Crossword Generator and Solver

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23 December 2020

1 Introduction and Overview

This project creates crossword grids and solves them using a user-specified dictionary. There are no clues or pre-filled letters involved: the solver produces all valid solutions for the crossword based on its dimensions and layout.

2 Crossword generation

The crossword grid generation loosely follows the New York Times' crossword construction constraints[1]:

1. Crosswords must have black square symmetry, which typically comes in the form of 180-degree rotational symmetry;
2. Crosswords must have all-over interlock;
3. Crosswords must not have unchecked squares (i.e., all letters must be found in both Across and Down answers);
4. All answers must be at least 3 letters long;
5. Black squares should be used in moderation.

Constraints 1 and 3 are fully implemented: our crosswords have 180-degree rotational symmetry, and all white squares are part of both horizontal and vertical entries. We implemented constraints 4 and 5 by allowing our crossword generator to take in a user-supplied minimum word length and target black square density. The rotational symmetry constraint made it very difficult to generate crosswords with odd dimensions that didn't violate the minimum word length, so we set our crossword generator to only generate grids with even dimensions. Our strategy also means that it is possible to generate grids with disconnected regions, which breaks constraint 2. See below for one of the example grids we generated, using a size of 10 and a black square density of 0.2.

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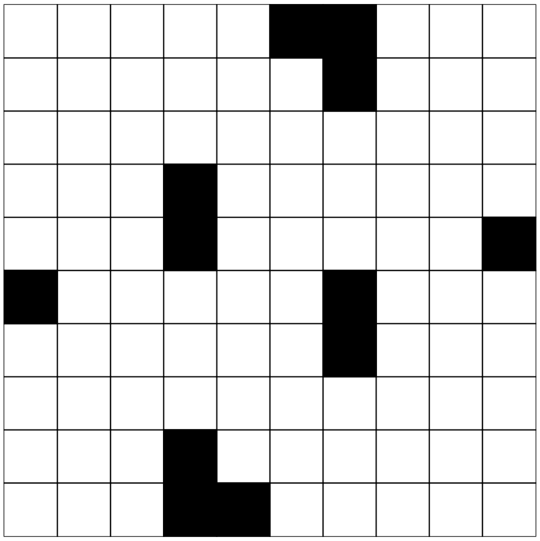
[1] https://www.nytimes.com/puzzles/submissions/crossword



3 Solving strategy

The solving strategy for the crossword is straightforward. Using a user-supplied dictionary, solution candidates are generated by filling each row of the grid with possible word combinations. The validity of the candidate is checked columnwise. Valid solutions are counted and printed.

Internally, grids are represented as two-dimensional lists of `Square`. Filling the crossword involves using list comprehensions to generate all possible solutions for each word, row, and grid. Checking the crossword is done by transposing each solution candidate and checking each column-turned-row to make sure it only contains sequences of letters found in the dictionary.

4 Dictionary selection

In our initial attempts, the dictionary was stored as a `Map Int (Set String)`, where integer keys representing word length mapped to sets of words with that number of letters. This was later changed to `Map Int (Set [Square])`, for reasons discussed later in this report. The `Map`/`Set` combination was chosen for efficient lookup.

The standard dictionary, located in `/usr/share/dict/words`, contains 102774 words. 63357 of these contain no special characters or captial letters, making them usable in our crosswords. However, testing with a dictionary this large proved to be too difficult – every machine we tested on, including a CLIC machine called Denhaag that boasts 125G of memory, failed to get past even the first layer of candidates for a 4x4 grid before getting killed. The solutions it did generate were often quite interesting (see Figure 1).

```
[a b l y]      [a c i d]      [a c t s]
[b e a u]      [b o r e]      [b o r e]
[e a r l]      [e p i c]      [e v e n]
[d u k e]      [d e s k]      [d e e d]
```

Figure 1: 4x4 puzzles generated using standard dictionary.

We instead chose to test smaller dictionaries of varying sizes. We started with a dictionary of 10000 most common English words[2], and trimmed the list by taking only the first 1000, 500, 200 or 100 words for different tests.

