Shape and Reflectance from an Image Sequence generated using Extended Sources

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Abstract
All existing shape extraction techniques that are based on photometric measurements rely on known surface reflectance properties. We present a method for determining the shapes of surfaces whose reflectance properties may vary from Lambertian to specular, without prior knowledge of the relative strengths of the Lambertian and specular components of reflectance. The object surface is illuminated using extended light sources and is viewed from a single direction. Surface illumination using extended sources makes it possible to estimate the direction of both Lambertian and specular reflections. Multiple source directions are used to obtain an image sequence of the object. An extraction algorithm uses the set of image intensity values measured at each surface point to compute orientation as well as relative strengths of the Lambertian and specular reflection components. The proposed method is called photometric sampling, as it uses samples of a photometric function that relates image intensity to surface orientation, reflectance, and light source characteristics. Experiments were conducted on Lambertian surfaces, specular surfaces, and hybrid surfaces, whose reflectance models are composed of both Lambertian and specular components. The results show high accuracy in measured orientations and estimated reflectance parameters.

1 Introduction
Shape from shading [4][6][9], photometric stereo [13][17][2], and local shape from specularities [3] are examples of techniques that extract three-dimensional shape information from photometric measurements. All of these techniques rely on prior knowledge of surface reflectance properties. The reflectance properties are either assumed, or measured using a calibration object known shape. In many real-world applications, such as those involving surfaces of different reflectance characteristics, the calibration approach is not a practical one. Therefore, the existing shape extraction methods are often used by assuming surface reflectance to be either Lambertian or specular. Many surfaces encountered in practice are hybrid in reflectance; their reflectance models are linear combinations of Lambertian and specular models. Therefore, Lambertian and specular models are only limiting instances of the hybrid model. It is desirable to have a method that is capable of extracting the shape of hybrid surfaces as well as Lambertian and specular ones.

In many industrial applications, surface polish and surface roughness are found to be important inspection criteria. In such cases, surface reflectance properties may be interpreted as measures of surface polish and roughness. Furthermore, reflectance properties may be used to segment an image into different regions; each region may be regarded as a different surface to suit the needs of inspection. For these reasons, it would be of great value to have a technique that could, in addition to determining shape, also estimate the reflectance properties of each surface point.

This paper presents a method for determining the shape of objects whose surfaces may be Lambertian, specular, or hybrid. Shape information is extracted without prior knowledge of the relative strengths of the Lambertian and specular reflection components. The method also computes parameters that are related to the reflectance properties of surface points.

2 Photometric Sampling
2.1 Basic Photometric Function
Consider the illumination of an object by a point source of light, as shown in Figure 1. The point source emits light in all directions. Light energy reflected by the surface in the direction of the camera causes an image of the surface to be formed. For a given orientation of the surface and direction of the point source, the amount of light energy reflected by the surface in a particular direction is determined by its reflectance properties. The reflectance model of a large number of surfaces is composed of two components, namely, the Lambertian (diffuse) component and the specular (gloss) component. Therefore, the intensity at any point in the image of the surface may be expressed as:

\[ I = I_L + I_S, \]

where \( I_L \) is the Lambertian intensity component and \( I_S \) is the specular intensity component.

Figure 1: Two-dimensional illumination and imaging geometry. A surface element with orientation \( n \), reflects light from the point source direction \( f \), into the camera.
We will express the two components of image intensity in terms of the parameters that describe the two-dimensional imaging and illumination geometry shown in Figure 1. In two dimensions, the source direction vector $\mathbf{s}$, surface normal vector $\mathbf{n}$, and viewing direction vector $\mathbf{v}$ lie in the same plane. Therefore, directions are represented by a single parameter, namely, the zenith angle $\theta$.

