Fundamentals of Computer Systems

Thinking Digitally

Stephen A. Edwards

Columbia University

Summer 2015

The Subject of this Class

0

The Subjects of this Class

0 1

But let your communication be, Yea, yea; Nay, nay: for whatsoever is more than these cometh of evil.

— Matthew 5:37

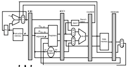
Computer Systems Work Because of Abstraction

Application Software

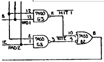
Operating Systems

```
;; voice 1 wave select
ld      a,(#CH1_W_NUM)
and     a
ld      a,(#CH1_W_SEL)
jr      nz,#00b4
ld      a,(#CH1_E_TABLE0)
```

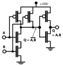
Architecture



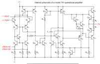
Micro-Architecture



Logic

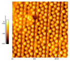


Digital Circuits



Analog Circuits

Devices

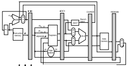
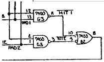
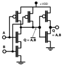
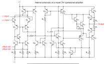
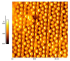


Physics

Computer Systems Work Because of Abstraction

	Application Software	COMS 3157, 4156, et al.
	Operating Systems	COMS W4118

`;; voice 1 wave select`
`ld a,(#CH1_W_NUM)`
`and a`
`ld a,(#CH1_W_SEL)`
`jr nz,#00b4`
`ld a,(#CH1_E_TABLE0)`
	Architecture	Second Half of 3827

	Micro-Architecture	Second Half of 3827
	Logic	First Half of 3827
	Digital Circuits	First Half of 3827
	Analog Circuits	ELEN 3331
	Devices	ELEN 3106
	Physics	ELEN 3106 et al.

Boring Stuff

http://www.cs.columbia.edu/~sedwards/classes/2015/3827-summer/

Prof. Stephen A. Edwards

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462 Computer Science Building

Lectures 5:30–8:40 PM, Tuesdays and Thursdays
825 Mudd
May 26–July 2

Weight	What	When
40%	Homeworks	See Webpage
60%	Final exam	July 2nd

Homework is due at the beginning of lecture.

Rules and Regulations

You may collaborate with classmates on homework.

Each assignment turned in must be unique; work must ultimately be your own.

List your collaborators on your homework.

Don’t be a cheater.

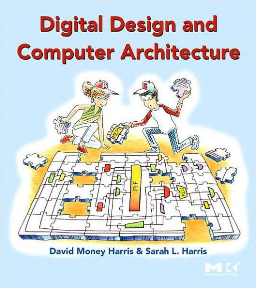
Final will be closed-book with a one-page “cheat sheet” of your own devising.

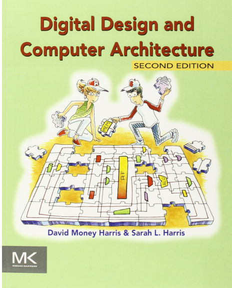
The Text(s): Alternative #1

No required text. There are two recommended alternatives.

- David Harris and Sarah Harris. *Digital Design and Computer Architecture.*

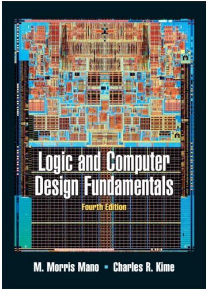
  Almost precisely right for the scope of this class: digital logic and computer architecture.