In terms of crossword filling, this does not yield very interesting solutions. Words like "the" and "that" appear often, but would not be not easy to write creative clues for in a real puzzle. In the 1000-word case, filling a 4x4 grid generated 39 solutions, 37 of which were diagonally symmetrical (same words down as across). The other 2 of were each others' transposes, and missed diagonal symmetry by only one letter.

```
[f a s t]      [s a l e]
[a u t o]      [a r e a]
[s t a y]      [l e s s]
[t o y s]      [e a s y]
```

Figure 2: 4x4 puzzles generated using 1000-common-word dictionary.

A more practical crossword generator would prefer words toward the middle of the frequency-ordered word list. Ideally, crossword words should be known to the average person but not necessarily used in the average sentence.

5 Initial Parallelization Attempts

Unless specified otherwise, all results in this section are from tests run using a dictionary of 500 randomly selected common words on the following 4x4 crossword:

$$\begin{bmatrix} @ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & @ \end{bmatrix}$$

where underscores represent blank white spaces and '@' represents a black square. While the solver can handle larger puzzles than this, increasing the size of the puzzle increases runtimes to the order of minutes rather than seconds, prohibiting analysis of event logs in Threadscope. In the following subsections, we will apply various parallelization techniques to the following (slightly squashed) code snippet:

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[2] https://github.com/first20hours/google-10000-english

```
solve :: Map Int (Set String) -> [[Square]] -> [[[Square]]]
solve dict grid = do
    let cands = map transpose (fill dict grid)
    filter (check dict) cands
```

The third line yields a list of transposed candidate solutions. The fourth cross-references them against the provided dictionary to verify that the columns contain valid words.

All figures reported in this section represent an average of five trials on an otherwise lightly loaded system.

Run sequentially, the program finds all valid solutions in 11.07 seconds.

5.1 Naive parBuffer

Our first attempt at parallelization was simply to parallelize the checking, expecting each thread to generate and then check their respective candidates independently, as follows:

```
solve :: Map Int (Set String) -> [[Square]] -> [[[Square]]]
solve dict grid = do
    let cands = map transpose (fill dict grid)
    filter (check dict) cands `using` parBuffer 100 rdeepseq
```

We use parBuffer here rather than parList in an attempt to mitigate the extreme memory requirements of loading the whole list of candidates at once and rdeepseq to force complete evaluation of each candidate in each thread. In practice, this implementation failed to parallelize at all. Zero threads are created, and the overhead of adding a second core (on which some of the garbage collection seems to be running) slows the runtime to 14.44s. The Threadscope graph is included in Figure 3. The culprit here appears to be filter. Unlike map, whose weak head normal form representation would consist of a concatenation of one thunk for applying the check to the first element of the list and then another for completing the map operation on the remainder of the list, filter must evaluate each check on each element completely in order to determine the shape of

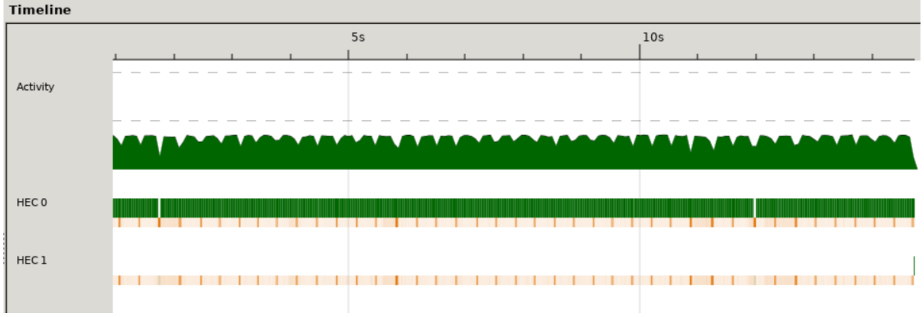

Figure 3: The naive parBuffer implementation, -N2

the list. This evaluation is performed sequentially before the parBuffer is even properly applied, completely negating the benefits of having multiple cores.

