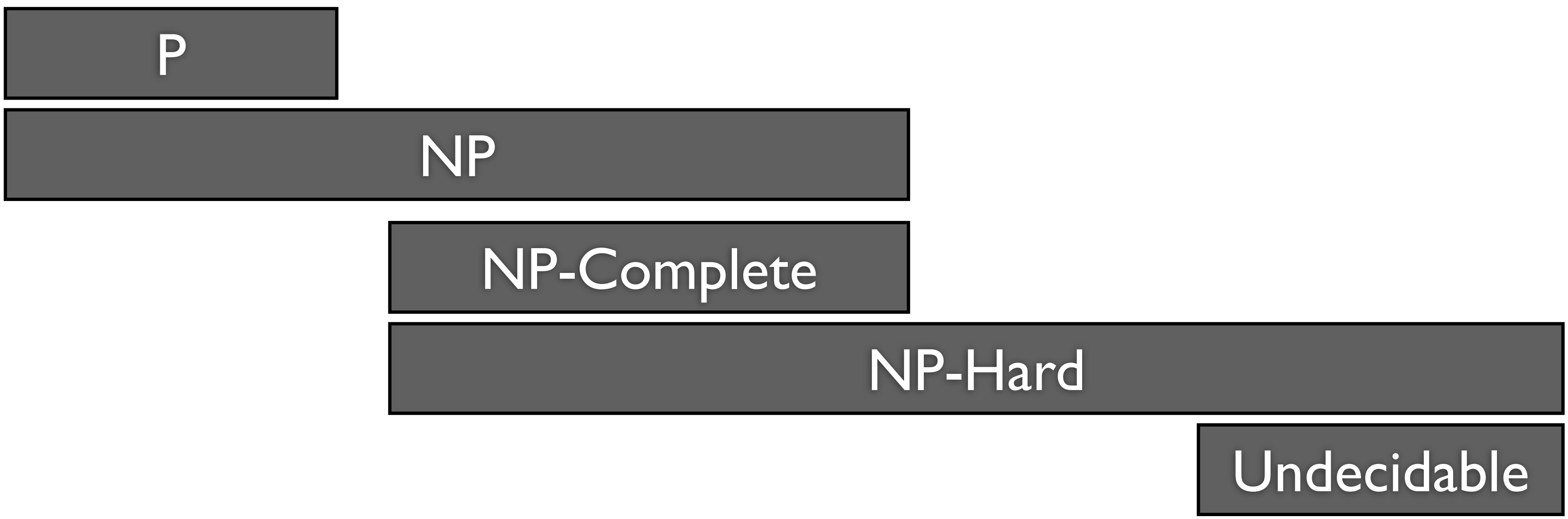
Three Graph Problems

- Euler Path: does a path exist that uses every **edge** exactly once?
- Hamiltonian Path: does a path exist that visits every **node** exactly once?
- Traveling Salesman: find the shortest path that visits every node exactly once?

Complexity Classes

- **P** - solvable in polynomial time
- **NP** - solvable in polynomial time by a nondeterministic computer
  - i.e., you can check a solution in polynomial time
- **NP-complete** - a problem in NP such that any problem in NP is polynomially reducible to it
- **Undecidable** - no algorithm can solve the problem

Probable Complexity Class Hierarchy



Polynomial Time **P**

- All the algorithms we cover in class are solvable in polynomial time
- An algorithm that runs in polynomial time is considered **efficient**
- A problem solvable in polynomial time is considered **tractable**

Nondeterministic Polynomial Time **NP**

- Consider a magical nondeterministic computer
  - infinitely parallel computer
- Equivalently, to solve any problem, check every possible solution in parallel
  - return one that passes the check

NP-Complete

- Special class of NP problems that can be used to solve any other NP problem
- Hamiltonian Path, Satisfiability, Graph Coloring etc.
- NP-Complete problems can be **reduced** to other NP-Complete problems:
  - polynomial time algorithm to convert the input and output of algorithms

NP-Hard

- A problem is NP-Hard if it is at least as complex as all NP-Complete problems
- NP-hard problems may not even be NP

Undecidable

- No algorithm can solve undecidable problems
- **halting problem -** given a computer program, determine if the computer program will finish
- Sketch of undecidability proof: Assume we have an algorithm that detects infinite loops
  - Write program LOOP(P) that runs P on itself
  - If P(P) halts, infinite loop, otherwise output

Halting Problem

```
LOOP(P):
  If P(P) will halt:    infinite loop
  If P(P) runs forever: output
```