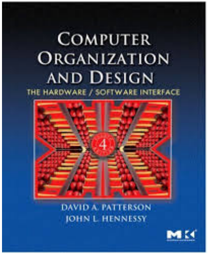


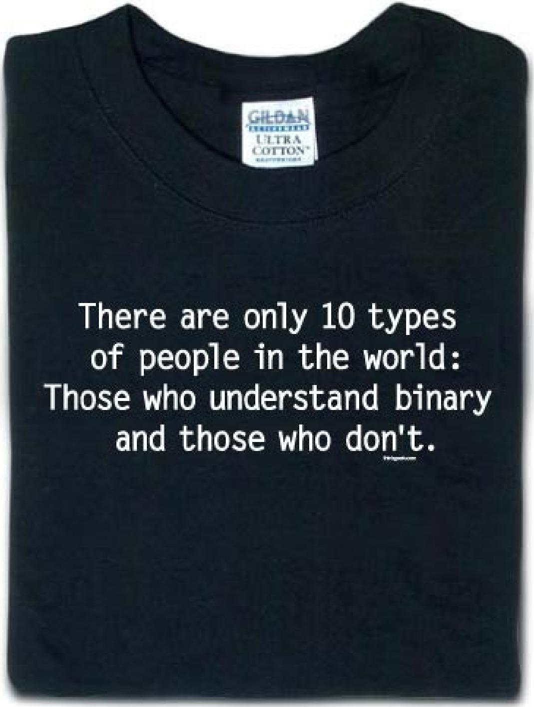
The Text(s): Alternative #2

- M. Morris Mano and Charles Kime. *Logic and Computer Design Fundamentals, 4th ed.*



- Computer Organization and Design, The Hardware/Software Interface, 4th ed. *David A. Patterson and John L. Hennessy*





thinkgeek.com

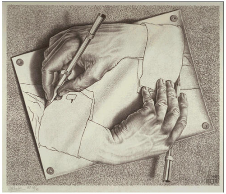
The Decimal Positional Numbering System

Ten figures: 0 1 2 3 4 5 6 7 8 9

$$7 \times 10^2 + 3 \times 10^1 + 0 \times 10^0 = 730_{10}$$

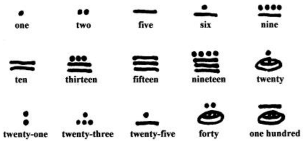
$$9 \times 10^2 + 9 \times 10^1 + 0 \times 10^0 = 990_{10}$$

Why base ten?

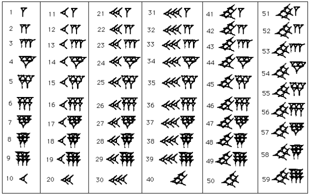


Which Numbering System Should We Use? Some Older Choices:

Roman: I II III IV V VI VII VIII IX X



Mayan: base 20, Shell = 0

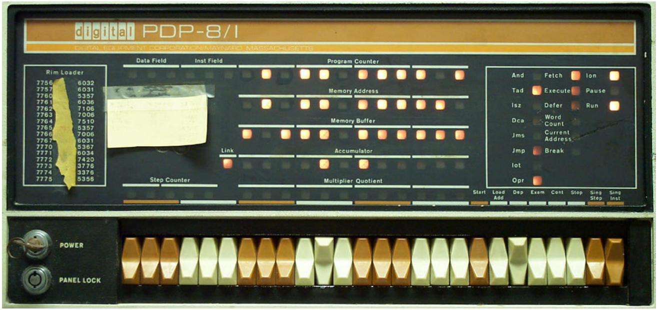


Babylonian: base 60

Hexadecimal, Decimal, Octal, and Binary

Hex	Dec	Oct	Bin
0	0	0	0
1	1	1	1
2	2	2	10
3	3	3	11
4	4	4	100
5	5	5	101
6	6	6	110
7	7	7	111
8	8	10	1000
9	9	11	1001
A	10	12	1010
B	11	13	1011
C	12	14	1100
D	13	15	1101
E	14	16	1110
F	15	17	1111

Binary and Octal



DEC PDP-8/I, c. 1968

Oct	Bin
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

$$\begin{aligned} \mathrm{PC} &= 0 \times 2^{11} + 1 \times 2^{10} + 0 \times 2^{9} + 1 \times 2^{8} + 1 \times 2^{7} + 0 \times 2^{6} + \\ &\quad 1 \times 2^{5} + 1 \times 2^{4} + 1 \times 2^{3} + 1 \times 2^{2} + 0 \times 2^{1} + 1 \times 2^{0} \\ &= 2 \times 8^{3} + 6 \times 8^{2} + 7 \times 8^{1} + 5 \times 8^{0} \\ &= 1469_{10} \end{aligned}$$

Hexadecimal Numbers

Base 16: 0 1 2 3 4 5 6 7 8 9 A B C D E F

Instead of groups of 3 bits (octal), Hex uses groups of 4.

