Potential Field Path Planning

Reference:

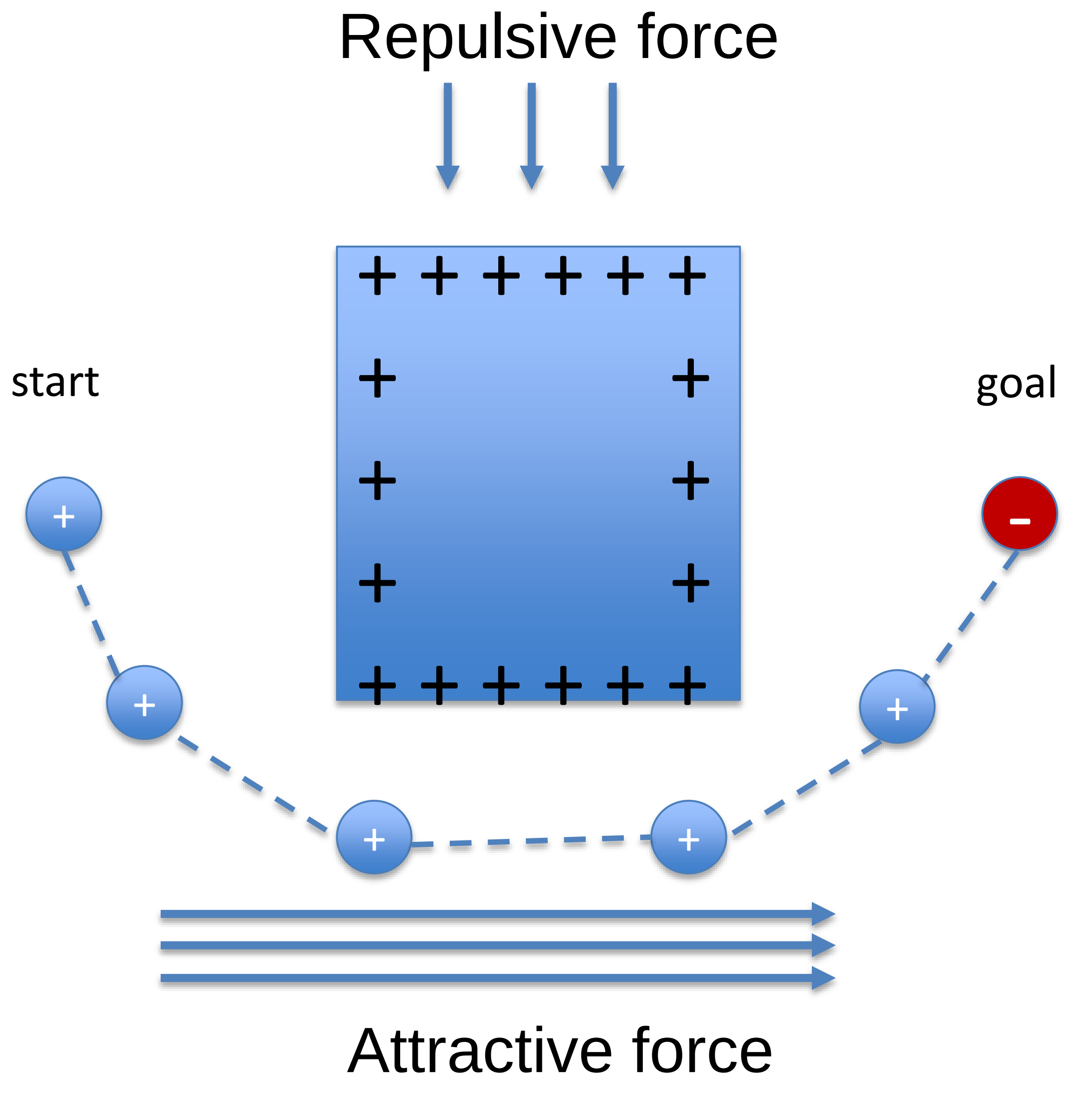
Principles of Robot Motion
H. Choset et.al.
Mit Press

Also: Siegwart text, section 6.3.2

Potential Field Path Planning

- Simple idea: Have robot “attracted” to the goal and “repelled” from the obstacles
- Think of robot as a positively charged particle moving towards negatively charged goal – attractive force
- Obstacles have same charge as robot – repelling force
- States far away from goal have large potential energy, goal state has zero potential energy
- Path of robot is from state of high energy to low (zero) energy at the goal
- Think of the planning space as an elevated surface, and the robot is a marble rolling “downhill” towards the goal