**Lambertian Reflection:** The Lambertian intensity $I_\text{L}$ represents the diffuse component of reflection. Surface that satisfies Lambert's law appear equally bright from all directions. Broadly speaking, here we are two mechanisms that produce Lambertian reflection. In one case, light rays that impinge on the surface are reflected many times between surface modulations before they are scattered into space. If the multiple reflections occur in a random manner, the incident energy is distributed in all directions, resulting in diffuse reflection. Another mechanism leading to diffuse reflection is the internal scattering of light rays. In this case, the light rays penetrate the surface and encounter microscopic inhomogeneities in the surface medium. The light rays are repeatedly reflected and refracted as boundaries of regions of differing refractive indices. Some of the scattered rays find their way back to the surface with a variety of directions, resulting in diffuse reflection. When diffuse reflection produced by either or both of the above mechanisms results in compact surface radiation in all directions, we have Lambertian reflection. The brightness of a Lambertian surface is independent of the viewing direction. However, the brightness of a Lambertian surface is proportional to the energy of incident light. As can be seen in Figure 1, the amount of light energy falling on a surface element is proportional to the area of the surface element as seen from the source position, often referred to as the foreshortened area. The foreshortened area is a function of the angle between the surface orientation direction $\mathbf{n}$ and the source direction $\mathbf{s}$. Therefore, the Lambertian intensity component $I_\text{L}$ may be written as:

$$I_\text{L} = A \cos(\theta),$$

where the constant $A$ describes the fraction of incident energy that is diffusely reflected. From here, we observed that the angle of incidence ($\theta$) is greater than $\pi/2$ and less than $\pi/2$, i.e., $I_\text{L} = A \cos(\theta)$ is always greater than zero.

**Specular Reflection:** Various specular reflection models have been proposed over the years. We will use the Beckmann and Spizzichino model (1) that is based on physical optics theory. The surface brightness is modeled as a continuous stationary random process with standard deviation $\sigma_\text{n}$, representing the physical displacement of the surface at any point on a surface with zero mean. Maxwellian distributions are used to determine how incident light waves are scattered by the surface at a given direction. Beckmann and Spizzichino have found specular reflection to be composed of two parts: one representing the specular lobe component and the other called the side components. The lobe component spreads around the specular direction and is defined as the specular intensity and the side component is zero in all directions except for a very narrow range around the specular direction. The strength of these two components is determined by the ratio $\gamma/\lambda$, where $\gamma$ is the wavelength of incident light. When $\gamma/\lambda \ll 1$, the side components have many orders of magnitude less intensity than the lobe component, and the surface reflects light like a mirror. It is the side components that are responsible for strong highlights that are often observed on smooth surfaces. At $\gamma/\lambda$ increases above unity, the side components start to dominate the surface radiation value. In this paper, we will assume that the surface is smooth ($\gamma/\lambda < 1.2$) and that the specular image intensity is determined solely by the side component. Beckmann and Spizzichino have found that the side component is

$$I_\text{S} = B(\theta) = B(\pi/2 - \theta),$$

The photonmetric function $I_\text{S}(\theta)$ relaxes image intensity to surface orientation, surface reflectance, and point source direction and may be written by substituting equations 2 and 3 into equation 1 to obtain:

$$I = A \cos(\theta)(\pi/2 - \theta) + B(\pi/2 - \theta).$$

The constants $A$ and $B$ in equation 4 represent the relative strengths of the Lambertian and specular components of reflection, respectively. We call $A$ and $B$ the reflectance parameters. We see that $A > 0$ and $B > 0$ for a purely Lambertian surface, $A = 0$ and $B > 0$ for a purely specular surface, and $A > 0$ and $B > 0$ for a hybrid surface.

Our objective is to determine orientation and reflectance at each surface point from a set of image intensities that result from changing the source direction $\theta$. As can be seen in Figure 1, by moving the source around, the object, we can vary the source direction without changing the orientation and reflectance parameters. Therefore, even though the orientation and reflectance parameters are unknown, we can treat them as constants in equation 4. For this reason, we will often refer to the basic photonmetric function $I_\theta(\theta)$, a relation between image intensity and source direction. Figure 2 shows a plot of the basic photonmetric function for a hybrid surface of given orientation.
ions [19]. Therefore, it is difficult to measure both components in the same image. Extended source illumination tends to make the image intensities due to Lambertian and specular reflections comparable to one another. A specular surface element of a given orientation will reflect light from a small area on the extended source into the camera. On the other hand, a Lambertian surface element of the same orientation reflects light from all points on the extended source. This feature of the proposed illumination scheme makes it possible to measure both Lambertian and specular reflections in the same image.

