Shape from Focus: An Effective Approach for Rough Surfaces

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Abstract

Rough surfaces pose a challenging shape extraction problem. Images of rough surfaces are often characterized by high frequency intensity variations, and it is difficult to preserve the shapes of these surfaces from their images. The shape-from-focus method described in this paper uses different focus levels to clarify a sequence of object images. The multi-modulated-Laplacian (SML) operator is developed to compute local measures of the quality of image focus. The SML operator is applied to the image sequence, and the focus surfaces obtained at each image point are used to compute local depth estimates. We present two algorithms for depth estimation. The first algorithm simply looks for the focus level that maximizes the distance function at each point. The second algorithm was a Gaussian model to incorporate the focus measures to obtain more accurate depth estimates. The algorithms were implemented and tested on a variety of different photomicrographs and photographs. The results indicate that the shape-from-focus method may be applied to a variety of industrial vision problems.

1 Introduction

1.1 Motivation

The advancement of three-dimensional machine vision is largely dependent on the development of efficient and reliable shape reconstruction methods. Shape extraction, in turn, requires a good understanding of various surface characteristics and the image formation properties that extracted methods use for their outputs. The existing methods have been developed for the past 30 years. The extraction algorithms assume a certain surface model, and surface reconstruction is performed under this assumption. All surfaces encountered in practical tasks are rough to some extent. In many vision applications, the spatial variations are comparable to dimensions in the viewing area of individual picture elements of the imaging sensor. Hence, image processing produced by such surfaces is not reproducible without some form of illumination or the next, and it is difficult to obtain dense and accurate surface shape information. In using existing algorithms, such as stereo, light, shape-from-stereo, etc. Three-dimensional and reliable solution to this rather difficult extraction problem is achievable.

1.2 Background

We propose to use focus analysis to recover the shape of surfaces. Previously, focus analysis has been used successfully for imaging systems or to obtain greater depth information from the observed scene. Here, (1) proposed focusing imaging systems by using the Fourier transform and assessing the frequency content in the image, (2) developed a general gradient-magnification extraction method and used the sharpness of edges to optimize focus quality. (3) described a two-modulus-difference that is computed by summing from the intensity difference between neighboring pixels and used as a measure of focus quality. (4) implemented and tested various automatically focused algorithms.

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Recent results, Nayar's [55] evaluated and compared the performance of different focus criterion functions. Nayar also propose a method to estimate the depth of an image by optimizing a focus measure by using a technique known as focus from motion. The accuracy and efficiency of such methods is greatly dependent on the focusing angle and sensitivity of the image formation system. The multi-modulated-Laplacian (SML) operator is developed to compare local measures of the quality of image focus. The SML operator is applied to the image sequence, and the focus surfaces obtained at each image point are used to compute local depth estimates. We present two algorithms for depth estimation. The first algorithm simply looks for the focus level that maximizes the distance function at each point. The second algorithm was a Gaussian model to incorporate the focus measures to obtain more accurate depth estimates. The algorithms were implemented and tested on a variety of different photomicrographs and photographs. The results indicate that the shape-from-focus method may be applied to a variety of industrial vision problems.

1.3 Proposed Approach

In this paper, we develop a shape-from-focus method. In contrast to previous work in this area, we used the following approaches:

- We will not attempt to estimate depth from a pair of images by exploiting local variations in the viewing angle. The accuracy of such a method is greatly dependent on the focusing angle and sensitivity of the image formation system. The multi-modulated-Laplacian (SML) operator is developed to compare local measures of the quality of image focus. The SML operator is applied to the image sequence, and the focus surfaces obtained at each image point are used to estimate depth information.

We restrict ourselves to visibly rough surfaces that produce certain images with high frequency intensity variations. We review the image formation process and show that a defocused imaging system plays the role of a low-pass filter. The shape-from-focus method moves the unsharp object with respect to the imaged system and obtains a sequence of images that correspond to different levels of object focus. The multi-modulated-Laplacian (SML) focus operator is developed to measure the relative changes in intensity and position. The operator is applied to the image sequence to obtain a set of depth images at each level. Each method is evaluated for depth estimation and the accuracy of the method is compared with the previous approaches. Our algorithm is implemented and tested on a variety of different photomicrographs and photographs. Experimental results indicate that the method is capable of extracting dense and accurate surface information with acceptable accuracy to enhance detail and type.