Potential Field Path Planning



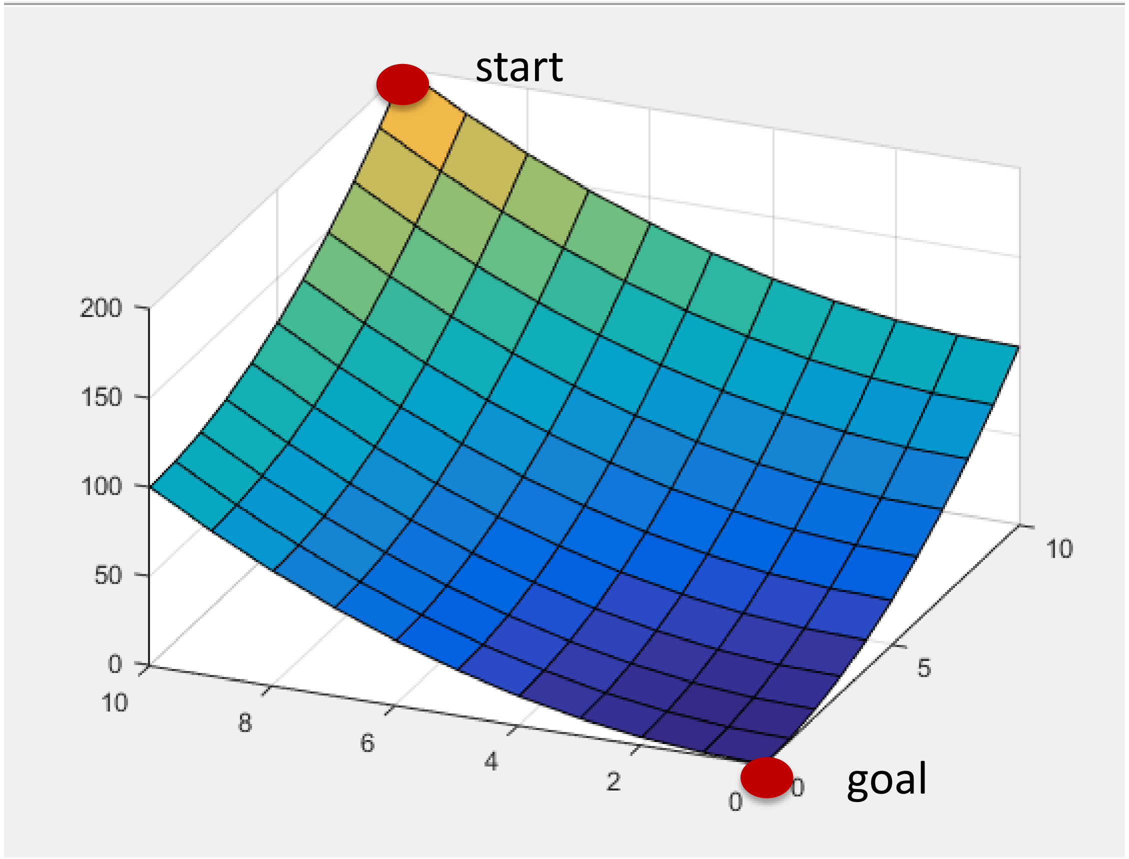
Potential Field Path Planning

- A potential function is a function that may be viewed as energy
- the gradient of the energy is force
- Potential function guides the robot as if it were a particle moving in a gradient field.
- Analogy: robot is positively charged particle, moving towards negative charge goal
- Obstacles have “repulsive” positive charge

- Potential functions can be viewed as a landscape
- Robot moves from high-value to low-value

  Using a “downhill” path (i.e negative of the gradient).
- This is known as gradient descent –follow a functional surface until you reach its minimum

Potential Field



- Attractive Potential Function is distance from goal
- High energy away from goal, Zero at goal
- Path is negative gradient, largest change in energy