In Appendix A, we have shown how extended sources are generated. The extended source, extended source function, \( L_0 \), is derived, and the parameters \( \alpha \) that determine the direction and limits of an extended source are defined. These results will be extensively used in the following discussions.

2.3 Photometric Function for Extended Sources

The photometric function for point source illumination (equation 4) needs to be modified for extended source illumination. An extended source may be thought of as a collection of point sources in which each point source has a radiant intensity that is dependent on its position on the extended source. The intensity of light reflected by a surface may be determined by computing the integral of the light energies reflected from all points on the extended source. Therefore, the modified photometric function \( F'(h) \) is determined by convolving the basic photometric function \( F(\theta) \) with the extended source function \( L_0(\theta, \alpha) \). This operation is illustrated in Figure 3. For a surface point of orientation \( h_0 \), the Lambertian component \( IL \) of the modified photometric function is determined as:

\[
IL = A \int_{\theta_0}^{\theta_{\text{max}}} L_0(\theta, \alpha) \cos(\theta - h_0) \, d\theta \tag{5}
\]

This expression represents the luminous flux per unit area emitted from an extended source into the direction \( h_0 \), where \( A \) is the area of the extended source.

The limits of the integral are determined by the width of the extended source (Appendix A). It can be shown [8] that \( IL \) is a cosine function of the angle between the surface orientation and the direction corresponding to the "center of mass" of the extended source. In our case, since the extended source is symmetric with respect to the source direction \( \theta \), the center of mass of the radiance function is in the direction \( h_0 \). Therefore, we obtain:

\[
IL = A \cos(h_0 - h_0) \tag{6}
\]

where the constant \( A \) represents the strength of the Lambertian component.

Similarly, the specular intensity component \( IS \) resulting from the extended source \( L_0(\theta, \alpha) \) is determined as:

\[
IS = B \int_{\theta_0}^{\theta_{\text{max}}} L_0(\theta, \alpha) \theta(\theta - 2\theta_0) \, d\theta \tag{7}
\]

or:

\[
IS = B L_0(h_0, \alpha) \tag{8}
\]

Strictly speaking, the result of the above integral is dependent on the exact shape of the specular spike. However, since the spike component is significant only in the specular direction, \( 2\theta_0 \), it is reasonable to assume that the specular intensity \( IS \) is proportional to \( L_0(\theta, \alpha) \), while the constant of proportionality is dependent on the exact shape of the spike. To this end, we will use the constant \( B \), rather than \( B \), to represent the strength of the specular component of the photometric function.

The modified photometric function relates image intensity \( I \) to extended source direction \( h_0 \), and is expressed as the sum of the components \( IL \) and \( IS \):

\[
F = A \cos(h_0 - h_0) + B L_0(h_0, \alpha) \tag{9}
\]

Since the parameters \( A \) and \( B \) are proportional to the parameters \( \alpha \) and \( \beta \), respectively, they may be used to represent the reflectance properties of the surface point.

2.4 Sampling

The process of measuring image intensities corresponding to different source directions is equivalent to sampling the modified photometric function \( F(h_0) \), as shown in Figure 4. Samples of the photometric function may be obtained by moving an extended source around the object and obtaining images of the object for different source positions. An alternate approach would be to distribute an array of extended sources around the object such that each source illuminates the object from a different direction. The entire array of extended sources may be sequentially scanned to obtain a photometric function.
3.2 Specular Surfaces

Now consider the case where the object surface is known to be purely specular, and the shape of the object is to be determined. The photometric samples for a specularity point may be written as:

\[ I = B(D, \theta, \phi) \]  

(14)