2 Visibly Rough Surfaces

In the study of rough, a rough surface is defined as one whose smallest spatial variations have dimensions that are much larger than the wavelength of the incident electromagnetic wave. This is an important property of our technique. In this work, we introduce the notion of visible roughness. A surface is considered to be rough if the dimensions of its spatial variations are comparable to the viewing area of individual pictures and are visible as a texture under a microscope or with a camera. The object shown in Fig. 1 is composed of a large num-
3 Focused and Defocused Images

In this section, we briefly review the image formation process and describe defocused images as processed versions of focused images. Fig. 2.5 shows the basic image formation properties. All light that is reflected by the object P and intercepted by the lens is refracted by the lens to converge at point Q, the image point. The relationship between the object distance, o, focal distance of the lens f, and the image distance i, is given by the Gaussian lens law:

\[ \frac{1}{f} = \frac{1}{o} + \frac{1}{i} \]  

(1)

Each point on the object plane is projected onto a single point on the image plane, thus causing a clear or focused image L(x,y) to be displayed on the image plane. If, however, the sensor plane does not coincide with the image plane and is displaced from the image plane, the image will be blurred. This is common in cases where the sensor plane is not parallel to the image plane.

No assumptions are made regarding the size of the scene. Scenes, for example, may not represent the microstructures defined in [15], [16].

There are many examples of what is meant by the term scene. Here, we define scenes as a variable structure to the inventory of conflicting image planes [18].

The images formed by rough surfaces may be periodic, quasi-periodic, or random. The wavefronts are made regarding the type of instance.

The shape of the patch also depends on the shape of the structure of the image plane. We are assuming the opposite to the opposite.
In order to overcome these problems, we propose to vary the degree of focus by moving the object \( z \) with respect to a fixed configuration of the optical system and sensor. This approach ensures that the focused areas of the image are always subjected to the same magnification.

4 Shape from Focus: An Overview

The shape-from-focus method is based on the observations made in the previous sections.

- At focus level magnification, rough surfaces produce images that are rich in texture.
- A defocused optical system plays the role of a low-pass filter.

Fig. 3 illustrates a rough surface of unknown shape placed on a translational stage. The reference plane shows the first-order positions of the stages. The configuration of the optics and sensor defines a single plane, the "focused" plane, that is perfectly focused onto the sensor plane. The distance \( d \) between the focused and reference planes, and the displacement \( d \) of the stage with respect to the reference plane, are shown by measurement. Consider the surface element, with \( x \) on the unknown surface, \( y \), if the stage is moved towards the focused plane, the image will gradually increase in its degree of focus (high-frequency content) and will be perfectly focused when \( x \) is on the focused plane. Further measurement of the element \( x \) will again decrease the defocusing of its image. If we observe the image area corresponding to \( y \) and move the stage displacement \( d \) as the moment of maximum focus, we can compute the height \( d(y) \) of \( y \) with respect to the stage in \( d \), \( d(y) = d(y) - d \). In fact, we can use \( d \) to determine the distance of \( y \) with respect to the focused plane, without modifying the coordinate system defined with respect to the imaging system. This approach may be applied independently to all surface elements to obtain the shape of the entire surface.

5 A Focus Measure Operator

To automatically detect the intensity of "best" focus, we will develop a focus measure operation. The operators must respond to high-frequency variations in image intensity, and ideally, must produce maximum response when the image area is perfectly focused. The high-frequency content of an image area can be determined using the Fourier transform. However, since Fourier transforms are expensive to compute without special purpose hardware, we seek an alternative approach.

A few focus measure operators have been proposed and used in our past [5]. Generally, the objective has been to find an operator that behaves in a stable and robust manner over a variety of images such as images of outdoor scenes, text, etc. Such an approach is essential while developing automatically focusing imaging systems that have to deal with "general" scenes. Realizing in mind that we are dealing with textured images, we develop an operator that is particularly well suited to such images. In the next section we will evaluate the performance of our operator.

