Probabilistic Roadmap Path Planning

Reference: Principles of Robot Motion

H. Choset et. al.

MIT Press

Probabilistic Roadmap Path Planning

- Explicit Geometry based planners (grown obstacles, Voronoi etc) impractical in high dimensional spaces.
- Exact solutions with complex geometries are provably exponential
- Sampling based planners can often create plans in high-dimensional spaces efficiently
- Rather than *Compute* the C-Space explicitly, we *Sample* it

Explicitly computing C-Space for more than 3 DOF is prohibitive!

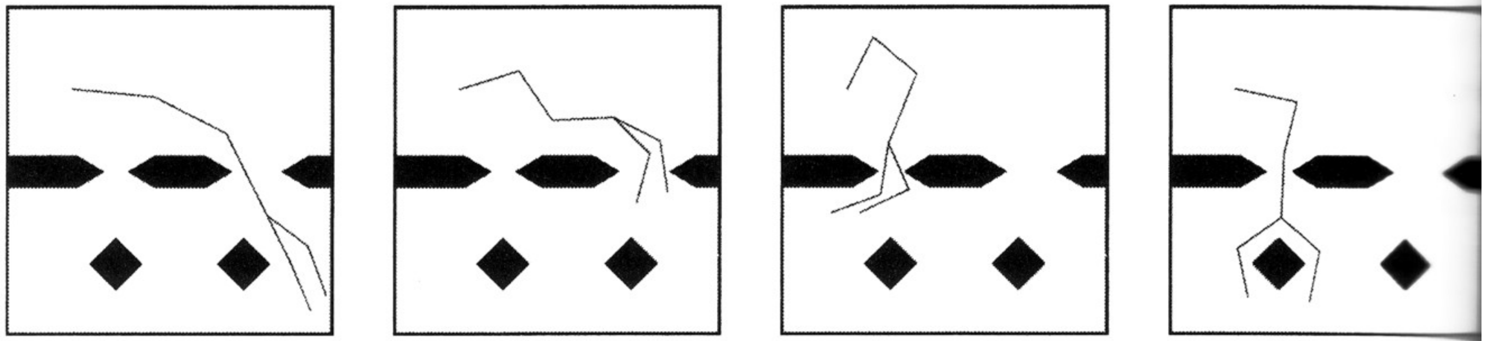
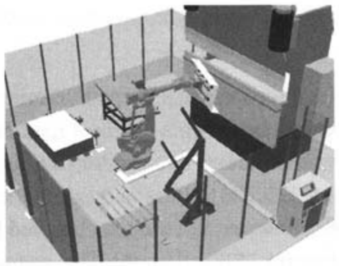


Figure 7.1 Snapshots along a path of a planar manipulator with ten degrees of freedom. manipulator has a fixed base and its first three links have prismatic joints—they can exten one and a half times their original length. (From Kavraki [221].)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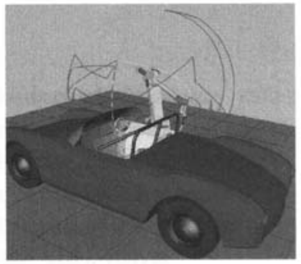
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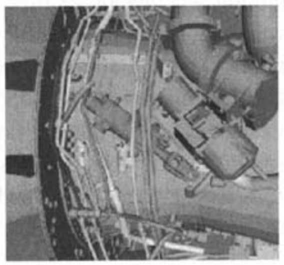
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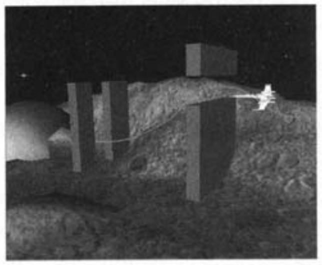
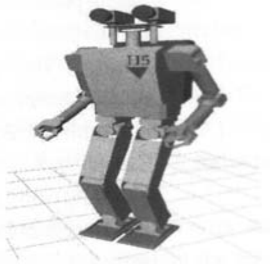

(e)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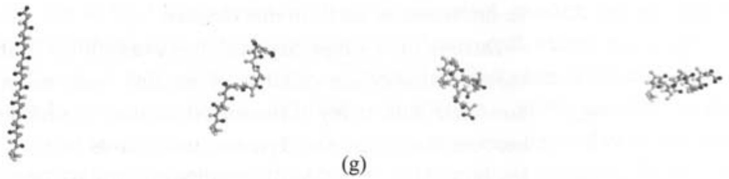

(g)

Figure 7.2 Path-planning problems. (a) Industrial manipulation. (b) Welding. (c) Planning removal paths for a part (the "robot") located at the center of the figure. (d) Computer animation. (e) Planning aircraft motion. (f) Humanoid robot. (g) Folding of a small peptide molecule. ((a) From Bohlin and Kavraki [54]; (b) from Hsu and Latombe [196]; (c) courtesy of Latombe; (d) from Koga, Kondo, Kuffner and Latombe [241]; (e) from Kuffner and LaValle [272]; (f) from Kuffner [248]; (g) from Amato [21].)

Notion of Completeness in Planning

- Complete Planner: always answers a path planning query correctly in bounded time
- Probabilistic Complete Planner: if a solution exists, planner will eventually find it, using random sampling (e.g. Monte Carlo sampling)
- Resolution Complete Planner: same as above but based on a deterministic sampling (e.g sampling on a fixed grid).

Sampling Based-Planners

- Do not attempt to explicitly construct the C-Space and its boundaries
- Simply need to know if a single robot configuration is in collision
- Exploits simple tests for collision with full knowledge of the space
- Collision detection is a separate module- can be tailored to the application
- As collision detection improves, so do these algorithms
- Different approaches for single-query and multi-query requests

PRM Planner

- Roadmap is a graph G(V,E)
- Robot configuration q→Q_free is a vertex
- Edge (q1, q2) implies collision-free path between these robot configurations
- A metric is needed for d(q1,q2) (e.g. Euclidean distance)
- Uses coarse sampling of the nodes, and fine sampling of the edges
- Result: a roadmap in Q_free

PRM Planner: Step 1, Learning the Map

- Initially empty Graph G
- A configuration q is randomly chosen
- If q→Q_free then added to G (collision detection needed here)
- Repeat until N vertices chosen
- For each q, select k closest neighbors
- Local planner Δ connects q to neighbor q’
- If connect successful (i.e. collision free local path), add edge (q, q’)

Algorithm 6 Roadmap Construction Algorithm

Input:

n : number of nodes to put in the roadmap

k : number of closest neighbors to examine for each configuration

Output:

A roadmap $G = (V, E)$

```
1: $V \leftarrow \emptyset$
2: $E \leftarrow \emptyset$
3: **while** $|V| < n$ **do**
4:     **repeat**
5:         $q \leftarrow$ a random configuration in $\mathcal{Q}$
6:     **until** $q$ is collision-free
7:     $V \leftarrow V \cup \{q\}$
8: **end while**
9: **for all** $q \in V$ **do**
10:     $N_q \leftarrow$ the $k$ closest neighbors of $q$ chosen from $V$ according to *dist*
11:     **for all** $q' \in N_q$ **do**
12:         **if** $(q, q') \notin E$ **and** $\Delta(q, q') \neq \texttt{NIL}$ **then**
13:             $E \leftarrow E \cup \{(q, q')\}$
14:         **end if**
15:     **end for**
16: **end for**
```

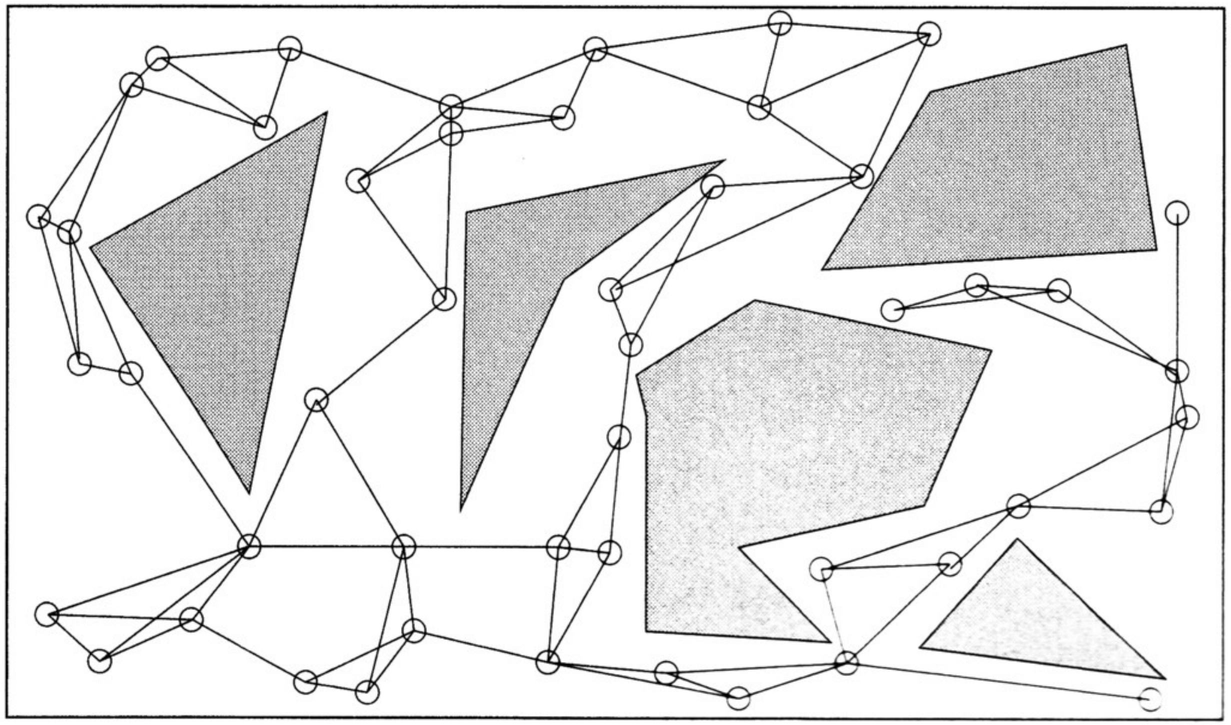


Figure 7.3 An example of a roadmap for a point robot in a two-dimensional Euclidean space. The gray areas are obstacles. The empty circles correspond to the nodes of the roadmap. The straight lines between circles correspond to edges. The number of k closest neighbors for the construction of the roadmap is three. The degree of a node can be greater than three since it may be included in the closest neighbor list of many nodes.

