**Problem 1** (10 points):
(a) Find the optimal decision regions for a minimum error rate classifier where:
- \( \Omega = \{ \omega_1, \omega_2 \} \)
- \( p(x | \omega_1) \sim N(2, 0.5) \) (The normal distribution is written as \( N(\mu, \sigma^2) \).)
- \( p(x | \omega_2) \sim N(1.5, 0.2) \)
- \( \omega_1 \) and \( \omega_2 \) are equally probable

(b) Now find the optimal decision regions with the following priors:
- \( P(\omega_1) = 2/3 \)
- \( P(\omega_2) = 1/3 \)

Note: Please give the exact solution as well as a numeric approximation (e.g. \( x = \sqrt{4 - \pi} \approx 0.9265 \)).

**Problem 2** (10 points):
Consider a two-class classification problem, where the feature is two-dimensional.

- \( p(x | \omega_1) \) is normal with mean \( \begin{bmatrix} 2 \\ 3 \end{bmatrix} \) and covariance matrix \( \begin{bmatrix} 0.1 & 0.3 \\ 0.3 & 1.0 \end{bmatrix} \)
- \( p(x | \omega_2) \) is normal with mean \( \begin{bmatrix} 4 \\ 4 \end{bmatrix} \) and covariance \( \begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.85 \end{bmatrix} \)
- The priors are \( P(\omega_1) = 3/5, P(\omega_2) = 2/5 \)

Give the most likely class (\( \omega_1 \) or \( \omega_2 \)) for these 5 samples. Justify your answer.
- \( x_1 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, x_2 = \begin{bmatrix} 3.7 \\ 3.2 \end{bmatrix}, x_3 = \begin{bmatrix} 3.1 \\ 3.5 \end{bmatrix}, x_4 = \begin{bmatrix} 2 \\ 3.8 \end{bmatrix}, x_5 = \begin{bmatrix} 3.5 \\ 3.2 \end{bmatrix} \)

**Problem 3** (10 points):
Write down and solve your own probability problem in which Bayes’ Rule is needed for the solution.
Problem 4 (required only for 6000 level) (10 points):
Find a scholarly paper in a field outside biometrics that uses Bayes' Rule. Give the citation and write a two- or three- paragraph summary, including an explanation of the author’s use of Bayes' Rule.