One way to high-pass filter an image is to determine its second derivative. For two-dimensional images, the Laplacian is very often used:

\[
\phi(x, y) = \frac{\partial^2}{\partial x^2} \phi(x, y) + \frac{\partial^2}{\partial y^2} \phi(x, y)
\]

where \( \phi(x, y) \) is the image intensity at the point \( x, y \). In frequency domain, applying the Laplacian \( \phi(x, y) \) to the defocused image \( f(x, y) \) (equation 5) gives:

\[
L(u, v) = H(u, v) f(u, v) = - (\Delta^2 - \lambda^2 u^2 - \lambda^2 v^2)
\]

where \( L(u, v) \), \( f(u, v) \), \( H(u, v) \) are Fourier transforms of the defocusing parameter \( \lambda \). The Laplacian acts as a high-pass filter and makes the effect of defocusing on the high-frequency prominent. Interestingly, for any given frequency \( (u, v) \), \( L(u, v) \) varies as a Gaussian function of the defocusing parameter \( \lambda \). In general, however, the result would depend on the frequency distribution of the image scene. Though our interest in random, it may be assumed to have a set of low-frequency envelopes. Then, loosely speaking, each frequency is attenuated by a Gaussian function of \( \lambda \) with a variance that is determined by the frequency. Therefore, the result of applying the Laplacian operator may be expressed as a sum of Gaussian functions in \( \lambda \). The result is expected to be maximum when \( \lambda = 0 \), i.e., when the image is perfectly focused and attenuation for all frequencies is minimum. Since the frequency distribution of the texture is random, the variances of the Gaussian functions are also random. Using con-

---

\[ f(z) \]

\[ \text{Figure 3: Shape from focus.} \]

\[ z \]

\[ x \]

\[ y \]

\[ d \]

\[ \text{surface element} \]

\[ \text{focused plane} \]

\[ \text{translational stage} \]

\[ \text{reference plane} \]

\[ d \]

\[ \phi(x, y) \]

\[ \frac{\partial^2}{\partial x^2} \phi(x, y) + \frac{\partial^2}{\partial y^2} \phi(x, y) \]

\[ L(u, v) = H(u, v) f(u, v) = - (\Delta^2 - \lambda^2 u^2 - \lambda^2 v^2) \]

\[ \text{Figure 4: The effect of defocusing and second-order differentiation in frequency domain.} \]

\[ \lambda = 0 \]

\[ \lambda = 0.05 \]

\[ \lambda = 0.10 \]

\[ \lambda = 0.15 \]

\[ \lambda = 0.20 \]
tual limit theorem, the net result may be assumed to be a Gaussian function of the defocusing parameter \( r_{\sigma} \). This general behavior is expected irrespective of the focus measure (or parameter) used. The focus measure operator basically selects the frequencies that will play a dominant role in this process. Both, our own experiments (Section 5) and Knudt's empirical evaluation of various focus criteria (5) support the steepest argument. As seen in (5), image noise that is time-varying will of course degrade the performance of any focus measure operator.

We note that the case of the Laplacian the second-derivative in the x and y direction has been used to illustrate this point. An example of such an instance is illustrated in Fig. 5; the

![Figure 5: A texture instance with zero Laplacian value.](image)

partial derivatives are equal in magnitude but opposed in sign, i.e. \( \Delta^2 f = 0 \). In the case of texture, such images and similar instances may occur frequently and the Laplacian is prone to behave in an unstable manner. We overcome this problem by defining the modified Laplacian as:

\[
\Delta^2_{M} f = \left[ \frac{\partial^2 f}{\partial x^2} \right] - \left[ \frac{\partial^2 f}{\partial y^2} \right]
\]

(10)

Note that the modified Laplacian is always greater or equal in magnitude to the Laplacian. In (11), we have empirically demonstrated the benefit of the above modification. However, the experimental results described in (12) also indicate that the response of the modified Laplacian is slightly more stable but not very different from that of the Laplacian.