PRM Planner: Step 2, Finding a Path

- Given q_init and q_goal, need to connect each to the roadmap
- Find *k* nearest neigbors of q_init and q_goal in roadmap, plan local path Δ
- Problem: Roadmap Graph may have disconnected components…
- Need to find connections from q_init, q_goal to same component
- Once on roadmap, use Dijkstra algorithm

Algorithm 7 Solve Query Algorithm

Input:

q_{init}: the initial configuration

q_{goal}: the goal configuration

k: the number of closest neighbors to examine for each configuration

$G = (V, E)$: the roadmap computed by algorithm 6

Output:

A path from q_{init} to q_{goal} or failure

```
1: $N_{q_{\text{init}}} \leftarrow$ the $k$ closest neighbors of $q_{\text{init}}$ from $V$ according to *dist*
2: $N_{q_{\text{goal}}} \leftarrow$ the $k$ closest neighbors of $q_{\text{goal}}$ from $V$ according to *dist*
3: $V \leftarrow \{q_{\text{init}}\} \cup \{q_{\text{goal}}\} \cup V$
4: set $q'$ to be the closest neighbor of $q_{\text{init}}$ in $N_{q_{\text{init}}}$
5: repeat
6:    if $\Delta(q_{\text{init}}, q') \neq$ NIL then
7:       $E \leftarrow (q_{\text{init}}, q') \cup E$
8:    else
9:       set $q'$ to be the next closest neighbor of $q_{\text{init}}$ in $N_{q_{\text{init}}}$
10:   end if
11: until a connection was succesful or the set $N_{q_{\text{init}}}$ is empty
12: set $q'$ to be the closest neighbor of $q_{\text{goal}}$ in $N_{q_{\text{goal}}}$
13: repeat
14:   if $\Delta(q_{\text{goal}}, q') \neq$ NIL then
15:      $E \leftarrow (q_{\text{goal}}, q') \cup E$
16:   else
17:      set $q'$ to be the next closest neighbor of $q_{\text{goal}}$ in $N_{q_{\text{goal}}}$
18:   end if
19: until a connection was succesful or the set $N_{q_{\text{goal}}}$ is empty
20: $P \leftarrow$ shortest path($q_{\text{init}}, q_{\text{goal}}, G$)
21: if $P$ is not empty then
22:    return $P$
23: else
24:    return failure
25: end if
```

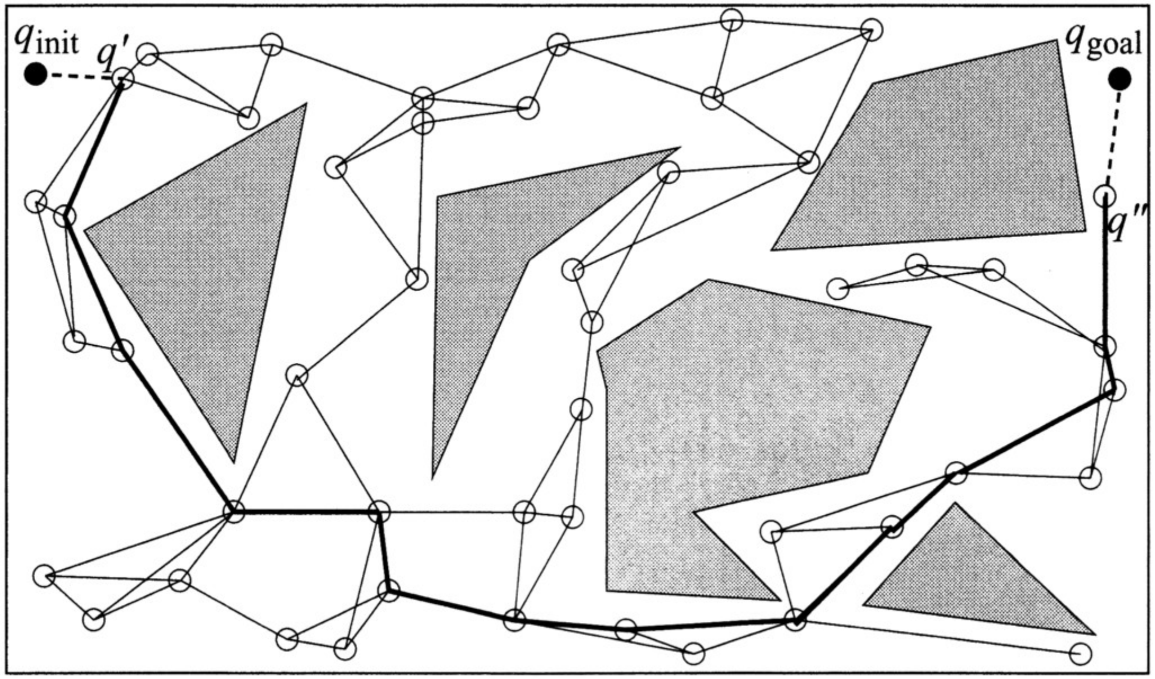


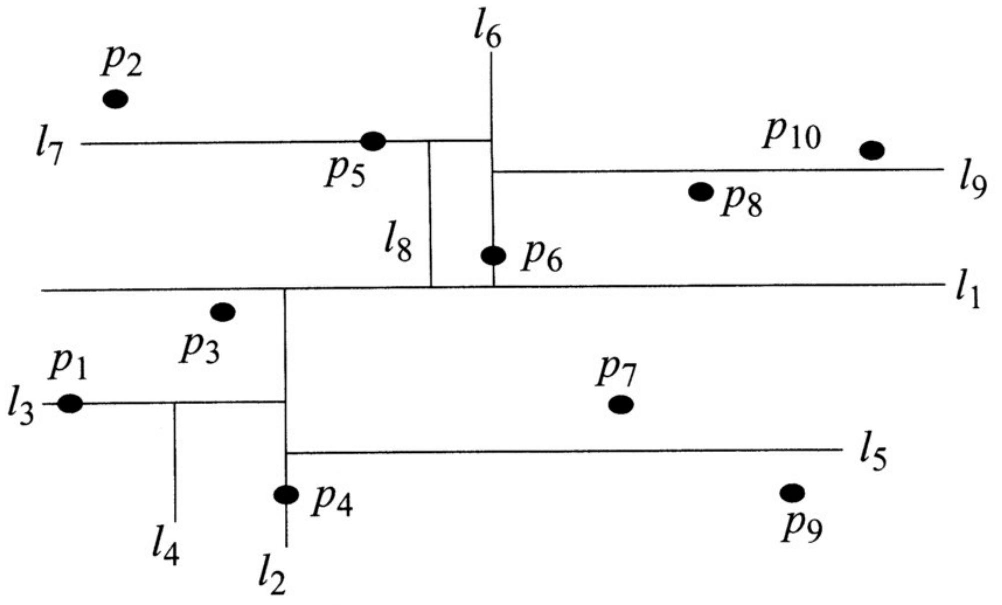
Figure 7.4 An example of how to solve a query with the roadmap from figure 7.3. The configurations q_{init} and q_{goal} are first connected to the roadmap through q' and q''. Then a graph-search algorithm returns the shortest path denoted by the thick black lines.

PRM Planner – unanswered questions

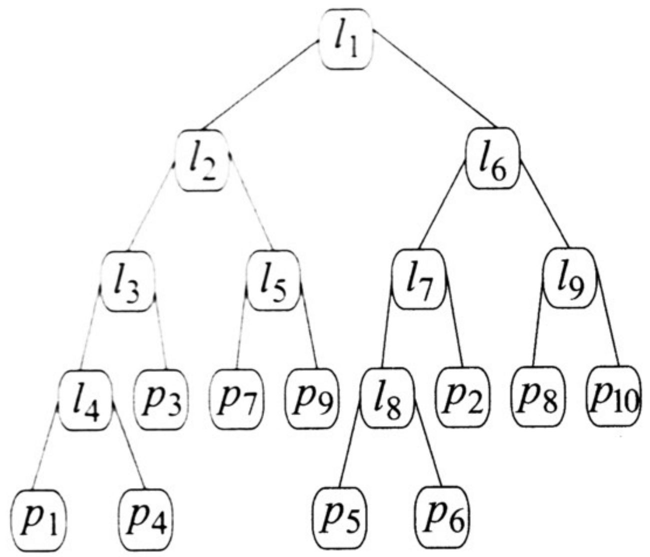
- How are random configurations chosen?
- How are closest neighbors found?
- How do we choose distance function?
- How are local path’s generated?