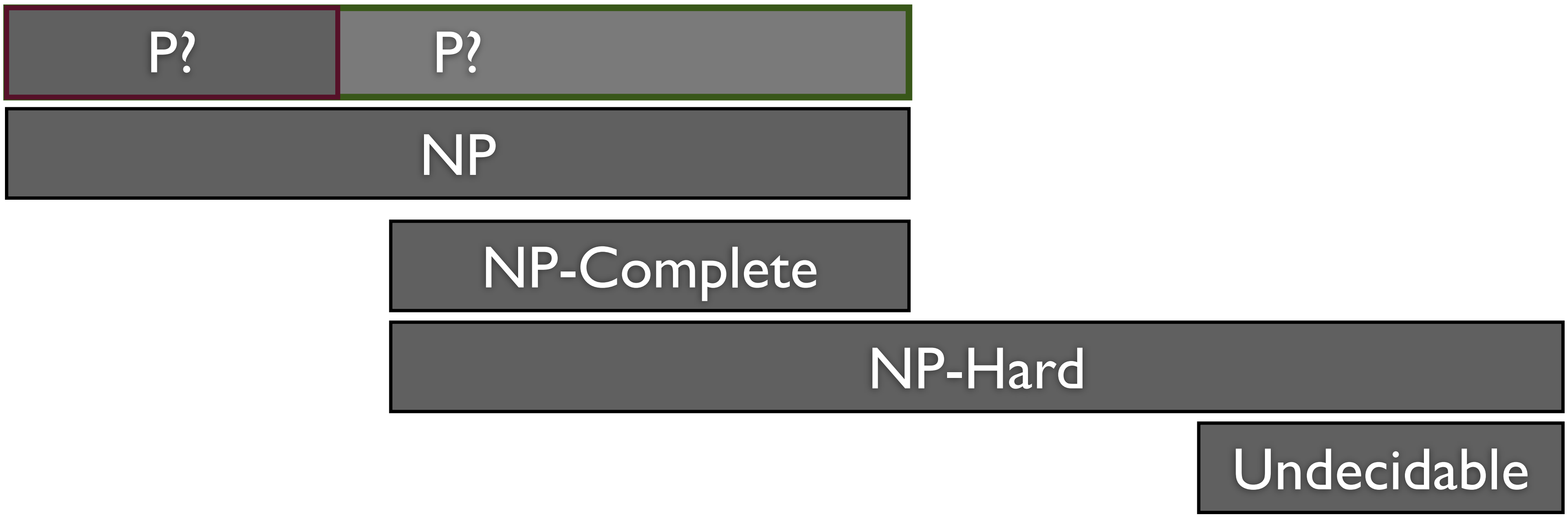
- What happens if we call LOOP(LOOP)?

```
LOOP(LOOP):
  If LOOP(LOOP) will halt:    infinite loop
  If LOOP(LOOP) runs forever: output
```

P vs. NP

- So far, nobody has proven a super-polynomial lower bound on any NP-Complete problems,
- nor has anyone found a polynomial time algorithm for an NP-complete problem
- Therefore no problem is proven to be in one class but not the other;
  - it is unknown if P and NP are the same

Complexity Class Hierarchy



P Problems

- Most problems we’ve solved
- Proving a polynomial (or logarithmic) bound on any algorithm proves a problem is in **P**
- Euler paths - does a path exist that uses every edge?
    - Return if there are zero or two nodes with odd degree.

Finding an Euler Circuit

- Run a partial DFS; search down a path until you need to backtrack (mark edges instead of nodes)
- At this point, you will have found a circuit
- Find first node along the circuit that has unvisited edges; run a DFS starting with that edge
- Splice the new circuit into the main circuit, repeat until all edges are visited

NP-Complete Problems Satisfiability

- Given Boolean expression of N variables, can we set variables to make expression true?
- First NP-Complete proof because Cook's Theorem gave polynomial time procedure to convert any NP problem to a Boolean expression
- I.e., if we have efficient algorithm for Satisfiability, we can efficiently solve any NP problem

NP-Complete Problems Graph Coloring

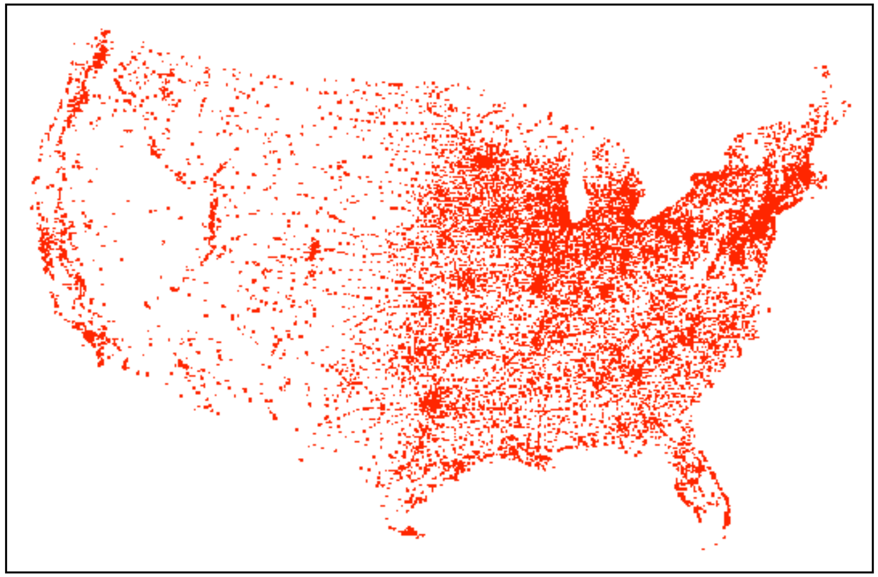
- Given a graph is it possible to color with **k** colors all nodes so no adjacent nodes are the same color?
- Coloring countries on a map
- Sudoku is a form of this problem. All squares in a row, column and blocks are connected. **k** = 9