The discrete approximation to the Laplacian is usually a 3x3 operator. In order to accommodate for possible variations in the size of texture elements, we compute the partial derivatives by using a variable spacing (step) between the points used to compute the derivatives. Hence, the discrete approximation to the modified Laplacian is expressed as:

\[
M(x_i, y_j) = |2(x_i + 1, y_j) - 2(x_i - 1, y_j) - 2(x_i, y_j + 1) + 2(x_i, y_j - 1) - 2(x_i, y_j)|
\]

(11)

Finally, the focus measure at a point (x, y) is computed as the sum of the modified Laplacian values, in a "small" window around (x, y), that are greater than a threshold value:

\[
F(x, y) = \sum_{i=1}^{N} \sum_{j=1}^{N} M(x_i, y_j) > \frac{T_F}{2} \geq T_F
\]

(12)

where the parameter \( N \) determines the window size used to compute the focus measure. In contrast to auto-focusing methods, we typically use a small window of size 3x3 or 5x5, \( N = 1 \) or \( N = 2 \). We shall refer to the above focus measure as the "normalized Laplacian" (NLM).

6 Evaluating the Focus Measure

We evaluate the SML focus measure by analyzing its behavior as a function of the distance between the observed surface and the focused plane. A detailed description of the experimental set-up is given in the latter section. In the following experiments, texture samples are

![Figure 6: SML focus measure function computed for two texture samples.](image)
7 Sampling the Focus Measure Function

We can interpret the focus measure function of an image point \((x, y)\) as \(F(x, y)\). Since depth estimation is a local operation, we will focus our attention on a single image point, bearing in mind that the same estimation method can be applied to all other image points. Therefore, we will denote the focus measure function as simply \(F(x, y)\). By using the arguments presented in section 5 and by studying the example result shown in Fig. 6, we assume that \(F(x, y)\) has a Gaussian distribution with mean value \(\overline{d}\) and standard deviation \(\sigma\) (Fig. 7). The mean \(\overline{d}\) corresponds to the stage displacement at which \(F(x, y)\) is maximum, i.e., \(\overline{d} = F(x, y)\). As the texture contrast of the surface element increases, \(F(x, y)\) increases and \(\sigma\) decreases. Each surface element, therefore, is expected to have its own \(\overline{d}\) and \(\sigma\) values.

If we use very small stage displacements \((\Delta d = 0)\), the number of images to be obtained and processed is too large from the perspective of practical implementation. Hence, we use large displacements to obtain a few images of different focal levels and use the Gaussian model to interpolate the small number of focus measures to obtain depth estimates. Computing focus measures at a fixed number of displacements is equivalent to sampling the function \(F(d)\). At each displacement \(\Delta d\), we compute the focus measure \(F(d)\) at the test axial position \(d = \overline{d} + \Delta d\). We show in the following section that a minimum of three focus measures are needed for the Gaussian interpolation. In other words, therefore, depth estimation may be obtained from only three images of the surface. However, since the Gaussian model only approximates the focus measure function, we use the condition \(\sigma \ll \Delta d \ll 2\sigma\) to ensure that \(\overline{d} = F(d)\) is valid for a minimum of three focus measures. Note that displacements are applied to all object points. Therefore, by applying the above condition to the image area that has maximum texture contrast, we can ensure that a few or many focus measures will be valid in the \(\Delta d \approx \overline{d}\) range of all object points.

We test that the value of \(\overline{d}\) also increases with the depth of the field of the imaging system. Therefore, for objects of large dimensions and, only a small number of images may be used by increasing the depth of field.

8 Depth Estimates from Focus Measures

In this section, we describe the estimation of depth of a surface point \((x, y)\) from the focus measure set \(F(x, y)\). We use the parameters \(\overline{d}\) to represent the depth of the surface point. For convenience, the notation \(F(d)\) is used to represent the focus measure value of depth \(d\). We show in the following section that a minimum of three focus measures is the best estimate of depth.

8.1 Coarse Resolution Depth Estimation

The first algorithm simply looks for the displacement value \(d\) that maximizes the focus measure and assigns that value to \(\overline{d}\).

Algorithm 1

Step 1: Let \(k = 1\), \(F_{\max} = 0\).
Step 2: If \(F_{k} > F_{\max}\), \(F_{\max} = F_{k}\) and \(\overline{d} = d_{k}\).
Step 3: If \(k < K\), \(k = k + 1\), go to step 2.
Step 4: If \(F_{\max} = F_{T_{1}}\), the point \((x, y)\) belongs to background. Stop.