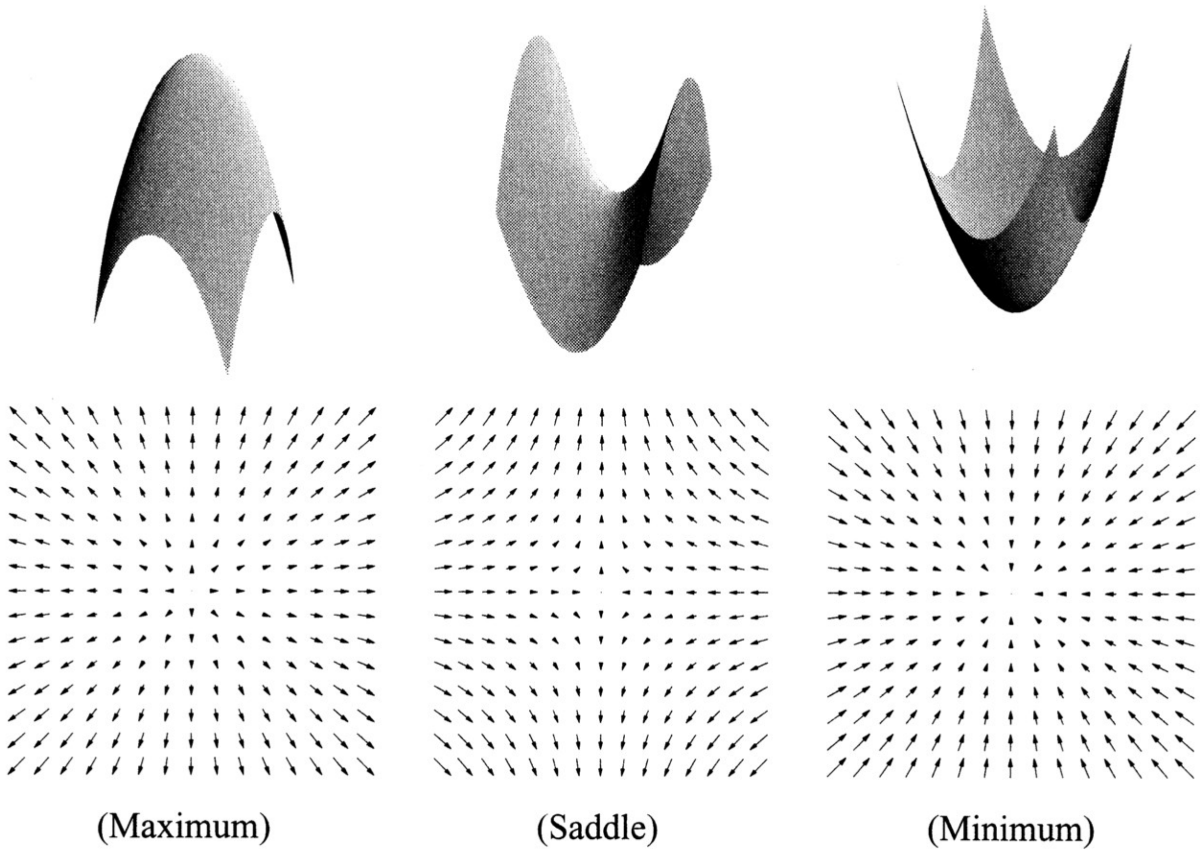
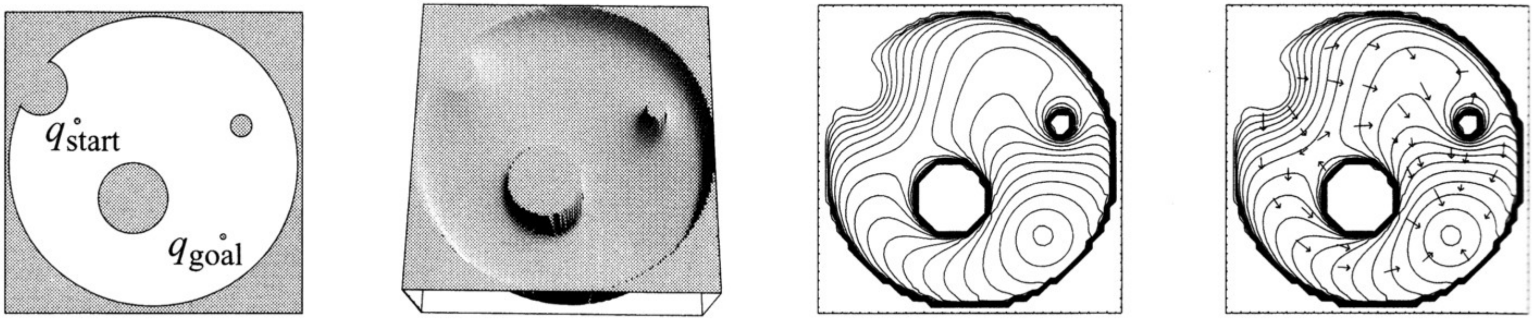


Figure 4.2 Different types of critical points: (Top) Graphs of functions. (Bottom) Gradients of functions.



(a) (b) (c) (d)

Figure 4.3 (a) A configuration space with three circular obstacles bounded by a circle. (b) Potential function energy surface. (c) Contour plot for energy surface. (d) Gradient vectors for potential function.