We want to determine the orientation \( \theta_s \) and the specular strength \( B \) from the intensity set \( I \). Let us assume that the specular direction \( D_s \) lies between the directions \( \theta_s \) and \( \theta_t \), of two adjacent extended sources. Further, let us assume that the photometric function is sampled using the minimum frequency, \( f_{min} \), and that \( \theta_s = \theta_t + \phi \). Then, since the surface is specular, only the samples \( I_s, I_t, \) and \( I_{st} \) will have zero values. We see that when \( \theta_s \) decreases, \( B \) increases, and \( I_t \) decreases, and \( I_{st} \) increases. Similarly, when \( \theta_s \) increases, \( B \) decreases, \( I_t \) increases, and \( I_{st} \) decreases. In other words, equation (12) tells us that the intensity ratio \( I_{st}/I_t \) is equal to the ratio \( L(D_s, \theta_t, \phi)/L(D_s, \theta_t, \phi) \).

Since the extended sources have decaying radiance functions (Appendix A), this ratio is a monotonous function of the angle \( \theta_s \). Since the radiance functions of the extended sources are known a priori, we can compute and store in memory the correspondence between \( I_{st}/I_t \) and \( \theta_s \).

Given the intensity set \( I \) as a specular surface point, the \( \omega \)-zero intensity is in the set of first derivative. If only a single intensity value, i.e., \( I_{st} \), is greater than zero, then we know that \( \theta_t \). It is given in equation (15): for \( I_{st} \), the \( \omega \)-zero intensity is the \( \omega \)-zero in the first derivative.

3.3 Hybrid Surfaces

The modified photometric function for hybrid surfaces is given by equation (15). At each surface point, we want to determine \( A \) and \( \gamma \) from the measured ensemble \( I = \{I_1, I_2, ..., I_N\} \) of the photometric functions. To this end, we develop an algorithm that attempts to separate the Lambertian and specular components of each measured image intensity and then computes surface orientation using the methods given above for Lambertian and specular surfaces.

The extraction algorithm is based on two constraints, namely, the sampling frequency constraint and the uniqueness constraint. By sampling the modified photometric function at the minimum sampling frequency \( f_{min} \), we can confirm that only two consecutive image intensities in the intensity set \( I \) contain non-zero specular components. For each \( k \) in the interval \( 0 \leq k < N \), and \( k \) is hypothesized to be the number of two intensities that have specular components. All remaining intensities in the set \( I = \{I_1, I_2, ..., I_N\} \) must represent only Lambertian components of reflectance. These intensities are used to compute for the surface orientation \( \theta_t \) and the hybrid surface strength \( A \) (Section 3.1). The Lambertian components \( L_s \) and \( L_{st} \) are determined and used to separate the specular components \( I_s \) and \( I_{st} \) from \( I_s \) and \( I_{st} \), respectively. The surface orientation \( \theta_s \) and specular strength \( B \) are computed from \( I_s \) and \( I_{st} \) (Section 3.4).
Selection because the surface orientation that is computed from intensities resulting from the pointer of the two reflection components is less sensitive to image noise and is, therefore, more reliable. An orientation error is found by comparing $\theta_0$ with $\theta_0$. Using the above approach, orientation errors are computed for all $k$, where $0 < k < M$. The orientation and reflectance parameters computed for the value of $k$ that minimizes the orientation error are assigned to the surface point under consideration. This process is repeated for all points on the object surface.

It is important to note that the extraction algorithm is not only capable of determining shape and reflectance properties of hybrid surfaces, but also purely Lambertian and purely specular surfaces.

Extraction Algorithm

Step 1: Let $k = 1$ and $a_1$ equal a large positive number.

Step 2: Assume that image intensities $I_{k1}$ and $I_{k+1}$ consist of specular components of reflection. All points $G_{k}$, where $k < k$ and $k_1 < k$, and the Lambertian model are used to compute the surface orientation $\theta_0$ and Lambertian strength $A_e$ (Section 3.1).

Step 3: The specular components $S_{k1}$ and $\hat{S}_{k+1}$ are separated from the image intensities $I_{k1}$ and $I_{k+1}$:

$$ S_{k1} = I_{k1} - A_e \cos(\theta_0 - \theta_e) $$
$$ \hat{S}_{k+1} = I_{k+1} - A_e \cos(\theta_0 - \theta_e) $$

If $S_{k1} < 0$ or $\hat{S}_{k+1} < 0$, set $k = k + 1$ and go to step 2.