PRM Sampling and Connectivity

- Sampling: Uniform random sampling of Q_free
- Can be multi-dimensional (e.g. translation and rotation, both 2-D or 3-D or higher)
- Connectivity: need to find nearest neighbors
- Naïve search is *O(n)*
- K-D trees are efficient ways to find nearest neighbors
- Cost: *O(sqrt(n)) for d=2*



(a) The way the plane is subdivided.



(b) The corresponding binary tree.

Figure 7.5 A kd-tree for ten points on a plane.

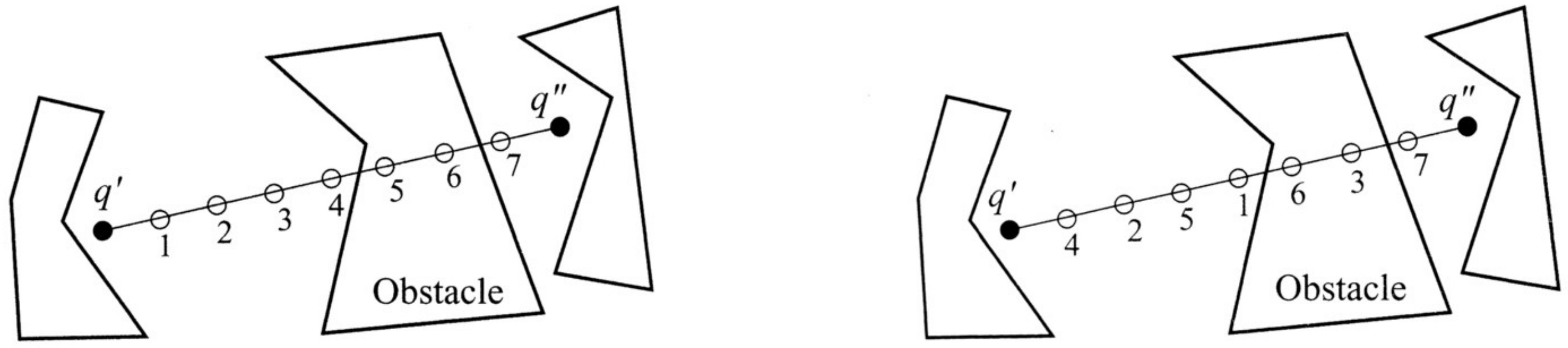
Applet: http://donar.umiacs.umd.edu/quadtree/points/kdtree.html

Local Planner

- Important aspect of PRM algorithm
- Tradeoff:
  - powerful planners are slow, but can find paths between relatively isolated nodes
  - fast planner is less accurate, more nodes need to be generated, called more often
- Local planner also needed for finding path from q_init and q_goal to roadmap

Local Planner

- Simplest: straight line planner. Connect q and q' by linear segment
- Now check the segment for collisions:
  - Incremental: use a small step size and iterate over the linear segment
  - Subdivision: use binary search decomposition to check for collisions



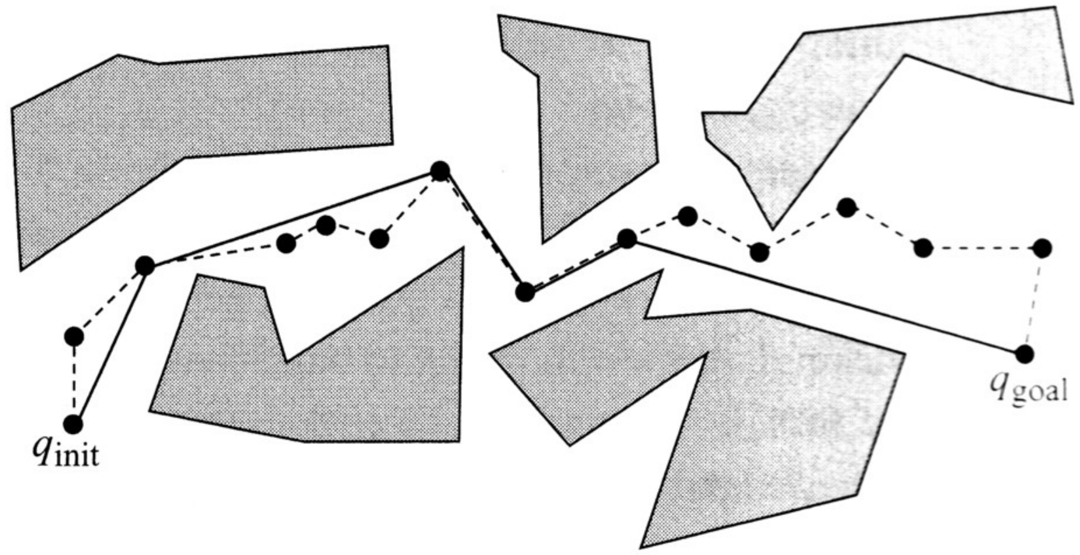
(a) Incremental: The algorithm returns failure after five collision checks.

(b) Subdivision: The algorithm returns failure after three collision checks.

Figure 7.6 Sampling along the straight line path between two configurations q' and q''. The numbers correspond to the order in which each strategy checks the samples for collision.

Postprocessing: Path Improvement

- Once a path is found, it can be optimized
- Try connecting non-adjacent configurations. Choose q_1 and q_2 randomly, try to connect.
- Greedy approach: try connecting points q_0, q_1, …q_n to q_goal.
- If q_k connects to q_goal, do the above with q_k as q_goal



Original Path - - - - - -

Shorter Path ———

Figure 7.7 Processing the path returned from PRM to get a shorter path with the greedy approach.

Example: 6-DOF Path Planning

- Robot: Rigid non-convex object in 3 space
- Obstacle: Solid wall with small opening
- Random configuration is chosen from *R3* for translation
- Axis and angle of rotation randomly chosen for rotation (quaternion representation)

Collision Detection

- Given configuration q and nearest neighbor q’ we can use straight line collision detection
- Each configuration q=(p,r)=(trans,quaternion)
- Check for collision by interpolating along line (p,p’) and along spherical interpolation (r,r’).

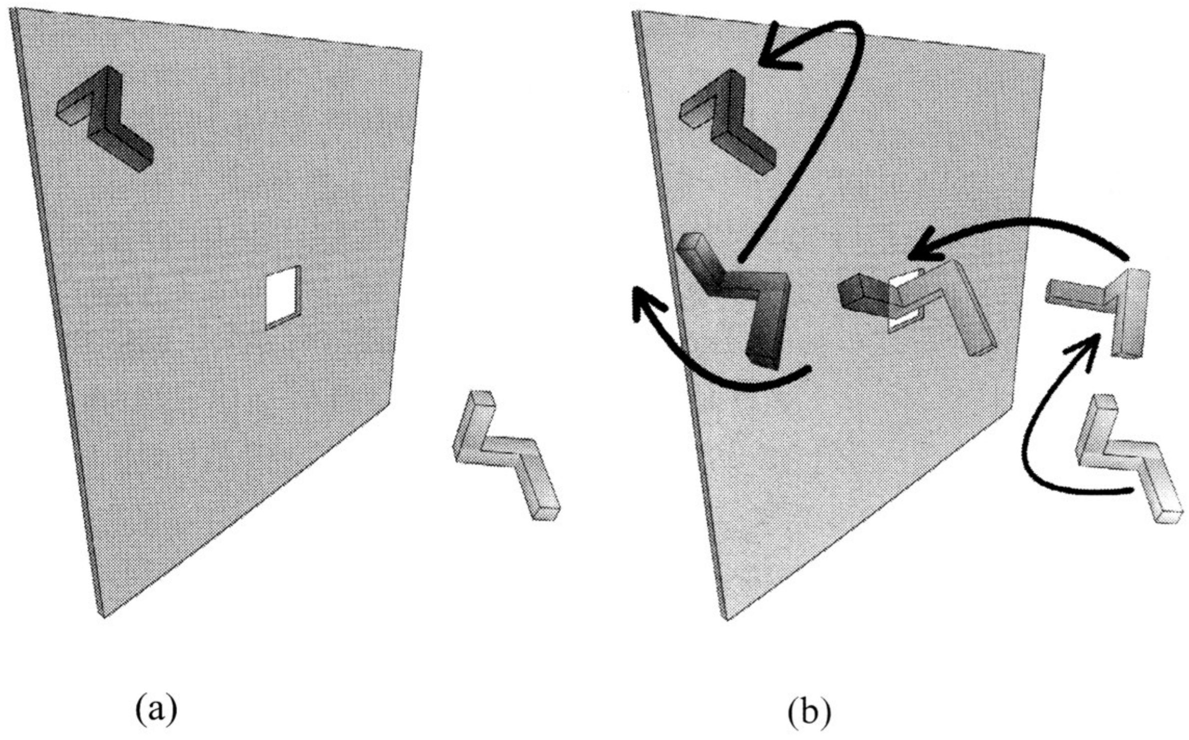


Figure 7.8 An example of a motion-planning problem where both the robot and the obstacles are a collection of polyhedral objects in three dimensions. Parts of the robot on the other side of the wall are indicated by the darker color. (a) The initial and goal configuration of the query. (b) A path produced from a PRM with $n = 1000$ and $k = 10$.