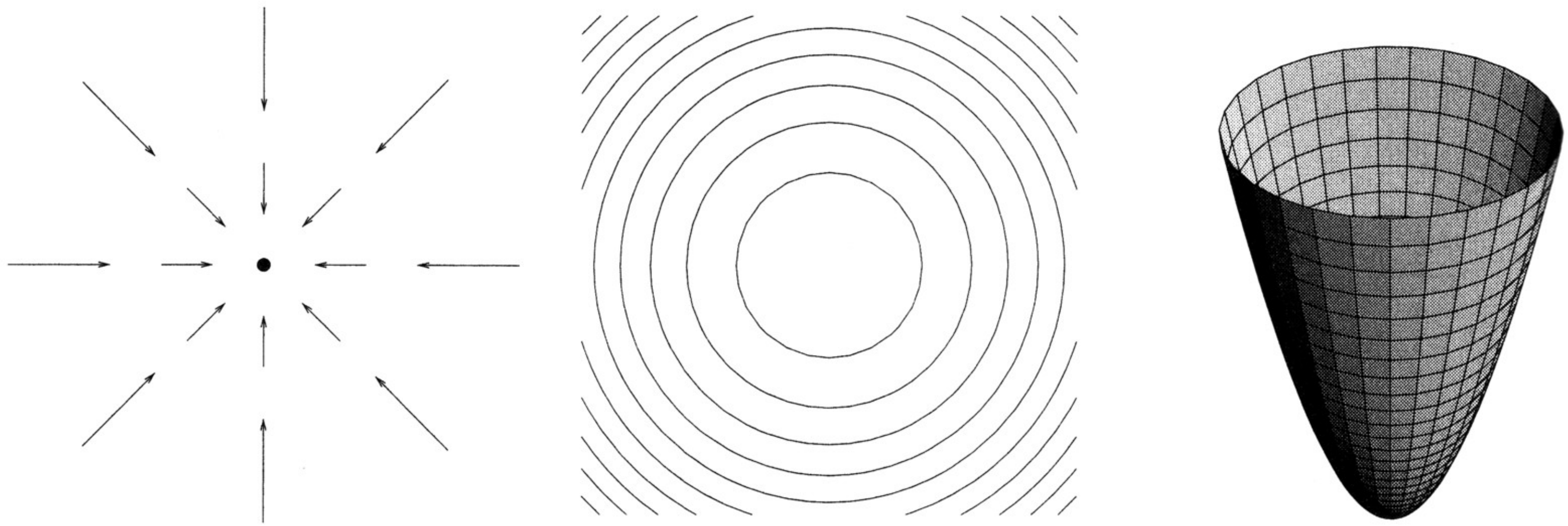
Computing Attractive Potential

- With point robot, we can use distance as a measure, and velocity becomes the gradient
- Simplest: scaled distance to goal. Gradient becomes a constant, undefined at goal
- Better: Use a continuously differentiable function – e.g. Quadratic Function of distance:

$$U_{\mathrm{att}}(q) = \frac{1}{2}\zeta d^2(q, q_{\mathrm{goal}}),$$

with the gradient

$$\begin{aligned}\nabla U_{\mathrm{att}}(q) &= \nabla\left(\frac{1}{2}\zeta d^2(q, q_{\mathrm{goal}})\right), \\ &= \frac{1}{2}\zeta \nabla d^2(q, q_{\mathrm{goal}}), \\ &= \zeta(q - q_{\mathrm{goal}}),\end{aligned}$$



(a) (b) (c)

Figure 4.4 (a) Attractive gradient vector field. (b) Attractive potential isocontours. (c) Graph of the attractive potential.

Computing Repulsive Potential

- Strength of repulsive force should increase as we near the obstacle
- Compute potential in terms of distance to closest obstacle
- Multiple obstacles: compute repulsive potential over all obstacles

$$U_{\text{rep}}(q) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{D(q)} - \frac{1}{Q^*}\right)^2, & D(q) \leq Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$

whose gradient is

$$\nabla U_{\text{rep}}(q) = \begin{cases} \eta\left(\frac{1}{Q^*} - \frac{1}{D(q)}\right)\frac{1}{D^2(q)}\nabla D(q), & D(q) \leq Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$