5.2 Parallelizing candidate creation

We can sidestep this problem by shifting the parallelization directly onto the code that generates candidate solutions:

```
solve :: Map Int (Set String) -> [[Square]] -> [[[Square]]]
solve dict grid = do
    let l = fill dict grid
    let cands = map transpose l `using` parBuffer 100 rdeepseq
    filter (check dict) cands
```

The event log tells us that this change was successful: The workload here appears extremely well-balanced. However, since about 67% of the 11694774 sparks created either fizzle or are garbage collected, sequential garbage collection for excess sparks short enough not to be visible at this scale in Threadscope drives the runtime all the way up to 24.09s, most of which comes in the form of additional garbage collection: we have achieved a “speedup” of 0.46x. This indicates that the parallelization is far too granular, making it inefficient: each spark is performing much less work per candidate grid than its computational upkeep.

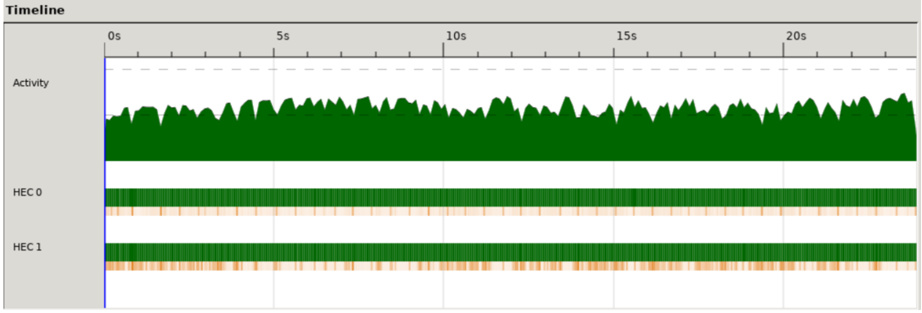

Figure 4: parBuffer applied to candidates, -N2

5.3 Parallelized candidate creation with chunking

Instead of parallelizing the output of map directly, we can group the candidates into chunks of more reasonable sizes and then hand off each one to be computed in parallel:

```
solve :: Map Int (Set String) -> [[Square]] -> [[[Square]]]
solve dict grid = do
    let l = fill dict grid
    let m = map transpose l
```

```
    let chunk n = Data.List.Split.chunksOf n
    let cands = chunk 10000 m `using` parBuffer 100 rdeepseq
    filter (check dict) $ concat cands
```

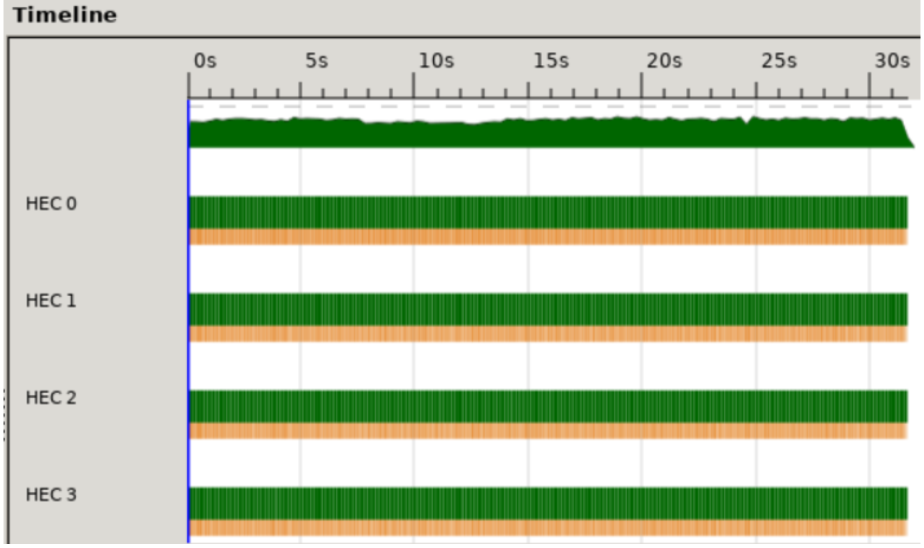

Figure 5: parBuffer applied to candidates w/ chunking, -N4

On this test, we see a 100% conversion rate: of all of the 19617 sparks created, none fail. The program spends much less time in the MUT phase of computation as a result, but the costs of garbage collection gives us our slowest runtime so far: 31.6s. Perturbing the chunk size to any of {10, 50, 100, 500, 1000, 10000, 50000, 100000} or changing the number of threads passed to parBuffer makes matters worse.