video

Distance Calculation for Rigid Object in 3D

- Distance function needed between 2 configurations q, q'
- Ideally, distance is the swept volume of the robot as it moves between configurations q and q'. Difficult to compute exactly
- Method I: Can approximate this distance with an embedding in a Euclidean metric space: **d(q,q') = || embed(q) - embed(q')||**
  - Choose set of p points on robot, concatenate them, and create a vector of size p x dimension of workspace.
  - Example of rigid object in 3D: Create vector of size 3p, choosing p points on the object. Intuitively, a "sampling" of the object's Euclidean domain.
  - For configuration q, **embed(q)** is the vector of p points transformed by the translation and rotation that is configuration q. Transform each of the p points into the vector **embed(q).**
  - Do the same for configuration q', create **embed(q').**
  - Distance is now Euclidean distance between the 3p vectors:

    d(q,q') = || embed(q) - embed(q')||
- How do you choose the p points?
- Cheaper solution: choose 2 points p1 and p2 of maximum extent on the object.
- Method II: Separate a configuration q into a translation **X** and a rotation **R**: **q=(X,R)**
- Calculate a weighted distance function **d(q,q') = w1||X-X'|| + w2 f(R,R').**
- Need to use a metric on rotations – quaternions are good for this
- Weights **w1** and **w2** need to be chosen, no real insight into this

OBPRM: Obstacle PRM

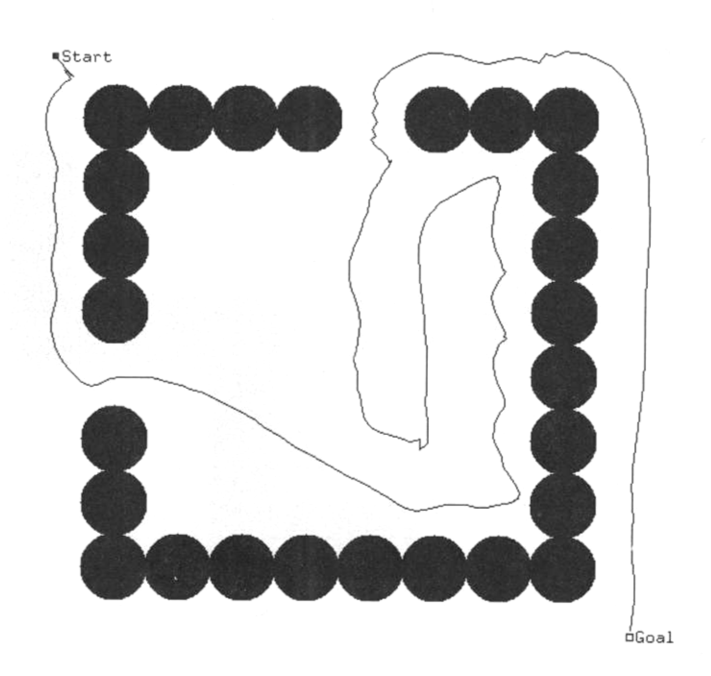
- If tight, small regions of the Cspace are needed to create a path (e.g. small opening in the wall), sampling may miss this
- Problem areas tend to be near the obstacles in tight spaces
- Solution: generate configuration q. If q in collision, choose random direction v and move q away from obstacle in direction v a small distance. If q now in Q_free, use this node
- Biases sampling near obstacles

Single-Query Sampling Based Planners

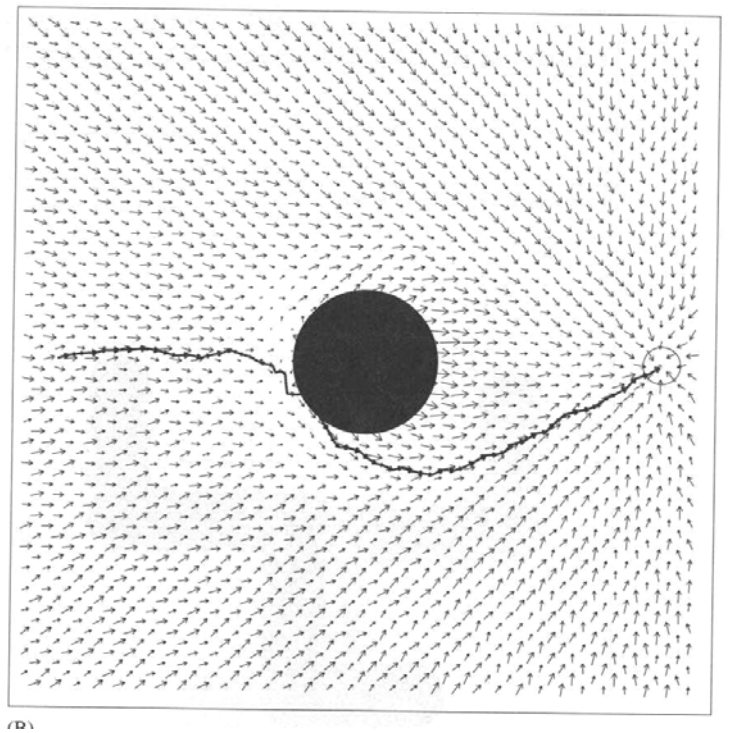
- PRM samples the entire space, plans paths anywhere
- Single query planners don’t explore all of Q_free, only relevant parts
- PRM can be used this way, inserting q_init and q_goal in Graph at beginning, then checking for a path

Random search

Often combined with potential field methods to escape minima



"Filling in" local minima



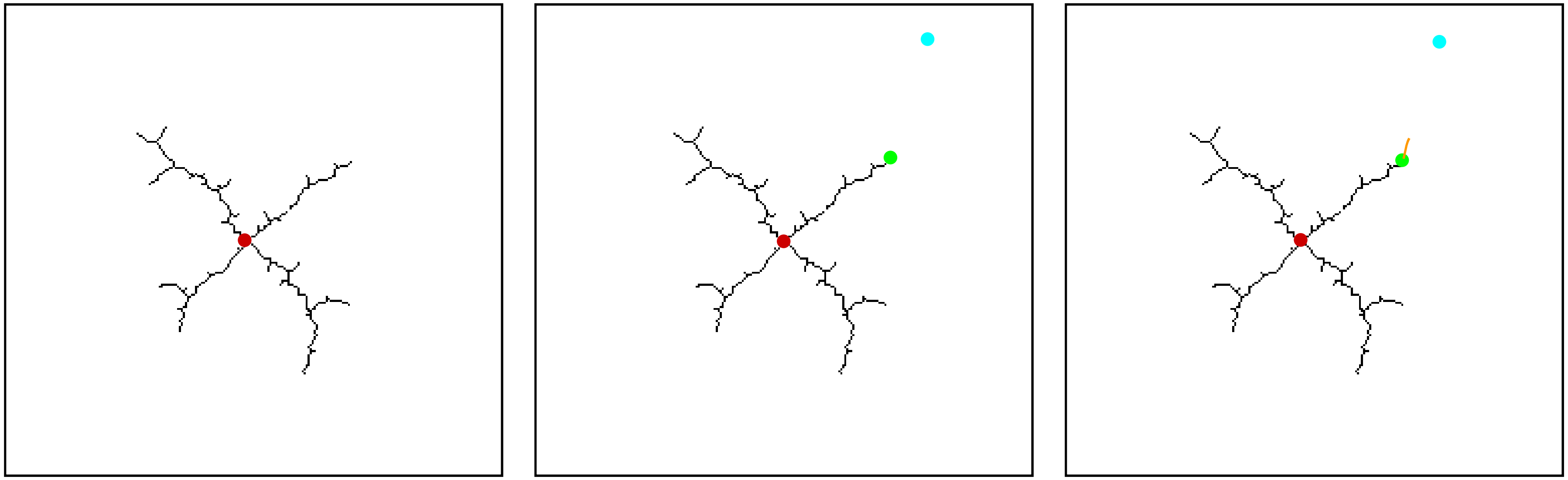
Random walks

random walks are not perfect...

RRT: Rapidly-exploring Random Trees

- Idea: sample Q_free for path from q_init to q_goal
- Use 2 trees, rooted at q_init and q_goal.
- As trees grow, the eventually share a common node, and are merged into a path

RRTs



1) Maintain a tree of configurations reachable from the starting point. ●

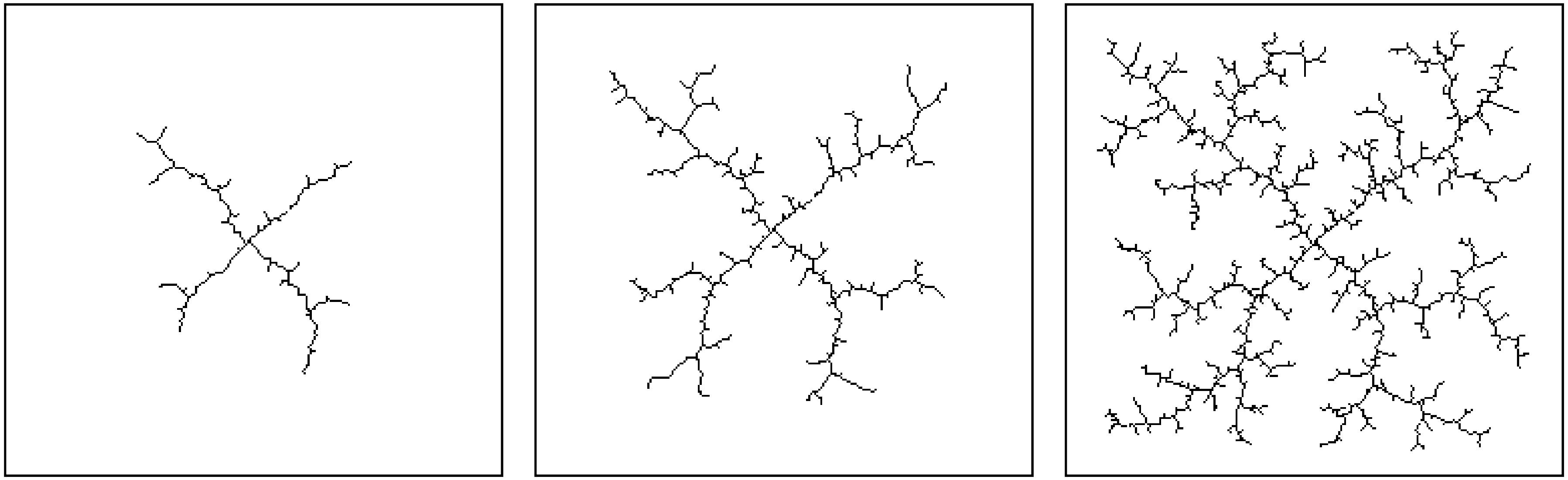
2) Choose a point at random from free space ●