This simple algorithm may be used to compute rough depth estimates. The performance of the algorithm is directly dependent on the selection of \(\Delta d\).

8.2 Depth Estimation by Gaussian Interpolation

The second algorithm uses the Gaussian distribution to model the focus measure function \(F(d)\) and interpolates the computed measure.

Figure 7: Gaussian interpolation of focus measures.

Values to obtain more accurate depth estimates. One approach is to fit all computed \(F(d)\) values to the Gaussian model. However, we feel that more accurate depth estimates can be obtained, while saving computations, by using the Gaussian distribution to model only the peak of \(F(d)\). The following algorithm uses only three focus measures, namely, \(F_{1}, F_{2}\), and \(F_{3}\), that lie on the largest mode of \(F(d)\), such that \(F_{1} > F_{2} > F_{3}\) (Fig. 7).

Using the Gaussian model, the focus measure function may be expressed as:

\[
F = F_{\max} \exp \left( -\frac{(d - \overline{d})^2}{2\sigma^2} \right)
\]

where \(\overline{d}\) and \(\sigma\) are the mean and standard deviation of the Gaussian distribution (Fig. 7). Using natural logarithmic expressions, we can rewrite Eq. 13 as:

\[
\ln F = \ln F_{\max} - \frac{(d - \overline{d})^2}{2\sigma^2}
\]

By substituting each of the three measures \(F_{1}, F_{2}\), and \(F_{3}\) and its corresponding displacement value in Eq. 14, we obtain three equations that can be solved for \(\overline{d}\) and \(\sigma\):

\[
\begin{align*}
\ln F_{1} - \ln F_{\max} &= -\frac{(d_{1} - \overline{d})^2}{2\sigma^2} \\
\ln F_{2} - \ln F_{\max} &= -\frac{(d_{2} - \overline{d})^2}{2\sigma^2} \\
\ln F_{3} - \ln F_{\max} &= -\frac{(d_{3} - \overline{d})^2}{2\sigma^2}
\end{align*}
\]

\[
\begin{align*}
\sigma^2 &= \frac{2\ln F_{\max} - \ln F_{1}}{(d_{1} - \overline{d})^2} \\
\sigma^2 &= \frac{2\ln F_{\max} - \ln F_{2}}{(d_{2} - \overline{d})^2} \\
\sigma^2 &= \frac{2\ln F_{\max} - \ln F_{3}}{(d_{3} - \overline{d})^2}
\end{align*}
\]

Using Eq. 15, we can find \(F_{\max}\) from Eq. 16 and \(\overline{d}\) as:

\[
F_{\max} = \exp \left( -\frac{(d - \overline{d})^2}{2\sigma^2} \right)
\]

If \(F_{\max}\) is large and \(\sigma\) is small, the focus measure function has a "spike" peak, indicating that the surface texture contrast is in the vicinity of the image point \((x, y)\). Thus, the above values can be used to acquire the observed scene into regimes of different texture contrast.

The following algorithm finds the measures \(F_{1}, F_{2}\), and \(F_{3}\) that correspond to the strongest peak of \(F(d)\), and then uses these measures to interpolate the depth \(\overline{d}\) by Gaussian interpolation.

Algorithm 2

Step 1: Let \(k = 3\), \(F_{1} = 0\), \(F_{2} = 0\), \(F_{3} = 0\), \(d_{1} = 0\).

*Due to image noise and variations in magnification, the focus measure image may be pre-sifted with a low-pass filter and then have various bands.

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9 Experiments

9.1 Experimental Set-up

Fig. 8 shows a photograph of the experimental set-up used to demonstrate the shape from-focus method. A microscope is used to magnify the object surface and images are obtained using a CCD camera with 235x236 pixels. Camera images are digitized and processes using a computer. The magnification of the imaging system can be varied from 3.5 to x160. The object is placed on a translational stage that is used to move the object through the focused plane of the imaging system. Stage displacements are monitored using an electronic displacement sensor that has an accuracy of within 0.1 µm. In all our experiments, the bright field illumination (15) of the microscope was used to illuminate the object surface.