Step 4: The surface orientation $\theta_0$ and the specular strength $A_e$ are determined by using specular intensities $S_{k1}$ and $\hat{S}_{k+1}$ and the speckled model (Section 3.2).

Step 5: The best estimate of surface orientation, for the $k$th iteration, is determined as:

$$ \theta_0 = \frac{A_e \hat{S}_{k+1} - \hat{S}_{k+1} \hat{S}_{k+1}}{A_e + \hat{S}_{k+1} \hat{S}_{k+1}} $$

The orientation error $\epsilon_{o}$ is determined as:

$$ \epsilon_{o} = \frac{A_e \hat{S}_{k+1} - \hat{S}_{k+1} \hat{S}_{k+1}}{A_e + \hat{S}_{k+1} \hat{S}_{k+1}} $$

Step 6: If $\epsilon_{o} < \epsilon_{o}$, then:

$$ \theta_0 = \theta_0, \quad A_e = A_e, \quad B' = B' $$

If $k < k_1$, set $k = k + 1$ and go to step 2. Otherwise, stop.

4 Experiments

4.1 Experimental Set-Up

We have conducted experiments to demonstrate the practical feasibility of the photometric sampling concept. A photograph of the experimental set-up used to implement photometric sampling is shown in Figure 5. A 14-inch diameter spot with shade is used as the spherical diffuse, and extended light sources are positioned on the diffuser's surface by illuminating it using condenser light bulbs. All light bulbs are assumed to have the same radiance intensity and are equidistant from the center of the diffuser. In our experiments, a source illumination angle of $\theta = 32$ degrees was used, and sampling was performed at the minimum frequency determined by equation 10. The object is placed at the center of the diffuser and is viewed by a camera through a 1-inch diameter hole in the top of the diffuser. The current setup uses a WV-22 model Panasonic CCD camera that has a 312 x 480 pixel resolution. The complete imaging system is comprised of cameras and can be physically isolated from the diffuser. The light bulbs, camera, and object are all placed in the same plane. This two-dimensional set-up is capable of measuring only the orientation of a surface normal vector that lies on a single plane in orientation space. For each extended source, an image of the object is digitized and stored in memory. The sequence of object images, generated by scanning the array of extended sources, is processed by a SUN SPARC workstation.

4.2 Experimental Results

The experimental set-up and the extraction algorithm were used to extract surface properties of a number of objects. Figures 6, 7, and 8 show the results of the extraction method applied to objects with different surface reflectance properties. For each object, a photograph of the object is collected by two reflectance images and a needle map produced by the extraction algorithm, and a depth map that is reconstructed from the needle map. The reflectance properties of the surfaces are given by two images: the Lambertian strength image and the specular strength image. The intensity at each pixel, in both of these images, is proportional to the strength of the reflectance model component, the image represents. The needle map is a representation of surface orientation. At each point on a needle map, the length of the needle is proportional to the tilt of the surface away from the viewing direction of the camera. The direction in which each needle point is determined by locating the starting point of the needle. All needles originate from the image that contains the resolution field of the needle map. To help evaluate the performance of the extraction algorithm, we have included a depth map of each object that is obtained by integrating the orientations in the needle map. Note that the reconstructed surfaces are displayed at some offset level in all the depth maps.

The object shown in Figure 6 is cylindrical and its surface is Lambertian. Figure 7 shows a prism-shaped object that has a highly specular surface. A major advantage of the

Figure 5: Photograph of the experimental set-up used to demonstrate the photometric sampling concept.
Nonmetric sampling method, over other existing shape extraction techniques, lies in its ability to determine the shape and reflectance of hybrid surfaces. The surfaces of many manufactured plastic objects seem to fall into this category. The Lambertian component is produced by the internal scattering mechanism, while the specular spike results from the smoothness of the surface. Figure 8 shows a photo of a cylindrical plastic object. As expected, non-zero Lambertian and specular strengths are seen in the reflectance images. The needle and depth maps of the object are consistent with the actual shape.