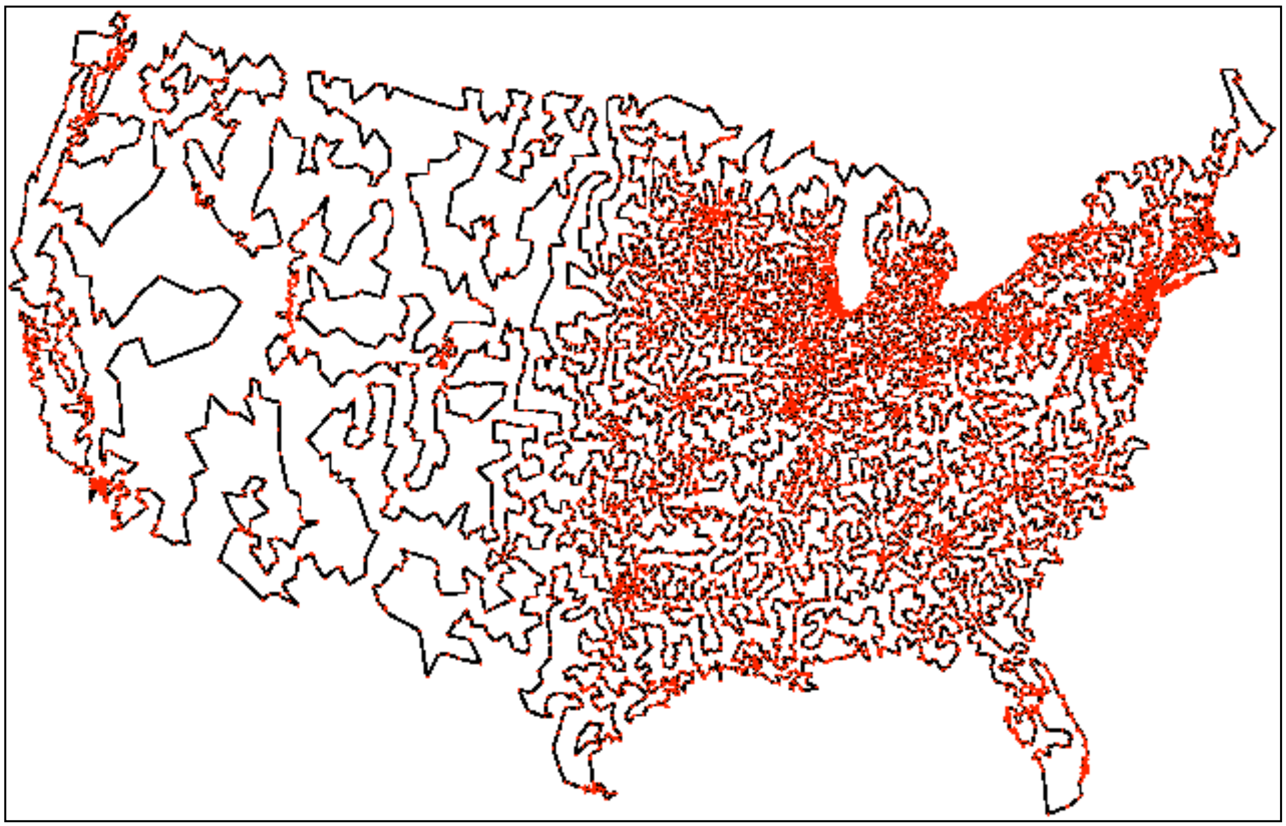
NP-Complete Problems Hamiltonian Path

- Given a graph with N nodes, is there a path that visits each node exactly once?

NP-Hard Problems Traveling Salesman

- Closely related to Hamiltonian Path problem
- Given complete graph **G**, find the shortest path that visits all nodes
- If we are able to solve TSP, we can find a Hamiltonian Path; set connected edge weight to constant, disconnected to infinity
    - TSP is NP-hard





http://www.tsp.gatech.edu/gallery/itours/usa13509.html

Reading

- Weiss 9.7
- http://www.tsp.gatech.edu/