$$\begin{aligned} \mathrm{CAFEF00D}_{16} &= 12 \times 16^7 + 10 \times 16^6 + 15 \times 16^5 + 14 \times 16^4 + \\ &\quad 15 \times 16^3 + 0 \times 16^2 + 0 \times 16^1 + 13 \times 16^0 \\ &= 3,405,705,229_{10} \end{aligned}$$

C	A	F	E	F	0	0	D	Hex

11001010111111101111000000001101 Binary

3	1	2	7	7	5	7	0	0	1	5	Octal

Computers Rarely Manipulate True Numbers

Infinite memory still very expensive

Finite-precision numbers typical

32-bit processor: naturally manipulates 32-bit numbers

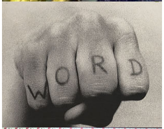
64-bit processor: naturally manipulates 64-bit numbers

How many different numbers can you represent with 5 { binary / octal / decimal / hexadecimal } digits?

Jargon

Bit	Binary digit: 0 or 1

Byte	Eight bits



Word	Natural number of bits for the processor, e.g., 16, 32, 64

LSB	Least Significant Bit (“rightmost”)

MSB	Most Significant Bit (“leftmost”)

Decimal Addition Algorithm

$$\begin{array}{r} 434 \\ +628 \\ \hline \end{array}$$

$$4 + 8 \quad = \quad 12$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} 1 \\ 434 \\ +628 \\ \hline 2 \end{array}$$

$$4 + 8 = 12$$

$$1 + 3 + 2 = 6$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} {\color{red}1} \\ 434 \\ +628 \\ \hline {\color{blue}62} \end{array}$$

$$\begin{aligned} 4 + 8 &= {\color{red}1}{\color{blue}2} \\ 1 + 3 + 2 &= {\color{blue}6} \\ 4 + 6 &= {\color{red}10} \end{aligned}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} 1\ \ 1 \\ 434 \\ +628 \\ \hline 062 \end{array}$$

$$\begin{aligned} 4+8 &= 12 \\ 1+3+2 &= 6 \\ 4+6 &= 10 \end{aligned}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} \color{red}{1\ \ 1} \\ 434 \\ +628 \\ \hline \color{blue}{1062} \end{array}$$

$$\begin{aligned} 4 + 8 &= \color{red}{1}\color{blue}{2} \\ 1 + 3 + 2 &= \color{blue}{6} \\ 4 + 6 &= \color{red}{10} \end{aligned}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Binary Addition Algorithm

$$\begin{array}{r} 10011 \\ +11001 \\ \hline \end{array}$$

$$1 + 1 \quad = \quad 10$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} 1 \\ 10011 \\ +11001 \\ \hline 0 \end{array}$$

$$1 + 1 = 10$$
$$1 + 1 + 0 = 10$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} \color{red}{11} \\ 10011 \\ +11001 \\ \hline \color{blue}{00} \end{array}$$

$$1 + 1 = \color{red}{1}\color{blue}{0}$$
$$1 + 1 + 0 = \color{red}{1}\color{blue}{0}$$
$$1 + 0 + 0 = \color{red}{0}\color{blue}{1}$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} 011 \\ 10011 \\ +11001 \\ \hline 100 \end{array}$$

$$1 + 1 = 10$$
$$1 + 1 + 0 = 10$$
$$1 + 0 + 0 = 01$$
$$0 + 0 + 1 = 01$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$
\begin{array}{r}
0011 \\
10011 \\
+11001 \\
\hline
1100
\end{array}
$$

$$
\begin{aligned}
1 + 1 &= 10 \\
1 + 1 + 0 &= 10 \\
1 + 0 + 0 &= 01 \\
0 + 0 + 1 &= 01 \\
0 + 1 + 1 &= 10
\end{aligned}
$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$
\begin{array}{r}
10011 \\
10011 \\
+11001 \\
\hline
101100
\end{array}
$$

$$1 + 1 = 10$$

$$1 + 1 + 0 = 10$$

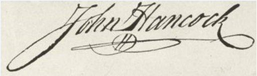
$$1 + 0 + 0 = 01$$

$$0 + 0 + 1 = 01$$

$$0 + 1 + 1 = 10$$

+	0	1
0	00	01
1	01	10
10	10	11

Signed Numbers: Dealing with Negativity



How should both positive and negative numbers be represented?