5.4 Parallelizing candidate creation & filtering

Instead of filtering candidates in a separate second step, we can check them at the same time:

```
solve :: Map Int (Set String) -> [[Square]] -> [[[Square]]]
solve dict grid = do
    let l = fill dict grid
    let m = map transpose l
    let f = filter (check dict)
    let chunk n = Data.List.Split.chunksOf n
    let chunks = chunk 20000 m
    concat $ map f chunks `using` parBuffer 100 rdeepseq
```

Note that we have heuristically increased the chunk size to 20000. This is a substantial improvement over the previous attempt; run on 4 cores. The conversion rate for the 981 sparks created is 100%. The MUT phase of the program runs in just 3.57s. Unfortunately, the greater the number of cores, the greater the time spent on garbage collection. The total runtime on 4 cores is 16.36s—faster than in the previous section, but still not close to the 11.07s achieved sequentially.

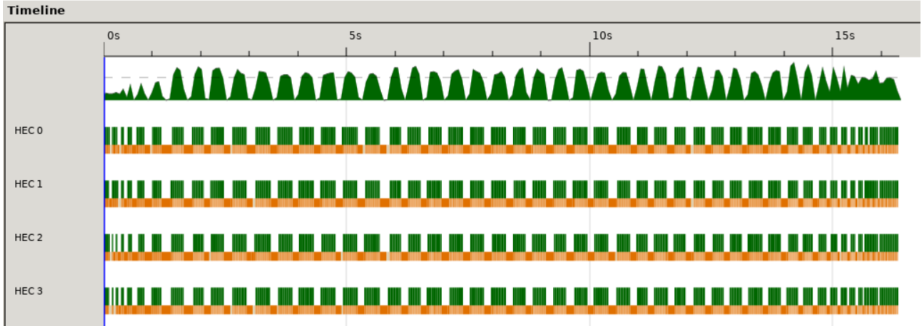

Figure 6: parBuffer applied to both candidate generation and filtering, -N4

Increasing or decreasing the core count, curiously enough, both have the effect of increasing the absolute time spent on garbage collection. The problem is only exacerbated on more complex tasks. When the size of the dictionary is increased to 1000, the parallel -N4 solver spends about 85 seconds in MUT and then another 285 in garbage collection, for a total of 370. The sequential solver using the same dictionary spends 242 seconds in MUT and then 52 in garbage collection, for a smaller total of 294.[3]. Figure 6 shows that, like Marlow's "kmeans_strat," our program performs computations in staggered chunks separated by periods of protracted garbage collection. Though we can decreasing the width of each individual stop by decreasing the chunk size, the total time spent on garbage collection remains fairly constant. Unlike Marlow, for whom the suspected culprit was expensive I/O operations, we make a point of not printing anything during execution. On the other hand, an inspection of Threadscope's "Raw events" tab reveals a number of heap overflows in individual threads, causing needless interruptions. We conclude that the issue must be related to inefficient memory constructions in the crossword source.

6 Speedup Attempts

We made several changes to our program in an attempt to fix the very large garbage collection overhead.