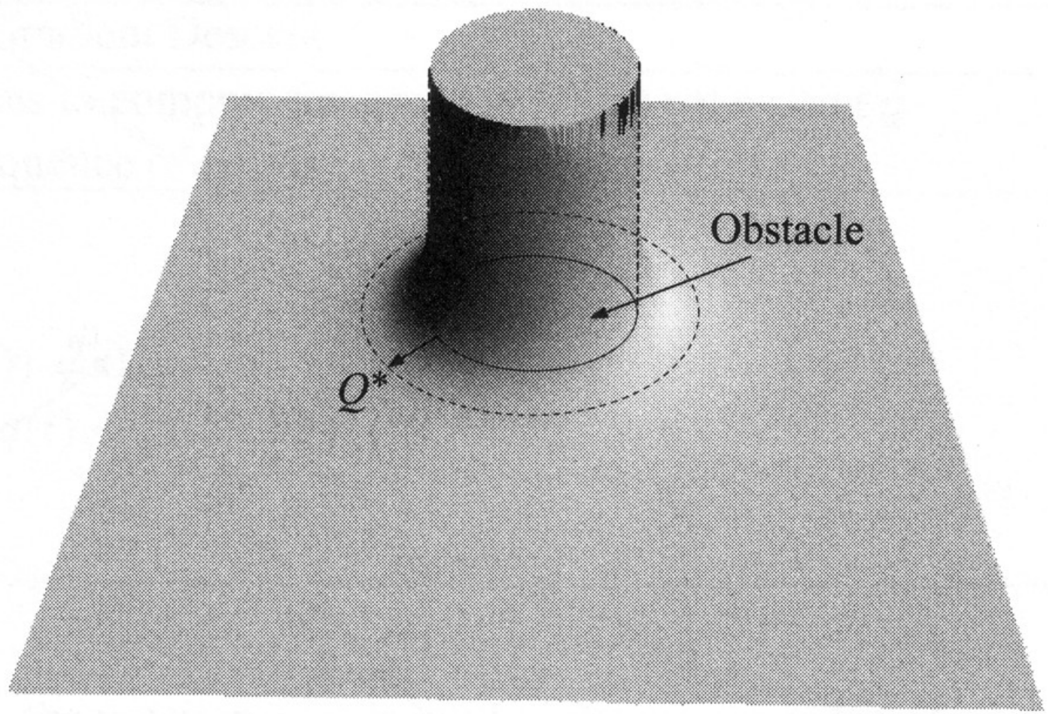
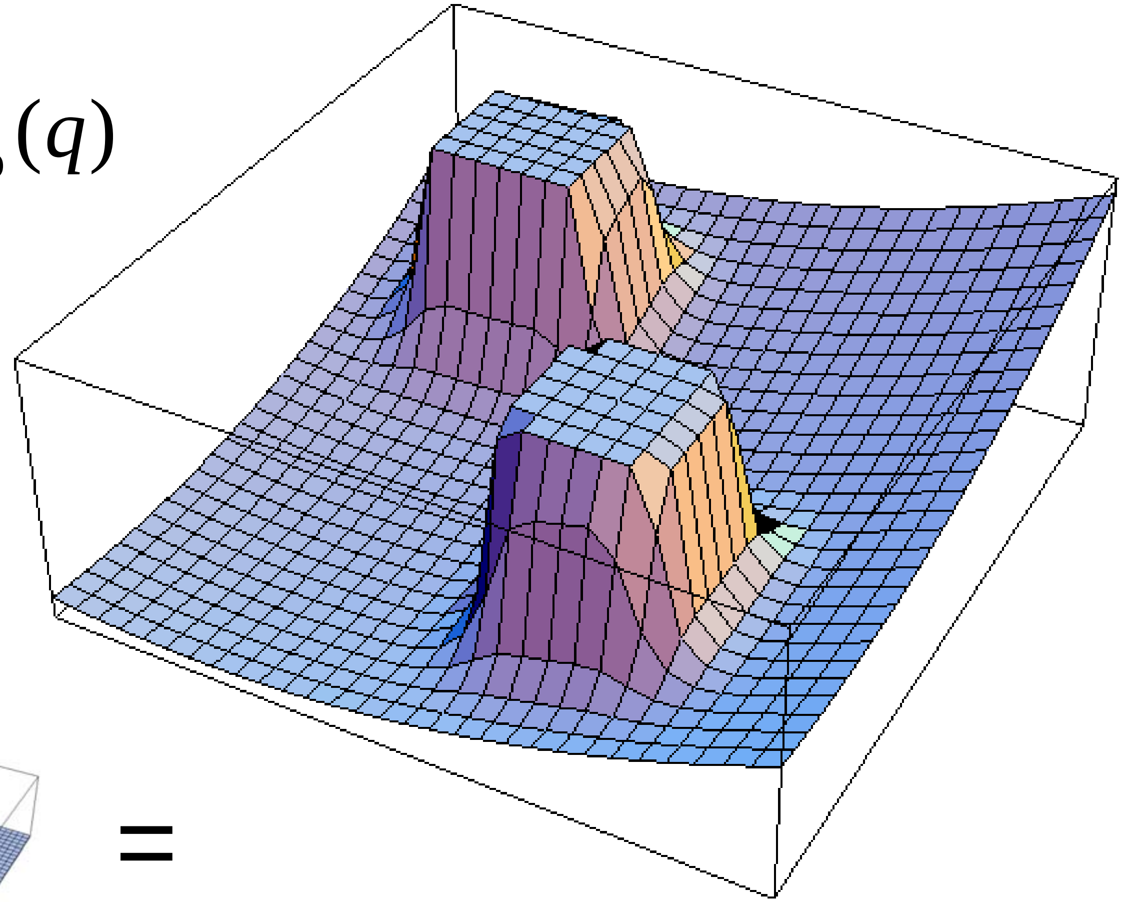


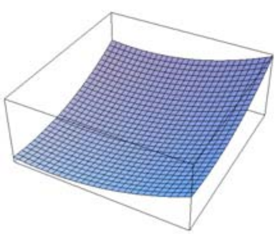
Figure 4.5 The repulsive gradient operates only in a domain near the obstacle.