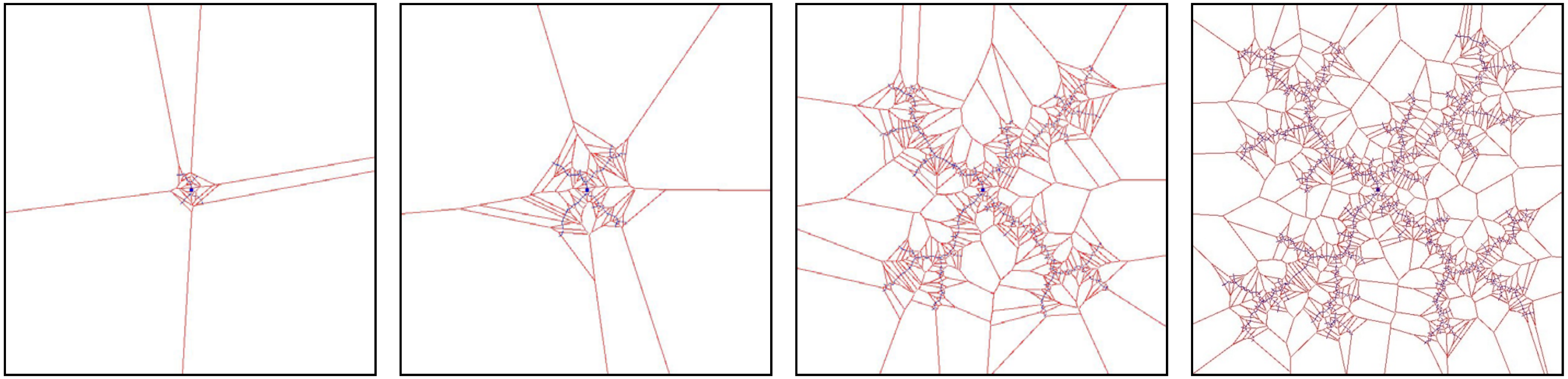
3) Find the closest configuration already in the tree ●

4) Extend the tree in the direction of the new configuration / EXTEND step

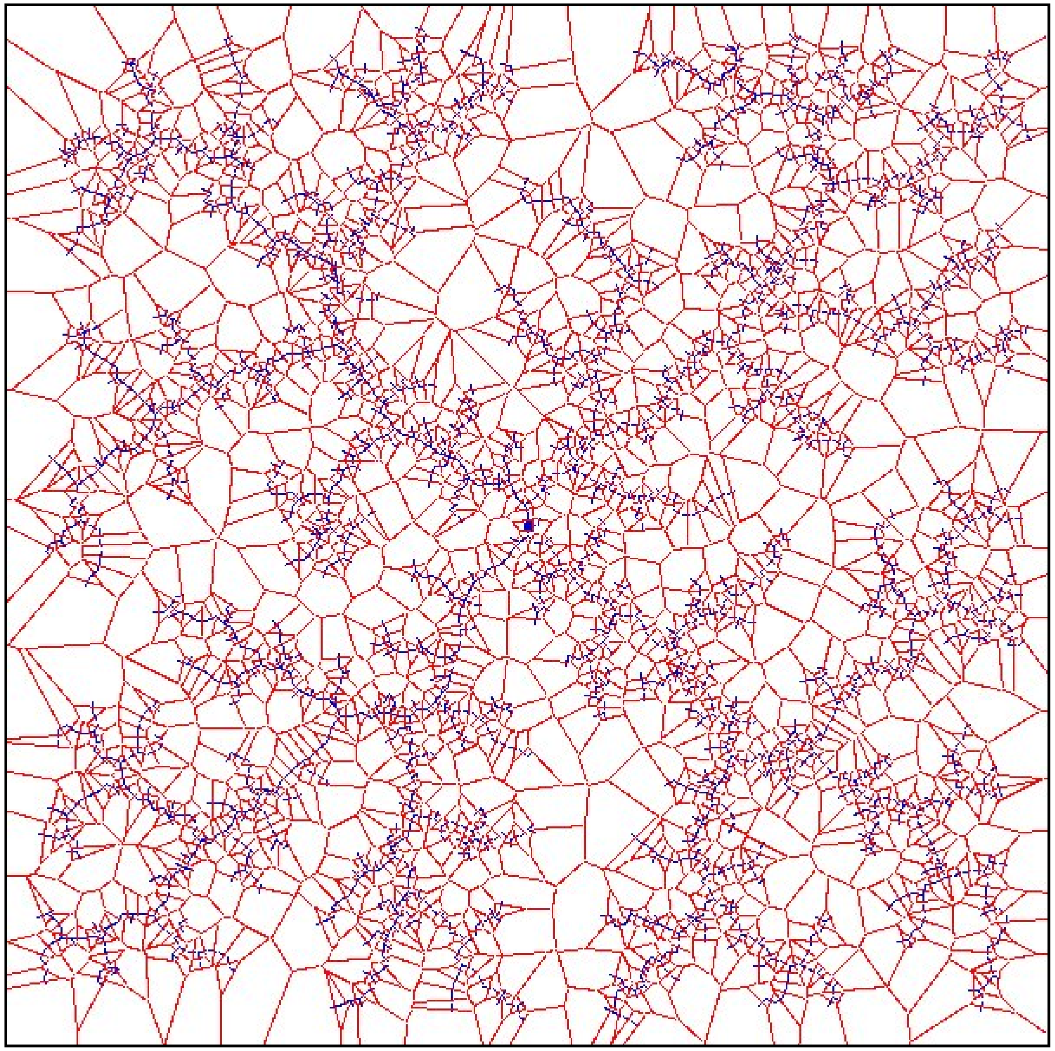
connects global & local information

Growth of an RRT


Example growth of an RRT - Biased toward the unexplored free space at each step.


Voronoi diagrams

A Mature RRT



RRT - blue

Voronoi - red

http://msl.cs.uiuc.edu/rrt
http://www.kuffner.org/james/humanoid/planning.php

Path Planning with RRT Algorithm

- 2 trees, T_init, T_goal, rooted at q_init, q_goal
- Each tree is expanded by:
  - q_rand is generated from uniform dist.
  - q_near is found, nearest tree node to q_rand
  - move step-size along line (q_near, q_rand) to q_new. If no collision, add q_new to tree
- If trees merge, path is found

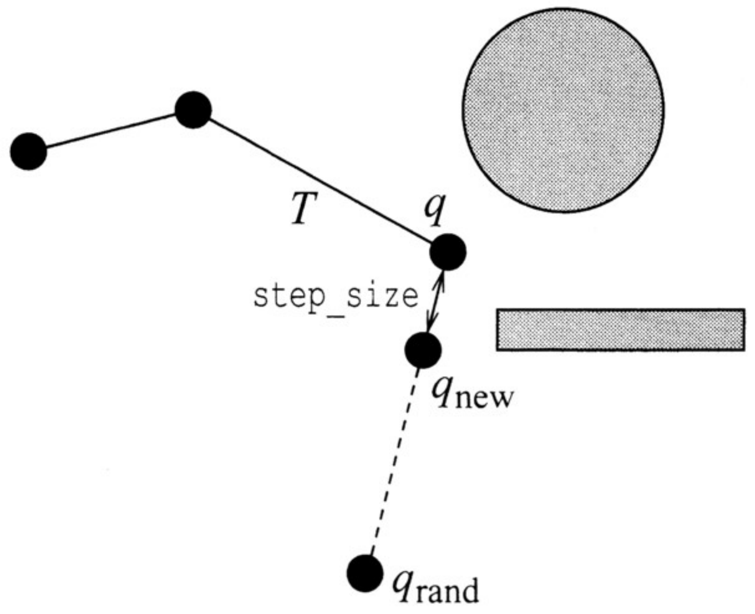


Figure 7.14 Adding a new configuration to an RRT. Configuration q_{rand} is selected randomly from a uniform distribution in $\mathcal{Q}_{\text{free}}$. Configuration q is the closest configuration in T to q_{rand} (this configuration is denoted as q_{near} in the algorithm). Configuration q_{new} is obtained by moving q by `step_size` toward q_{rand}. Only q_{new} and the edge (q, q_{new}) are added to the RRT.