![Figure 8: Photograph of the experimental set-up.](image)

9.2 Results

The accuracy of the shape from-focus algorithm was analyzed using a steel ball sample that was 139µm in diameter. The ball was sandpapered to give it a rough surface. A camera image of the ball under bright field illumination is shown in Fig.9a. Incremental displacement of ∆d = 100 µm were used to obtain 15 images of the ball, and a 5x5 SML operator was applied to the image sequence to obtain focus measures. Depth maps of the ball, generated by the curve evolution and Gaussian interpolation algorithms, are shown in Fig.9b and 9c, respectively. This allows size and location of the ball were used to obtain error maps by subtracting a smooth ball from the two depth maps. It is difficult to define the accuracy of the method as it depends on many factors: the surface texture, depth of field of the imaging system, and the incremental displacement ∆d. The results shown in Fig.9d shows the error statistics computed from the error maps corresponding to the two algorithms. A total of 32252 image pixels lie within the boundary of the ball. The number of depth values computed by each algorithm depends on the values selected for the thresholding T, T1, T2, and T3.

Fig.10 and 11 show samples with different surface reflectance and roughness properties. As it is difficult to perceive the shapes of these samples from their camera images, we have also included scanning electron microscope (SEM) images of the samples. We hope that these images will provide sufficient shape cues to the reader. Both samples are approximately 100 µm in width and an incremental displacement of ∆d = 10 µm was used in both cases to obtain sequences of about 10 images each. Depth maps of the samples were obtained using the Gaussian interpolation algorithm. A 5 x 5 median filter was used to get rid of a few erroneous depth estimates that result from the lack of texture in some image areas.

Fig.10a and 10b show the camera and SEM images of a tungsten plate filling in a via-hole on a ceramic substrate (16) that is used to establish electrical connections between different components. Conditions such as coarser filling and lack of filling causes electrical defects such as short and open circuits. The sample shown in Fig.10a has a bump on its surface, indicating excess filling. The specular reflectance and variable size of the tungsten particles gives the surface a random texture. The white background (Fig.10a) is the substrate area that has very weak texture. For this sample, we selected the threshold values to classify the substrate area as background. An arbitrary depth value is assigned to the background region.

Fig.11 shows another via-hole sample. In this case, the substrate and filling are hardbaked by baking. The baking process changes the reflectance and texture of the filling and also increases the texture content of the substrate. From the SEM image we are that the via-hole is not sufficiently filled with tungsten paste. For this sample, the algorithm threshold values were selected to obtain the depth of the substrate area too. To accommodate for the large size of substrate texture elements, a step size of 2 µm was used. Two different views of the sample's depth map are shown in Fig.11c.

The above experiments indicate that the Gaussian interpolation algorithm performs stably over a wide range of textures. Errors in computed depth estimates result from factors such as image noise, Gaussian approximation of the SML focus measure function, and weak texture in some image areas. In the current implementation, object displacement and image acquisition are manually initiated. In a high-speed implementation, the object can be moved continuously while images are obtained at fixed intervals of time. By using commercial hardware, the SML focus operator can be applied to each image frame-by-frame and the object movement can be implemented by using look-up tables. We estimate that a high-speed implementation of the method can generate surface depth maps in less than 1 second.

10 Conclusion

In this paper, we have presented a shape-from-focus as a new method of extracting the shapes of rough surfaces.

- To measure the quality of image focus we developed the SML operator. By evaluating the SML operator, we found that it is particularly suited for measuring depth of field.

- We developed and tested two depth estimation algorithms and found, through numerous experiments, that the Gaussian interpolation algorithm produces accurate results for a variety of textures.

- The local nature of the depth estimation technique enables it to adapt to substantial variations in image texture.

- Although we have concentrated on rough surfaces in this paper, the shape-from-focus method can be directly applied to smooth textured surfaces. Smooth non-textured surfaces can also be handled by using special discrimination methods (15).

- This method may be more precisely applied to a number of industrial machine vision problems. Our experiments demonstrate one such application, namely, the inspection of via-hole fillings on ceramic substrates.
(a) Camera image.

(b) Depth map: coarse resolution.

(c) Depth map: Gaussian interpolation.

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<th>Diameter of Test Sphere: 1500 μm</th>
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(d) Error statistics.

Figure 9: Steel ball.

Figure 10: Via-hole filling.
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