Total Potential Function

$$U(q) = U_{\text{att}}(q) + U_{\text{rep}}(q)$$

$$F(q) = -\nabla U(q)$$





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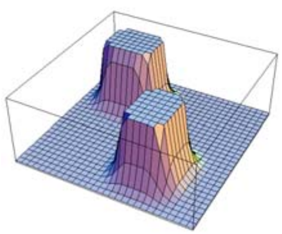

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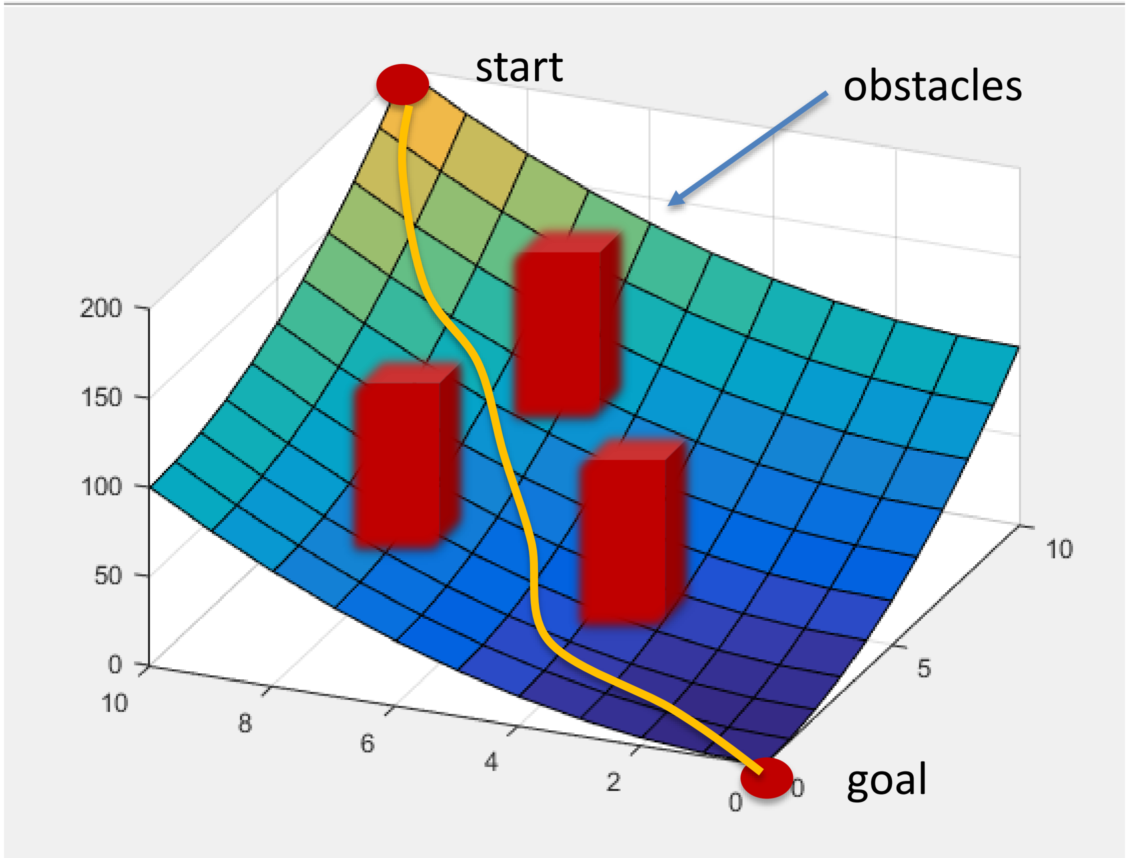
Potential Field

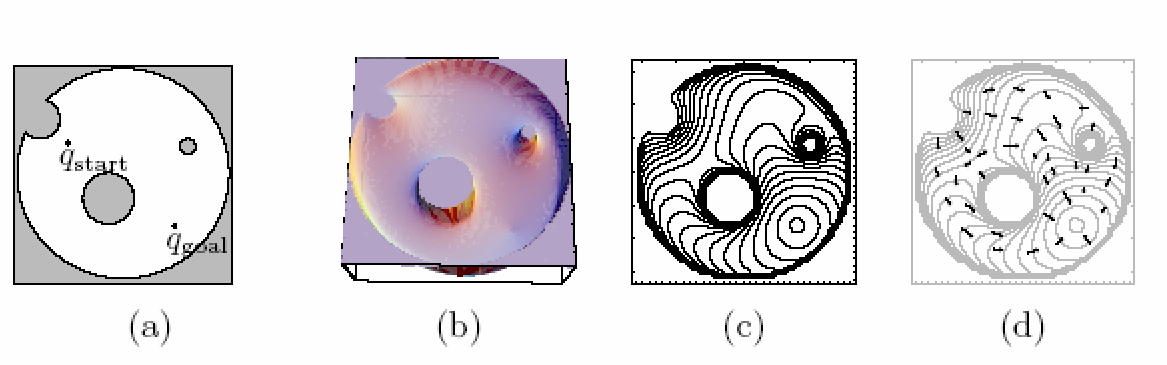


- Obstacles create high energy barriers
- Gradient descent follows energy minimization path to goal