Algorithm 10 Build RRT Algorithm

Input:

q_0: the configuration where the tree is rooted

n : the number of attempts to expand the tree

Output:

A tree $T = (V, E)$ that is rooted at q_0 and has $\leq n$ configurations

```
1: $V \leftarrow \{q_0\}$
2: $E \leftarrow \emptyset$
3: **for** $i = 1$ to $n$ **do**
4:    $q_{\text{rand}} \leftarrow$ a randomly chosen free configuration
5:    extend RRT $(T, q_{\text{rand}})$
6: **end for**
7: **return** $T$
```

Algorithm 11 Extend RRT Algorithm

Input:

$T = (V, E)$: an RRT

q: a configuration toward which the tree T is grown

Output:

A new configuration q_{new} toward q, or NIL in case of failure

```
1: $q_{\text{near}} \leftarrow$ closest neighbor of $q$ in $T$
2: $q_{\text{new}} \leftarrow$ progress $q_{\text{near}}$ by step_size along the straight line in $Q$ between $q_{\text{near}}$
   $q_{\text{rand}}$
3: **if** $q_{\text{new}}$ is collision-free **then**
4:    $V \leftarrow V \cup \{q_{\text{new}}\}$
5:    $E \leftarrow E \cup \{(q_{\text{near}}, q_{\text{new}})\}$
6:    **return** $q_{\text{new}}$
7: **end if**
8: **return** NIL
```

RRT Algorithm

- Algorithm sensitive to step-size
- How far do we move along line (q_near, q_rand)?
- Can a greedier algorithm work better?
- Why not move all the way to q_rand?

Algorithm 12 Connect RRT Algorithm

Input:

$T = (V, E)$: an RRT

q: a configuration toward which the tree T is grown

Output:

connected if q is connected to T; failure otherwise

```
1: repeat
2:     $q_{\text{new}} \leftarrow$ extend RRT $(T, q)$
3: until ($q_{\text{new}}$= q or $q_{\text{new}}$= NIL)
4: if $q_{\text{new}} = q$ then
5:     return connected
6: else
7:     return failure
8: end if
```

RRT Tradeoffs

- If step-size is small, many nodes generated, close together
- As number of nodes increases, nearest-neighbor computation slows down
- May be better to only add the last sample along the line (q_near, q_rand)

Shaping the RRT

- q_rand determines what direction we go
- What if q_rand == q_goal?
- Very greedy algorithm. Introduces too much bias
- Becomes a potential field planner that gets stuck in local minima
- Idea: use uniform q_rand with occasional q_rand ==q_goal (maybe we get lucky?)
- Introducing just .05 bias towards goal, results improve

Merging RRT's

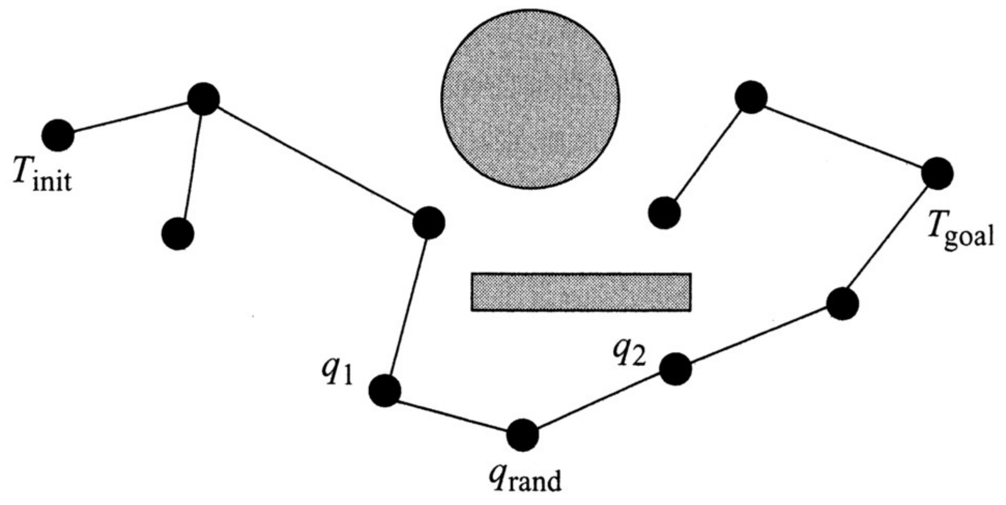


Figure 7.15 Merging two RRTs. Configuration q_{rand} is generated randomly from a uniform distribution in $\mathcal{Q}_{\text{free}}$. Configuration q_1 was extended to q_{rand}. q_2 is the closest configuration to q_{rand} in T_{goal}. It was possible to extend q_2 to q_{rand}. As a result, T_{init} and T_{goal} were merged.

Algorithm 13 Merge RRT Algorithm

Input:

T_1: first RRT

T_2: second RRT

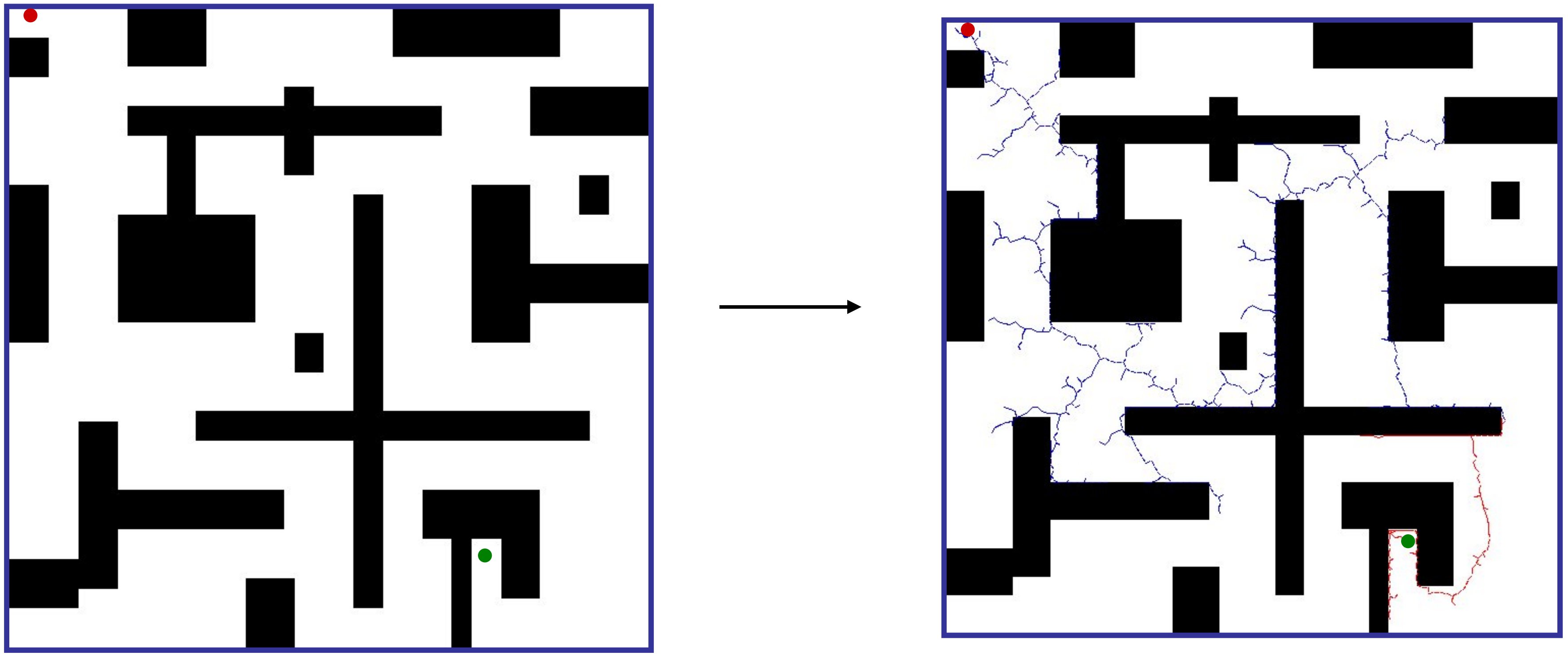
ℓ: number of attempts allowed to merge T_1 and T_2

Output:

merged if the two RRTs are connected to each other; failure otherwise

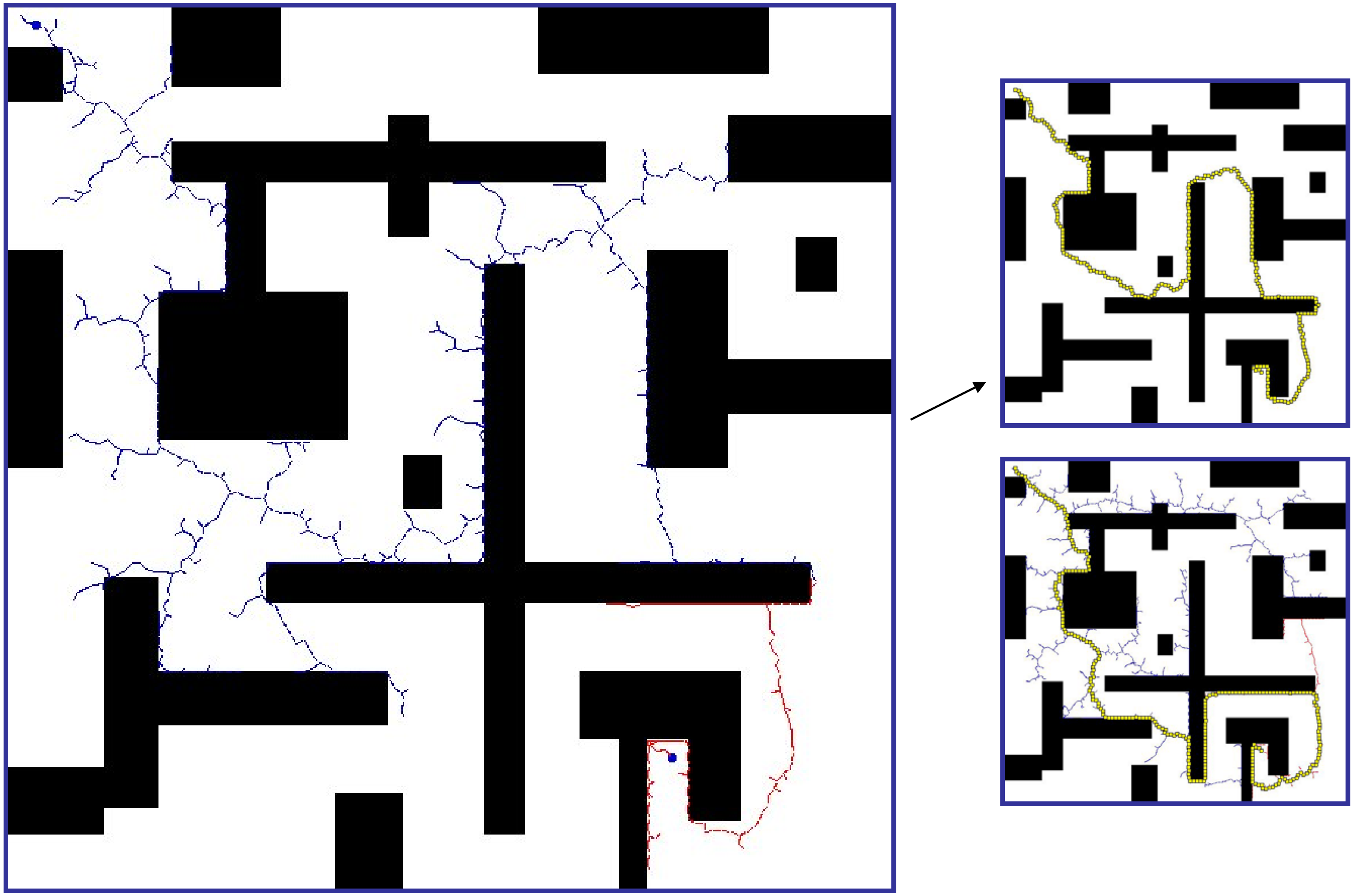
```
1:  for $i = 1$ to $\ell$ do
2:    $q_{\text{rand}} \leftarrow$ a randomly chosen free configuration
3:    $q_{\text{new},1} \leftarrow$ extend RRT $(T_1, q_{\text{rand}})$
4:    if $q_{\text{new},1} \neq$ NIL then
5:      $q_{\text{new},2} \leftarrow$ extend RRT $(T_2, q_{\text{new},1})$
6:      if $q_{\text{new},1} = q_{\text{new},2}$ then
7:        return merged
8:      end if
9:      SWAP$(T_1, T_2)$
10:   end if
11: end for
12: return failure
```