An important feature of all the above results is that the surface properties at a pixel are computed solely from the intensity recorded at that pixel. The needle maps and reflectance images have not been subjected to any filtering operations. A simple error analysis was conducted to estimate the measurement accuracy of the current set-up. In the results obtained so far, measured surface orientations were found to be within 4 degrees of the actual orientation values, and an average error of 3 degrees in orientation was estimated.

5 Conclusions

We conclude this paper with the following remarks:

- The photometric sampling method is capable of determining the shape and the reflectance parameters of Lambertian, specular, and hybrid surfaces.
- The method is local in that the orientation and reflectance of a surface pixel are computed solely from image intensities recorded at that pixel.
- Active surface illumination, using extended light sources, makes it possible to capture both Lambertian and specular reflectances in the image intensities.
- Accurate orientation estimates are obtained by using both Lambertian and specular components of the image intensities.

We are currently in the process of extending the theory and experimental set-up to three dimensions.

![Figure 6: Cylindrical painted object with a Lambertian surface.](image)
Figure 7: Prism-shaped metallic object with a specular surface.

Figure 8: Cylindrical plastic object with a hybrid surface.
A Generating Extended Sources

There are numerous ways of generating extended sources. In this section, we present the approach that we have chosen to use. An extended light source can be generated by illuminating a sheet of light-diffusing material with a point light source. Figure 9 illustrates this, the illumination of a section of a circular diffuser of radius \( R \). The point source is placed at a distance \( H \) from the diffuser's surface, and the viewed object is placed at the center of the circle. Let us assume that the diffuser is "ideal," i.e., that incident energy is scattered equally in all directions. Then, the radiance \( L(\theta, \phi) \) of the outer surface of the diffuser is proportional to the irradiance \( E(\theta, \phi) \) of the outer surface of the diffuser:

\[
L(\theta, \phi) = C E(\theta, \phi),
\]

(20)

where \( C \) is a constant of proportionality. The analytic expression for the surface irradiance \( E(\theta, \phi) \) may be derived from the basics of radiometry as:

\[
E(\theta, \phi) = \frac{I(\theta, \phi)}{C} = \frac{C(R + H + R C o s(\theta - \Theta) - R)}{(R + H - R C o s(\theta - \Theta))^2 + (R S i n(\theta - \Theta))^2}.
\]

(21)

where \( I \) is the radiation intensity of the point source \( S \). The radiance of the extended source \( S \) may be determined by expressing the variables \( \theta \) and \( \phi \) in equation 21 in terms of the parameters \( R, H \), and \( \Theta \) of the illumination geometry:

\[
L(\theta, \phi) = \frac{C(R + H + R C o s(\theta - \Theta) - R)}{(R + H - R C o s(\theta - \Theta))^2 + (R S i n(\theta - \Theta))^2}.
\]

(22)

Throughout this paper, the position of an extended source will be described by the angle \( \Theta \) of the spot source used to generate the extended source. The radiance function \( L(\theta, \phi) \) is symmetric, or even, with respect to the source direction (\( \theta = \Theta \)), and its magnitude decreases as \( \theta \) deviates from \( \Theta \). Points on the diffuser that lie in the interval \( \theta = \Theta \), \( R < \theta < \Theta + \phi \) receive light from the point source \( S \). Points that lie outside this interval are occluded from the point source \( S \) by points in the interval. Thus, \( L(\theta, \phi) = 0 \) for \( \theta < \Theta - \phi \) and \( \theta > \Theta + \phi \). The source

\[\text{radiance is defined as the flux emitted per unit of surfaces d} \text{rivative surface area per unit solid angle. Radiance is restrained in watts per square meter per steradian (W m}^{-2})\text{.} \]

\[\text{radiance is defined as the incident flux density and is restrained in watts per square meter (W m}^{-2})\text{.} \]

\[\text{radiance of a source is defined as the flux emitted per unit solid angle and is restrained in watts per steradian (W m}^{-2})\text{.} \]

The term \( \phi \) is given by Figure 9 as:

\[
\phi = 2 \sin^{-1} \frac{R}{R + H}.
\]

(23)

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