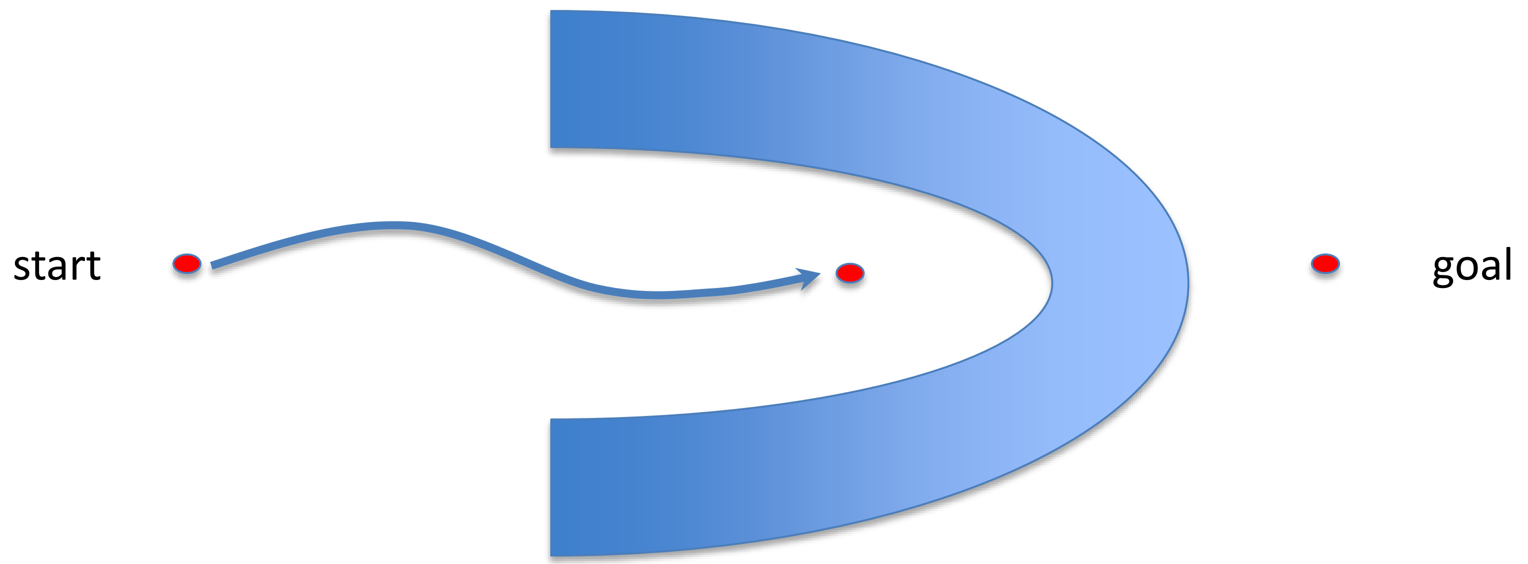
Gradient Descent

Gradient Descent:

- $q(0)=q_{start}$
- $i = 0$
- while $\| \nabla U(q(i)) \| > \varepsilon$ do
  - $q(i+1) = q(i) - \alpha(i) \nabla U(q(i))$
  - $i=i+1$