Using RRTs



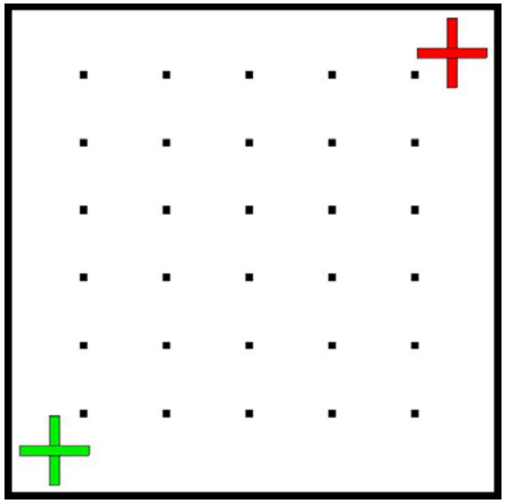
http://msl.cs.uiuc.edu/rrt/

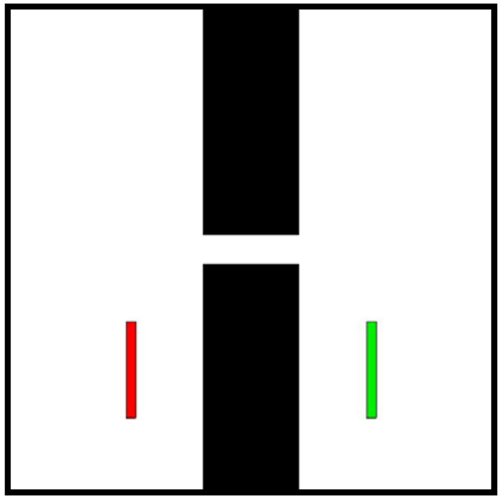
Bidirectional search



Additional complexity

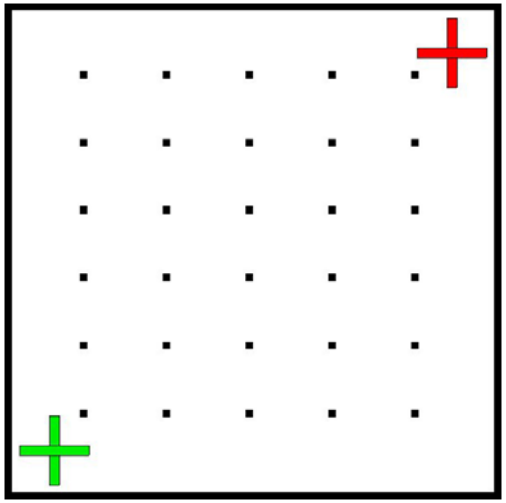
additional degrees of freedom

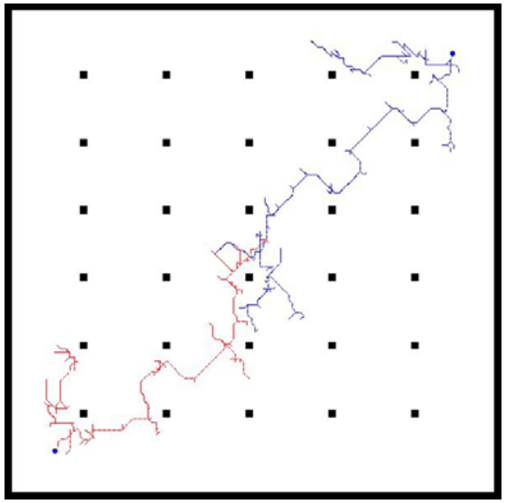


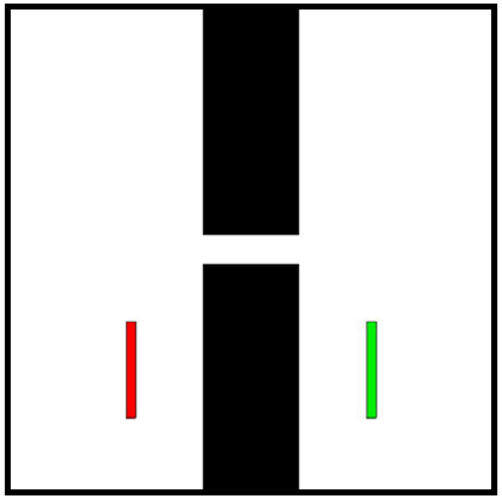


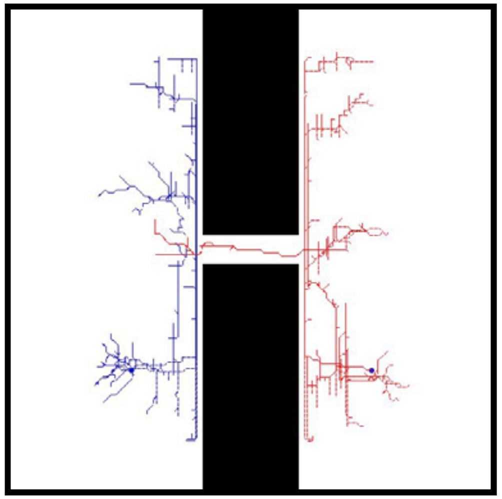
Additional complexity

additional degrees of freedom





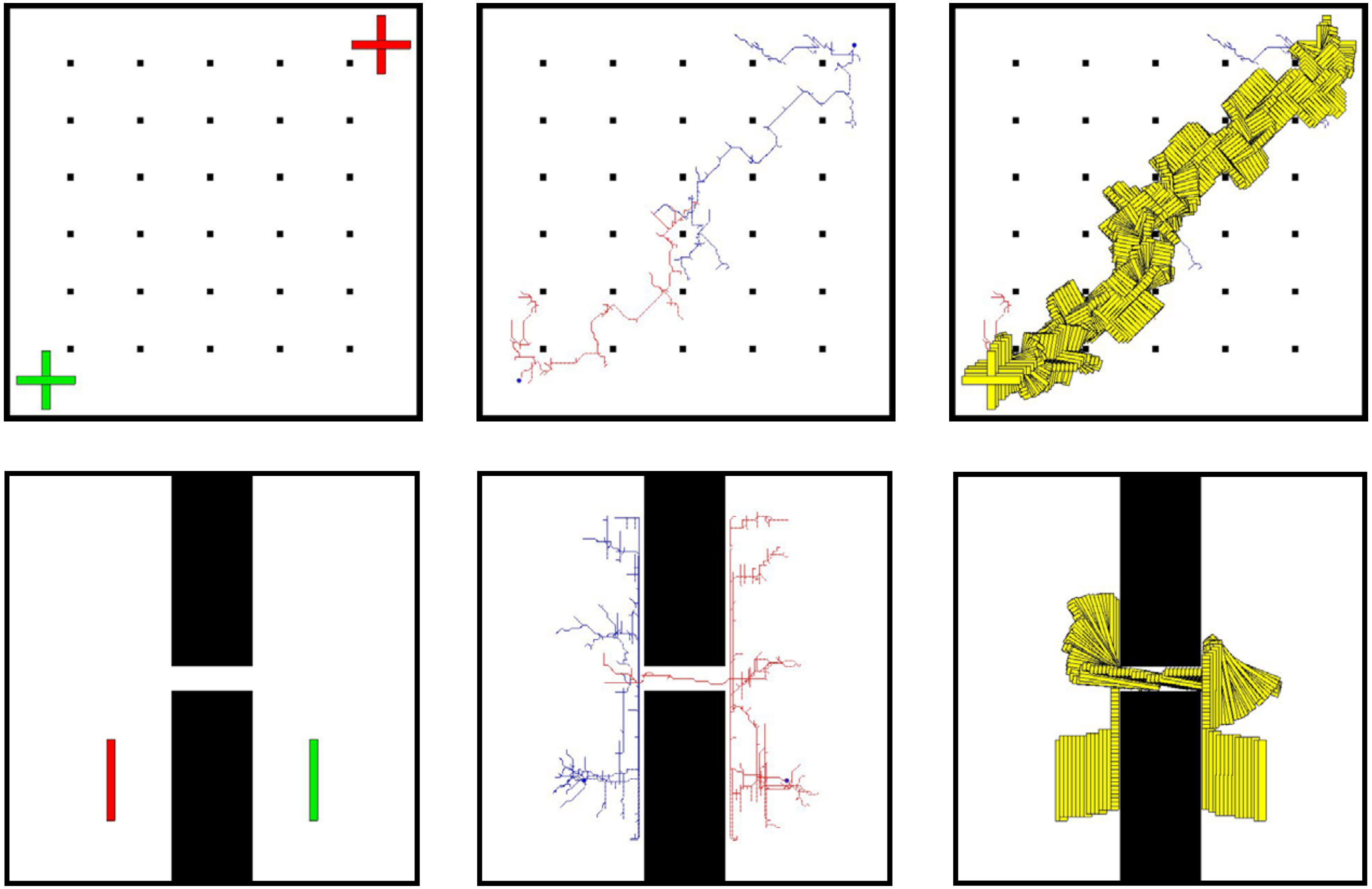




xy projections

Additional complexity

additional degrees of freedom

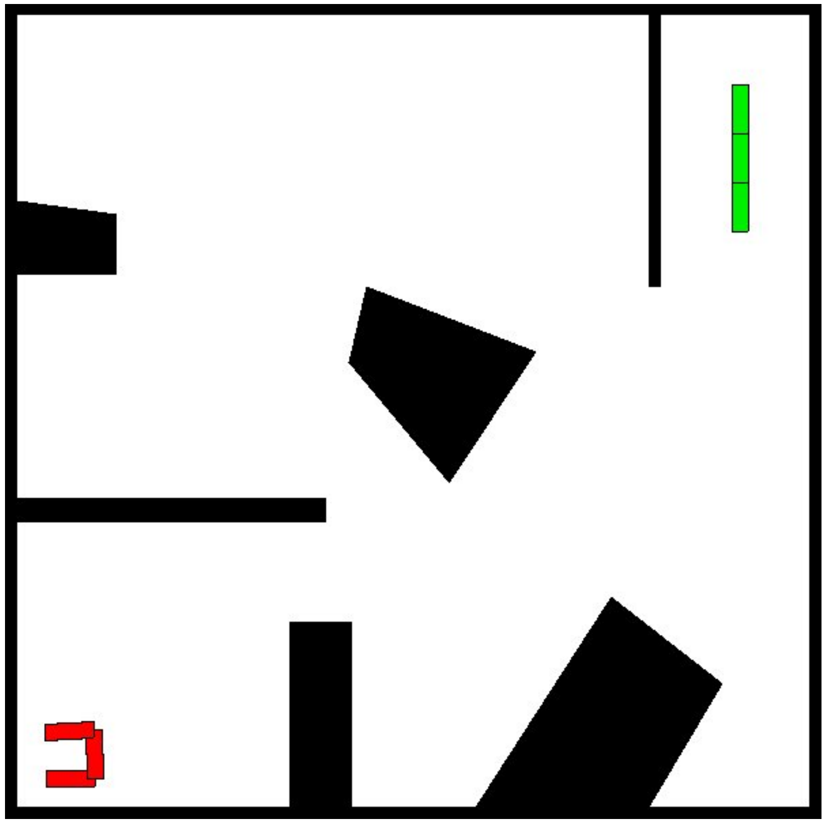


xy projections

time-lapse paths

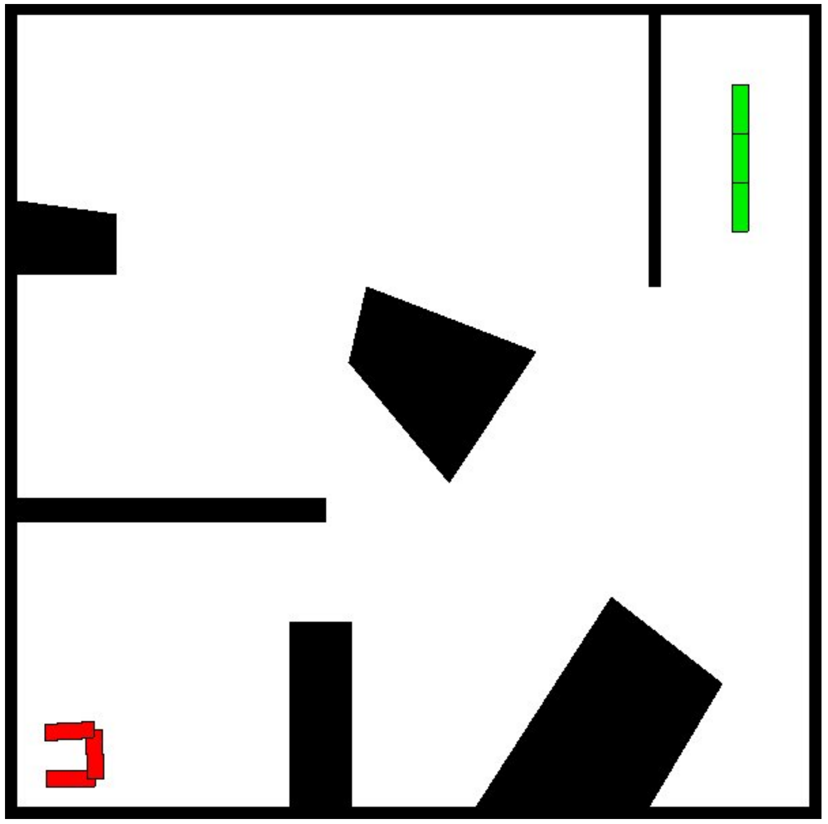
Additional complexity

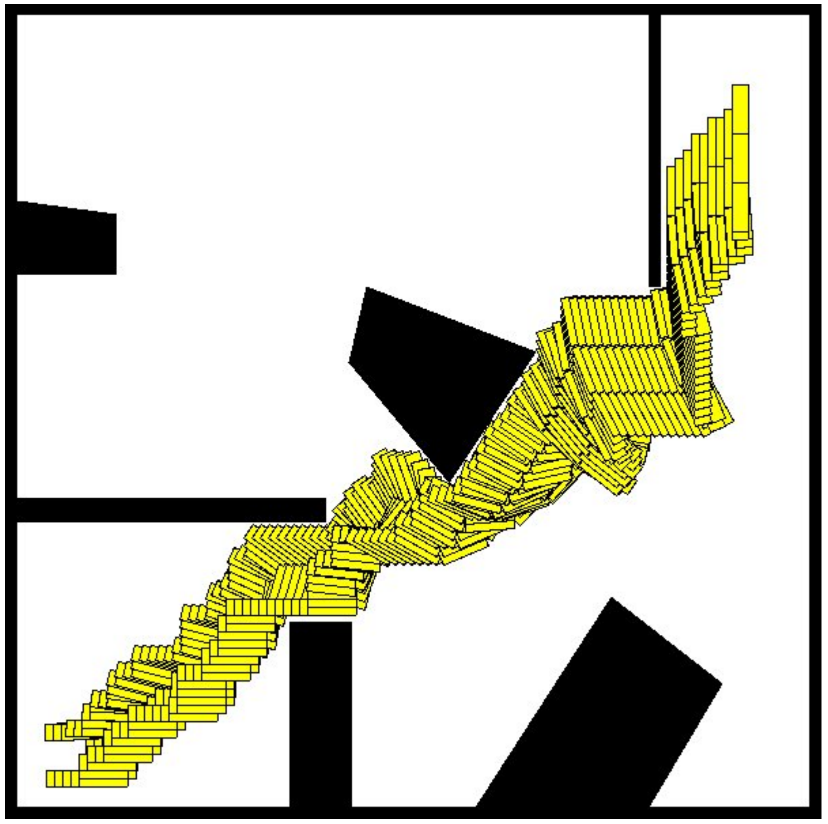
articulated linkages



Additional complexity

articulated linkages





http://msl.cs.uiuc.edu/rrt
http://www.kuffner.org/james/humanoid/planning.html