Potential Field Path Planning

Reference:

Principles of Robot Motion
H. Choset et.al.
Mit Press

Also: Siegwart text, section 6.3.2
Potential Field Path Planning

• Simple idea: Have robot “attracted” to the goal and “repelled” from the obstacles
• Think of robot as a positively charged particle moving towards negatively charged goal – attractive force
• Obstacles have same charge as robot – repelling force
• States far away from goal have large potential energy, goal state has zero potential energy
• Path of robot is from state of high energy to low (zero) energy at the goal
• Think of the planning space as an elevated surface, and the robot is a marble rolling “downhill” towards the goal
Potential Field Path Planning

Repulsive force

Attractive force

start

goal
Potential Field Path Planning

- A potential function is a function that may be viewed as energy
- the gradient of the energy is force
- Potential function guides the robot as if it were a particle moving in a gradient field.
- Analogy: robot is positively charged particle, moving towards negative charge goal
- Obstacles have “repulsive” positive charge
• Potential functions can be viewed as a landscape
• Robot moves from high-value to low-value
  Using a “downhill” path (i.e. negative of the gradient).
• This is known as gradient descent – follow a functional surface until you reach its minimum
• Attractive Potential Function is distance from goal
• High energy away from goal, Zero at goal
• Path is negative gradient, largest change in energy
Figure 4.2  Different types of critical points: (Top) Graphs of functions. (Bottom) Gradients of functions.
Figure 4.3  (a) A configuration space with three circular obstacles bounded by a circle. (b) Potential function energy surface. (c) Contour plot for energy surface. (d) Gradient vectors for potential function.
Computing Attractive Potential

• With point robot, we can use distance as a measure, and velocity becomes the gradient
• Simplest: scaled distance to goal. Gradient becomes a constant, undefined at goal
• Better: Use a continuously differentiable function – e.g. Quadratic Function of distance:

\[ U_{\text{att}}(q) = \frac{1}{2} \zeta d^2(q, q_{\text{goal}}), \]

with the gradient

\[ \nabla U_{\text{att}}(q) = \nabla \left( \frac{1}{2} \zeta d^2(q, q_{\text{goal}}) \right), \]

\[ = \frac{1}{2} \zeta \nabla d^2(q, q_{\text{goal}}), \]

\[ = \zeta (q - q_{\text{goal}}), \]
Figure 4.4  (a) Attractive gradient vector field. (b) Attractive potential isocontours. (c) Graph of the attractive potential.
Computing Repulsive Potential

- Strength of repulsive force should increase as we near the obstacle
- Compute potential in terms of distance to closest obstacle
- Multiple obstacles: compute repulsive potential over all obstacles

\[
U_{\text{rep}}(q) = \begin{cases} 
\frac{1}{2} \eta \left( \frac{1}{D(q)} - \frac{1}{Q^*} \right)^2, & D(q) \leq Q^*, \\
0, & D(q) > Q^*, 
\end{cases}
\]

whose gradient is

\[
\nabla U_{\text{rep}}(q) = \begin{cases} 
\eta \left( \frac{1}{Q^*} - \frac{1}{D(q)} \right) \frac{1}{D^2(q)} \nabla D(q), & D(q) \leq Q^*, \\
0, & D(q) > Q^*, 
\end{cases}
\]
Figure 4.5  The repulsive gradient operates only in a domain near the obstacle.
Online Distance Computation

Figure 4.7  Local minima of rays determine the distance to nearby obstacles.
Total Potential Function

\[ U(q) = U_{\text{att}}(q) + U_{\text{rep}}(q) \]

\[ F(q) = -\nabla U(q) \]
• Obstacles create high energy barriers
• Gradient descent follows energy minimization path to goal
Gradient Descent:

- $q(0) = q_{\text{start}}$
- $i = 0$
- while $\| \nabla U(q(i)) \| > \varepsilon$ do
  - $q(i+1) = q(i) - \alpha(i) \nabla U(q(i))$
  - $i = i + 1$
Potential Field Limitations

Local minimum:
attractive force (goal) = repulsive force (obstacles)
Local minimum: attractive force = repulsive force
Solution: Take a random walk – perturb out of minima
Need to remember where you have been!
Potential Fields Summary

• More than just a path planner: Provides simple control function to move robot: gradient descent
• Allows robot to move from wherever it finds itself
• Can get trapped in local minima
• Can be used as online, local method:
  – As robot encounters new obstacles, compute the Potential Function online
  – Laser/sonar scans give online distance to obstacles