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[3] Due to their length, the 1000-word trials were only run once each

6.1 List Comprehension

One of our attempts to improve our solution involved changing how we generated solution candidates. Originally, we used a list comprehension and recursion when filling the grid.

```
fill_crossword dict (l:ls) =
                    [ line : rest | line <- fill_line dict l,
                                    rest <- fill_crossword dict ls ]
```

At the suggestion of Prof. Edwards, we switched order of the list comprehension generators in the `fill crossword` and `fill line` functions. This has the effect of staggering memory-intensive recursive calls to `fill crossword` and reducing the likelihood of expensive heap overflows.

```
fill_crossword dict (l:ls) =
                    [ line : rest | rest <- fill_crossword dict ls,
                                    line <- fill_line dict l ]
```

As will be shown in section 7, this change led to a speedup for both solvers, particularly the parallel one.

Another attempt involved generating candidates without a list comprehension at all. Instead, we used the `sequence` function, which takes lists as a parameter and returns their Cartesian product, to generate all possible solution candidate combinations.

```
fill_crossword dict grid = sequence result_grid
       where result_grid = Prelude.map (fill_line dict) grid
```

This modification led the program to consume even more memory and spend more time in garbage collection; we reverted it to the flipped list comprehension above.

6.2 Set and List Comparison

Another possible source of problems we identified was how words were being picked from the dictionary when building candidate grids. Instead of `Map Int (Set String)`, we changed the dictionary passed to the `fill` functions to be of type `Map Int [String]`, in order to avoid possible overhead from the `Set toList` function. This did not affect the speed or memory consumption, perhaps because the results of Set toList are already being efficiently cached.

6.3 Dictionary Word Storage

We attempted to speed up dictionary lookups by eliminating the conversion step from `String` to `Square` and back. This involved changing the dictionary type from `Map Int (Set String)` to `Map Int (Set [Square])`. This did not end up changing much, but it was one of the few cases in which the parallel version ran slightly faster than the sequential version.

7 Improved Parallelization Results

After implementing these improvements and dramatically shrinking the size of the chunks being passed to our sparks (from 10000 → 50), we observe the results shown in Figure 7.

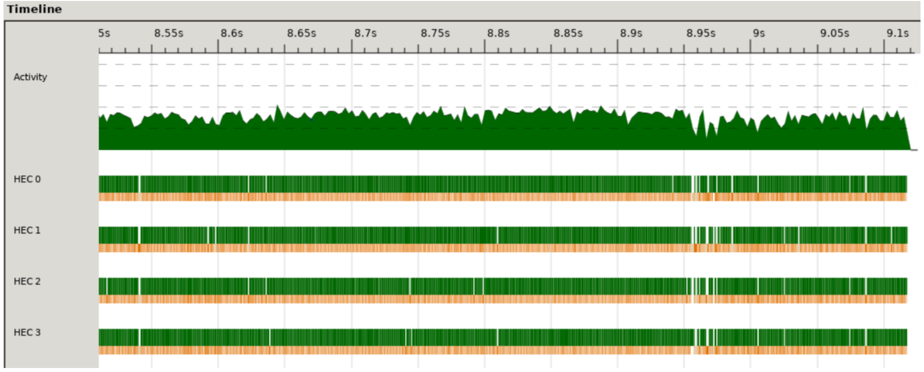

Figure 7: Parallelization strategy from 5.4 with improvements, -N4

Note the consistently higher activity on the cores and the absence of gaps as wide as those in Figure 6. Since the chunk size is so much smaller than the previous test, we see a dramatically increased number of sparks: about 392,000. As in earlier tests, however, all but 16 GC'd sparks and 24 fizzles successfully converted. For the first time, we are able to observe a speedup relative to the sequential version. The optimizations in Section 6 also benefited the latter: it now runs the task in 10.70s. However, the parallel version is even faster, finishing in 9.05s, 7 seconds faster than in the previous test. This buys us a final speedup of about x1.18. Granted, though the two allocate a similar number of bytes over their lifetimes—about 40 billion for the sequential version and 36 for the parallel—the productivity of the sequential version is much higher. It spends 97.1% of its time in MUT where the parallel version spends just 24.5%, leaving apparent room for future optimizations.

8 Discussion

While we were able to achieve high degrees of parallelization, the memory requirements of our program overwhelmed most of the accompanying performance benefits. However, the performance benefits we did see underneath the garbage collection overhead lead us to believe that further improvements are still possible.

