Data Structures in Java

Session 11

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Announcements

- Homework 2 solutions posted
- Homework 3 due 10/20
- Midterm Exam, open book/notes 10/22
  - see theory problems for examples
- My office hours this week 4-6 PM

Review

- Amortized Running time
  - Splay Trees
- Tries

Priority Queues

- New abstract data type Priority Queue
  - Insert: add node with key
  - deleteMin: delete the node with smallest key
  - findMin: access the node with smallest key
  - (increase/decrease priority)

Tradeoffs

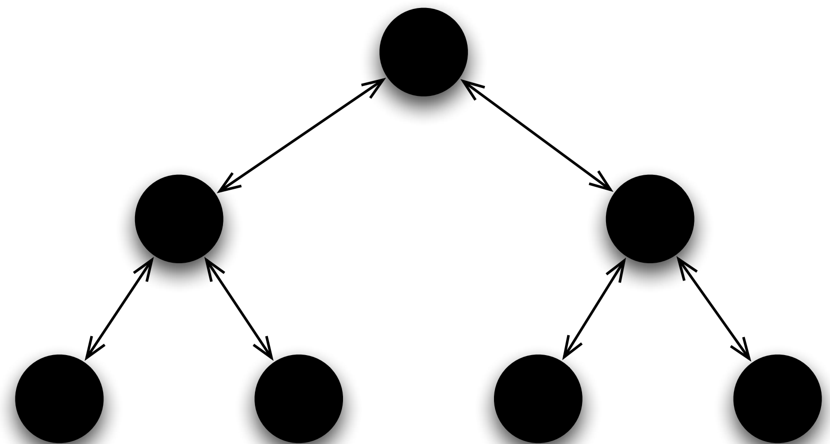
- Binary search trees contain full order information (inorder returns sorted list)
- Priority queues only maintain efficient method to find minimum element
- Loss in functionality is worth it for gain in speed

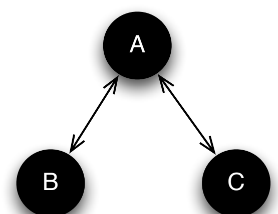
Simple Implementations

- Use a list
  - O(1) insert, O(N) deleteMin/findMin
- Use a balanced BST
  - O(log N) insert/deleteMin*/findMin
  - deleting min from BST leads to imbalance

Heap Implementation

- Binary tree with special properties
- Heap Structure Property: all nodes are full*
- Heap Order Property: any node is smaller than its children





$$A < B$$
$$A < C$$
$$C \; ? \; B$$

Array Implementation

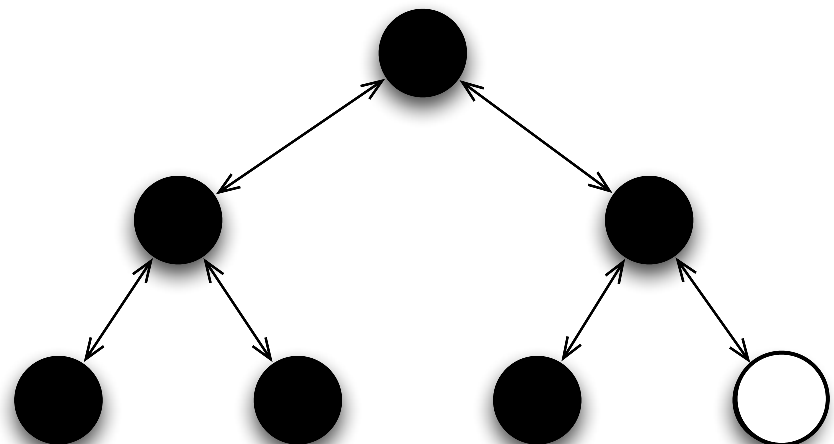
- A full tree is regular: we can store in an array
  - Root at **A[1]**
  - Root's children at **A[2]**, **A[3]**
  - Node **i** has children at **2i** and **(2i+1)**
  - Parent at **floor(i/2)**
- No links necessary, so much faster (but only constant speedup)

Array Implementation

- A full tree is regular: we can easily store in an array
  - Root at **A[0]**
  - Root's children at **A[1]**, **A[2]**
  - Node **i** has children at **2(i+1)-1** and **2(i+1)**
  - Parent at **floor((i+1)/2)-1**
- No links necessary, so faster (in most languages)

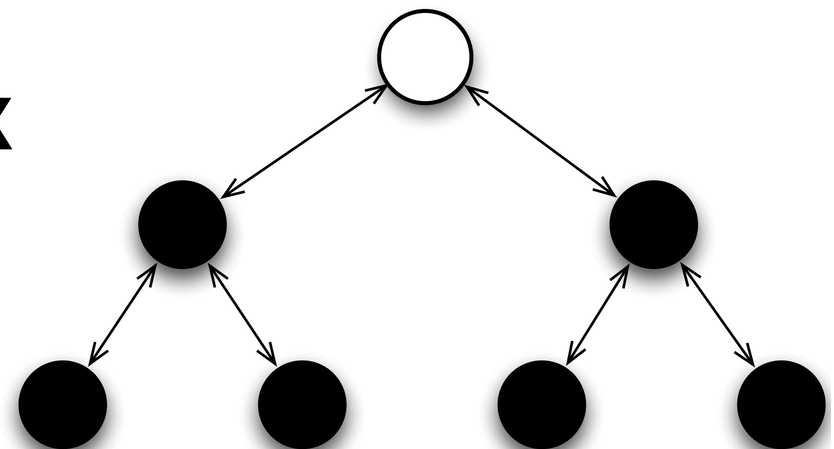
Insert

- To insert key **X**, create a hole in bottom level
- **Percolate up**
  - Is hole's parent is less than **X**
    - If so, put **X** in hole, heap order satisfied
    - If not, swap hole and parent and repeat



DeleteMin

- Save root node, and delete, creating a hole
- Take the last element in the heap **X**
- **Percolate down**:
  - is X is less than hole's children?
    - if so, we're done
    - if not, swap hole and smallest child and repeat



Changing a key

- Assuming you allow direct access to elements in heap
- decreaseKey: lower key, percolate up
- increaseKey: raise key, percolate down

Running times

- Insert/deleteMin O(log N)
- findMin O(1)
- Where's the big gain?
  - buildHeap: given N items, creates a heap in linear time

Reading

- This class and next: Weiss 6.1-6.3