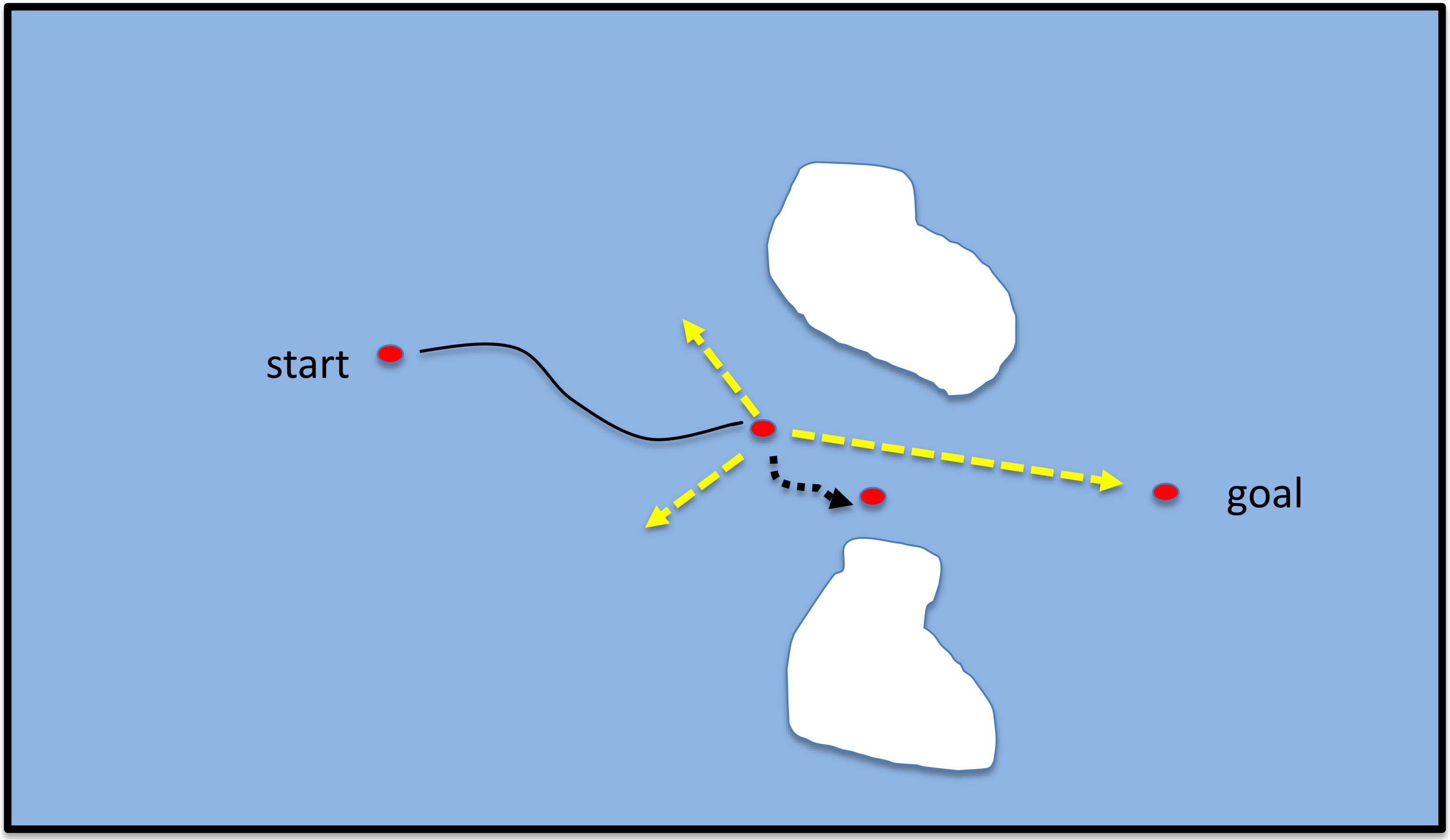


Potential Field Limitations



Local minimum:
attractive force (goal) = repulsive force (obstacles)

Potential Field Methods



Local minimum: attractive force = repulsive force
Solution: Take a random walk – perturb out of minima
Need to remember where you have been!

<u>Online</u> Distance Computation

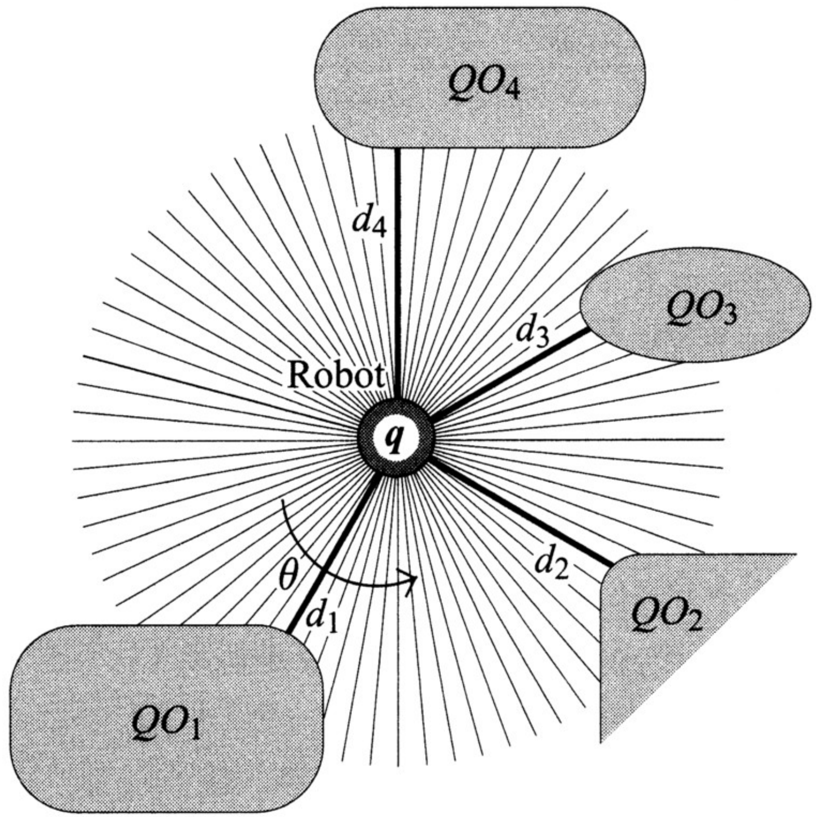


Figure 4.7 Local minima of rays determine the distance to nearby obstacles.

Potential Fields Summary

- More than just a path planner: Provides simple control function to move robot: gradient descent
- Allows robot to move from wherever it finds itself
- Can get trapped in local minima
- Can be used as online, local method:
  - As robot encounters new obstacles \
    compute the Potential Function onlir
  - Laser/sonar scans give online
    distance to obstacles