A Source Code

A.1 Main.hs

```
import Data.Map
import Data.Set
import Data.List
import Data.List.Split
import Control.Parallel.Strategies
import System.Random
import System.Environment(getArgs, getProgName)
import System.Exit(die)
import DictUtil
import SquareDefs
import GenerateGrid

fill_and_check_s :: Map Int (Set [Square]) -> [[Square]] -> [[[Square]]]
fill_and_check_s dict grid = do
                             let grids = fill_crossword dict grid
                             let candidates = Prelude.map transpose grids
                             Prelude.filter (check_crossword dict) candidates

fill_and_check_p :: Map Int (Set [Square]) -> [[Square]] -> [[[Square]]]
fill_and_check_p dict grid = do
                             let f = Prelude.filter (check_crossword dict)
                             let grids = fill_crossword dict grid
                             let m = Prelude.map transpose grids
                             let c = Data.List.Split.chunksOf 50 m
                             let p = parBuffer 100 rseq
                             let sols = Prelude.map f c `using` p
                             concat sols

main :: IO ()
main = do args <- getArgs
          case args of
              [dimS, mS, threshS, dictPath, deterministic, parallel] -> do
                  seed <- case deterministic of
                                "y" -> return $ mkStdGen 15
                                _   -> newStdGen
                  let dim = read dimS :: Int
                  let m = read mS :: Int
                  let thresh = read threshS :: Float
                  let result = generateGrid dim m thresh seed
                  let peeled = maybe [[]] (\x -> x) $ result
                  mapM_ (putStrLn) $ Prelude.map show peeled
                  dictStream <- readFile dictPath
                  let dict = build_dict dictStream
```

```haskell
                  filled <- case parallel of
                               "y" -> return $ fill_and_check_p dict peeled
                               _   -> return $ fill_and_check_s dict peeled
                  mapM_ (printGrid) filled
                  let num = show (length filled)
                  putStrLn $ "found " ++ num ++ " ways to fill the given grid"
              _ -> do pn <- getProgName
                      die $ "Usage: "++pn++" <dimension> <min_word_length>\
                                              \ <probability_threshold> \
                                              \ <dict_filepath> \
                                              \ <deterministic> \
                                              \ <parallel>"
```

A.2 SquareDefs.hs

```haskell
module SquareDefs where

import Control.DeepSeq

data Square = Black | White Char

instance Show Square where
  show Black = "@"
  show (White c) = [c]

instance Eq Square where
  (==) Black Black = True
  (==) (White a) (White b) = (a == b)
  (==) _ _ = False

instance NFData Square where
  rnf s = s `seq` ()

instance Ord Square where
  Black `compare` Black = EQ
  (White a) `compare` (White b) = (a `compare` b)
  Black `compare` _ = GT
  _ `compare` Black = LT
```

