COMS W3261
Computer Science Theory
Lecture 23: December 7, 2011
Final Review

Final will be held in 209 Havemeyer, 1:10-2:25pm, Monday, December 12, 2011.

1. Which of these strings are generated by each of the regular expressions (a) through (e): (i) *b*, (ii) *ba*, (iii) *bab*, (iv) *baba*, (v) *babab*? (Show all strings for each regular expression.)

   a) $(ab^*a + b)^*$
   b) $a(a + b)^*a$
   c) $b^*(a^* + bbb^*)^*b^*$
   d) $(aa^*b + bb^*a)(a + b)^*$
   e) $(aa + bb + (ab + ba)(aa + bb)^*(ba + ab))^*$

2. a) Informally describe in words an algorithm to determine whether two deterministic finite automata are equivalent.
   b) Let M be the DFA ($\{A, B\}$, $\{a, b\}$, δ_1, A, $\{A\}$) where δ_1 is given by

State	Input Symbol	
	a	*b*
A	*A*	*B*
B	*A*	*B*

   Let N be the DFA ($\{C, D, E\}$, $\{a, b\}$, δ_2, C, $\{C, D\}$) where δ_2 is given by

State	Input Symbol	
	a	*b*
C	*D*	*E*
D	*D*	*E*
E	*C*	*E*

   Use your algorithm to determine whether M and N are equivalent.

3. Define the operator $swap(w) = yx$ where $|x| = |y|$ and $w = xy$. That is, *swap* interchanges the first and last halves of an even-length string. For a language L of even-length strings define $swap(L) = \{ swap(w) \mid w \text{ is in } L \}$. Show that the regular languages are not closed under the *swap* operator. *Hint*: Find a regular language L such that $swap(L)$ is a nonregular language.

4. Which of these strings are generated by each of the context-free grammars (a) through (e): (i) *a*, (ii) *ab*, (iii) *aba*, (iv) *abab*, (v) *ababa*?

   a) $S \rightarrow aSa \mid bSb \mid a \mid b$
   b) $S \rightarrow aSbS \mid \varepsilon$
   c) $S \rightarrow aSbS \mid bSaS \mid \varepsilon$
   d) $S \rightarrow SbS \mid a$
   e) $S \rightarrow aSS \mid b$

5. Show that if *L* is a regular language of even-length strings defined by a finite automaton *A*, then *swap*(*L*) is a context-free language. *Hint*: Briefly explain in words how *swap*(*L*) can be recognized by a nondeterministic PDA that incorporates *A* in its finite-state control.

6. Transform the following CFG into an equivalent Chomsky Normal Form grammar:

   $S \rightarrow Ac$
   $A \rightarrow aAbA \mid \varepsilon$

7. Use the CYK algorithm to find a parse tree for the input string *baaba* according to the following context-free grammar:

   $S \rightarrow AB \mid BC$
   $A \rightarrow BA \mid a$
   $B \rightarrow CC \mid b$
   $C \rightarrow AB \mid a$

8. If *L* is a context-free language, is *swap*(*L*) context free? Justify your answer. *Hint*: Choose a CFL *L* contained in a^*b^* such that swap(*L*) is not context free.

9. True or false? Briefly justify your answer. (*A* and *B* are languages.)

   a) If the diagonal language is reducible to *A*, then *A* is not recursive.
   b) If SAT is reducible in polynomial time to *A*, then *A* is NP-complete.
   c) If Post's Correspondence Problem is reducible to *A*, then *A* is undecidable.
   d) If the universal language is reducible to *A*, then the complement of *A* is recursive.
   e) If *A* is NP-complete and *A* is reducible in polynomial time to *B* which is in P, then P = NP.

10. Consider the lambda-calculus expression $(\lambda x.\ \lambda y.\ y)(\ (\lambda z.\ zz)\ (\lambda z.\ zz)\ )$.

    a) What is normal order evaluation?
    b) Evaluate this expression using normal order evaluation.
    c) What is applicative order evaluation?
    d) Evaluate this expression using applicative order evaluation.

11. The game PEBBLES is played on an $k \times n$ chessboard. Initially each square of the chessboard has a black pebble, or a white pebble, or no pebble. You play the game by removing pebbles one at a time. You win the game if you can end up with a board in which each column contains only pebbles of a single color and each row contains at least one pebble.

    a) Show that the set of winnable PEBBLES games is in NP. *Hint*: Describe a nondeterministic polynomial-time algorithm to determine whether a given PEBBLES board is winnable.
    b) Given a boolean expression *E* in 3-CNF with *k* clauses and *n* variables, construct the following $k \times n$ board: If literal x_i is in clause c_j, put a black pebble in column x_i, row c_j. If literal $\neg x_i$ is in clause c_j, put a white pebble in column x_i, row c_j. Show that *E* is satisfiable if and only if this PEBBLES game is winnable.
    c) What can you conclude from showing (a) and (b)?