A.3 DictUtil.hs

```haskell
module DictUtil where

import Data.Set
```

```haskell
import Data.Map
import Data.Char
import Data.List
import SquareDefs


blockedSq :: Char
blockedSq = 'X'

set_unwrap :: Maybe (Set [Square]) -> Set [Square]
set_unwrap (Just x) = x
set_unwrap Nothing  = Data.Set.empty

getWordSet :: Map Int (Set [Square]) -> Int -> Set [Square]
getWordSet dict len = set_unwrap $ Data.Map.lookup len dict

pick_word :: Set [Square] -> [[Square]]
pick_word dict_slice = Data.Set.toList dict_slice

fill_word :: Map Int (Set [Square]) -> [Square] -> [[Square]]
fill_word _ [] = [[]]
fill_word dict single_word = pick_word candidateSet
                where candidateSet = getWordSet dict $ length single_word

fill_line :: Map Int (Set [Square]) -> [Square] -> [[Square]]
fill_line _ [] = [[]]
fill_line dict xs = [ w ++ blocked ++ r | w <- fill_word dict firstWord,
                                          r <- fill_line dict restOfLine ]
                        where (word, suffix) = break isBlocked xs
                              (blocked, restOfLine) = span isBlocked suffix
                              isBlocked Black = True
                              isBlocked (White _) = False


fill_crossword :: Map Int (Set [Square]) -> [[Square]] -> [[[Square]]]
fill_crossword _ [] = [[]]
fill_crossword dict (l:ls) = [ c : r | c <- fill_crossword dict ls,
                                       r <- fill_line dict l ]

-- An alternative fill_crossword. In our tests, it did not perform as well
{- sequence result_grid
   where result_grid = Prelude.map (fill_line dict) unfilled_grid -}

check_word :: Map Int (Set [Square]) -> [Square] -> Bool
check_word _ [] = True
check_word dict single_word = Data.Set.member single_word candidateSet
                where candidateSet = getWordSet dict $ length single_word
```

```haskell
check_line :: Map Int (Set [Square]) -> [Square] -> Bool
check_line _ [] = True
check_line dict xs = check_word dict firstWord && check_line dict restOfLine
                where (firstWord, suffix) = break isBlocked xs
                      (_, restOfLine) = span isBlocked suffix
                      isBlocked Black = True
                      isBlocked (White _) = False

check_crossword :: Map Int (Set [Square]) -> [[Square]] -> Bool
check_crossword dict grid = Prelude.foldl (&&) True m
                      where m = Prelude.map (check_line dict) grid

check_crosswords :: Map Int (Set [Square]) -> [[[Square]]] -> Bool
check_crosswords dict grids = Prelude.foldl (&&) True m
                      where m = Prelude.map (check_crossword dict) grids

valid_dict_word :: String -> Bool
valid_dict_word w = Prelude.foldl (&&) True $ Prelude.map isAcceptable w
                    where isAcceptable c = isAscii c && isLower c

get_valid_words :: String -> [String]
get_valid_words streamIn = Prelude.filter valid_dict_word $ words streamIn

insert_word :: Map Int (Set [Square]) -> String -> Set [Square]
insert_word dict single_word = Data.Set.insert squarified_word len_set
                    where len_set = getWordSet dict $ length single_word
                          squarified_word = Prelude.map toSquare single_word
                          toSquare c = White c

update_set :: Map Int (Set [Square]) -> String -> Map Int (Set [Square])
update_set dict single_word = Data.Map.insert word_len updated_set dict
                              where word_len = length single_word
                                    updated_set = insert_word dict single_word

build_dict_rec :: Map Int (Set [Square]) -> [String] -> Map Int (Set [Square])
build_dict_rec dict [] = dict
build_dict_rec dict (w:ws)= update_set (build_dict_rec dict ws) w

build_dict :: String -> Map Int (Set [Square])
build_dict streamIn = build_dict_rec Data.Map.empty valid_words
                      where valid_words = get_valid_words streamIn

fill_and_check :: Map Int (Set [Square]) -> [[Square]] -> [[[Square]]]
fill_and_check dict emptyGrid = Prelude.filter (check_crossword dict) candidates
                      where candidates = Prelude.map transpose f
                            f = fill_crossword dict emptyGrid
```

A.4 GenerateGrid.hs

```haskell
module GenerateGrid where

import Data.List (sortBy)
import Data.Map.Strict (Map, (!))
import qualified Data.Map.Strict as Map
import Data.Set (Set)
import qualified Data.Set as Set
import System.Random
import SquareDefs

type Index = (Int, Int)
type Constraint = (Int, Int, Int, Int)
type Constraints = Map Index Constraint

probs :: RandomGen g => Int -> g -> [Float]
probs 0 _ = []
probs n s = r : probs (n-1) s'
           where
               (r, s') = randomR (0.0::Float, 1.0::Float) s

indices :: Int -> Int -> Set Index
indices h w = Set.fromList $ [(i,j) | i <- [0..(h - 1)], j <- [0..(w - 1)]]

initConstraints :: Int -> Int -> Constraints
initConstraints h w = Map.fromSet (\_ -> (0, w - 1, 0, h - 1)) (indices h w)

updateConst :: Index -> Index -> Constraint -> Constraint
updateConst (bi, bj) (i, j) (l,r,t,b) = (nl, nr, nt, nb)
                            where
                                (nl, nr) | not sameRow = (l, r)
                                         | left = (max l bj + 1, r)
                                         | right = (l, min r bj - 1)
                                         | otherwise = (i, j)
                                (nt, nb) | not sameCol = (t,b)
                                         | above = (max t bi + 1, b)
                                         | below = (t, min b bi - 1)
                                         | otherwise = (i, j)
                                sameRow = bi == i
                                (left, right) = (bj < j, bj > j)
                                sameCol = bj == j
                                (above, below) = (bi < i, bi > i)

adjustWithKeyList :: Ord k => (k -> a -> a) -> [k] -> Map k a -> Map k a
adjustWithKeyList _ [] m = m
adjustWithKeyList f (x:xs) m = Map.adjustWithKey f x $ adjustWithKeyList f xs m
```

```haskell

addBS :: Index -> Constraints -> Constraints
addBS (i,j) con = updateRow $ updateCol con
                       where
                         updateCol c = adjustWithKeyList f col c
                         updateRow c = adjustWithKeyList f row c
                         f = updateConst (i,j)
                         col = filter (\(_,b) -> b == j) keyList
                         row = filter (\(a,_) -> a == i) keyList
                         keyList = Map.keys con

checkSquare :: Index -> Constraints -> Int -> Bool
checkSquare (i,j) c m | i - top < m && i - top /= 0 = False
                      | bot - i < m && bot - i /= 0 = False
                      | j - left < m && j - left /= 0 = False
                      | right - j < m && right - j /= 0 = False
                      | otherwise = True
                         where
                          (left, right, top, bot) = c ! (i,j)

spiralIndices :: Int -> Int -> [Index]
spiralIndices 0 _ = []
spiralIndices 1 w = [ (0, i) | i <- [0..(w - 1)]]
spiralIndices h w = (++) (f 0 0) $ adj $ spiralIndices (h - 2) (w - 2)
                        where
                          f i j = (i,j) : l i j
                          l i j | j < w - 1 && i == 0 = f i (j + 1)
                                | j == w - 1 && i < h - 1 = f (i + 1) j
                                | j > 0 && i == h - 1 = f i (j - 1)
                                | j == 0 && i > 1 = f (i - 1) j
                                | otherwise = []
                          adj spiral = map (\(a, b) -> (a + 1, b + 1)) spiral

fillIn :: [Index] -> [Bool] -> Constraints -> Int -> [Square]
fillIn [] _ _ _ = []
fillIn _ [] _ _ = []
fillIn (i:is) (b:bs) c m | not b = white
                         | checkSquare i c m = black
                         | otherwise = white
                             where
                               white = (White '_') : fillIn is bs c m
                               black = Black : fillIn is bs (addBS i c) m

unravel :: Int -> [(Index, Square)] -> [[Square]]
unravel h s = map getRow [0..(h - 1)]
                where
                  getRow i = map snd $ sortBy my_compare $ filter (isRow i) s
                  my_compare ((_,y1),_) ((_,y2),_) = compare y1 y2
```

```
                  isRow i ((x, _), _) = x == i

generateGrid :: RandomGen g => Int -> Int -> Float -> g -> Maybe [[Square]]
generateGrid dim m thresh seed | even dim = Just even_grid
                               | otherwise = Nothing
                                where
                                  even_grid = half ++ rev_half
                                  rev_half = reverse $ map reverse half
                                  half = unravel half_h $ zip s $ squares
                                  squares = fillIn s b c m
                                  s = spiralIndices half_h dim
                                  c = initConstraints half_h dim
                                  b = map (\x -> x <= thresh) p
                                  p = probs (half_h * dim) seed
                                  half_h = div dim 2

printGrid :: [[Square]] -> IO ()
printGrid grid = do mapM_ (putStrLn) $ map show grid
                    putStrLn ""
```