Signed Magnitude Numbers

You are most familiar with this:
negative numbers have a leading $-$

In binary, a leading 1 means negative:

$0000_2 = 0$

$0010_2 = 2$

$1010_2 = -2$

$1111_2 = -7$

$1000_2 = -0$?

Can be made to work, but addition is annoying:

If the signs match, add the magnitudes and use the same sign.

If the signs differ, subtract the smaller number from the larger; return the sign of the larger.

One's Complement Numbers

Like Signed Magnitude, a leading 1 indicates a negative One's Complement number.

To negate a number, complement (flip) each bit.

$0000_2 = 0$

$0010_2 = 2$

$1101_2 = -2$

$1000_2 = -7$

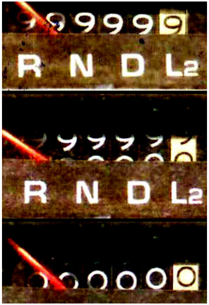
$1111_2 = -0?$

Addition is nicer: just add the one's complement numbers as if they were normal binary.

Really annoying having a -0: two numbers are equal if their bits are the same or if one is 0 and the other is -0.


0 CALORIE COLA
pepsi MAX
zero calories
maximum taste
NOT ALL ZEROS ARE CREATED EQUAL
ZERO CALORIES. MAXIMUM PEPSI® TASTE.
pepsi MAX
zero calories
PEPSI, PEPSI MAX and the Pepsi Globe are registered trademarks of PepsiCo, Inc.

Two’s Complement Numbers



Really neat trick: make the most significant bit represent a *negative* number instead of positive:

$$1101_2 = -8 + 4 + 1 = -3$$

$$1111_2 = -8 + 4 + 2 + 1 = -1$$

$$0111_2 = 4 + 2 + 1 = 7$$

$$1000_2 = -8$$

Easy addition: just add in binary and discard any carry.

Negation: complement each bit (as in one’s complement) then add 1.

Very good property: no -0

Two’s complement numbers are equal if all their bits are the same.

Number Representations Compared

Bits	Binary	Signed Mag.	One's Comp.	Two's Comp.
0000	0	0	0	0
0001	1	1	1	1
⋮				
0111	7	7	7	7
1000	8	−0	−7	−8
1001	9	−1	−6	−7
⋮				
1110	14	−6	−1	−2
1111	15	−7	−0	−1

Smallest number

Largest number

Fixed-point Numbers

How to represent fractional numbers? In decimal, we continue with negative powers of 10:

$$\begin{aligned}31.4159 &= 3 \times 10^1 + 1 \times 10^0 + \\ &\quad 4 \times 10^{-1} + 1 \times 10^{-2} + 5 \times 10^{-3} + 9 \times 10^{-4}\end{aligned}$$

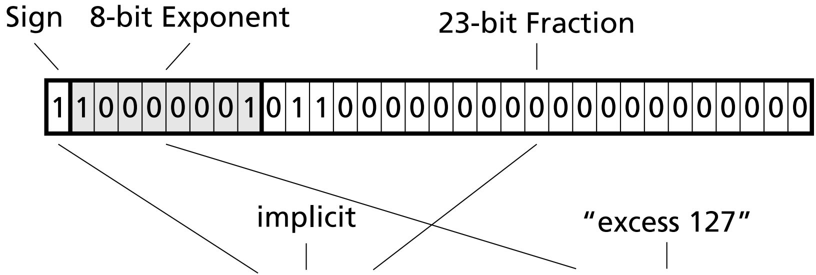
The same trick works in binary:

$$\begin{aligned}1011.0110_2 &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + \\ &\quad 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} \\ &= 8 + 2 + 1 + 0.25 + 0.125 \\ &= 11.375\end{aligned}$$

Floating-Point Numbers: “Scientific Notation”

Greater dynamic range at the expense of precision
Excellent for real-world measurements

IEEE 754 Single-Precision (32-bit)



$$= -\ 1.0110000_2 \times 2^{10000001_2 - 127}$$

$$= -1.375 \times 2^2$$

$$= -5.5$$

Characters and Strings? ASCII

	0	1	2	3	4	5	6	7
0	NUL	DLE	space	0	@	P	`	p
1	SOH	DC1 XON	!	1	A	Q	a	q
2	STX	DC2	"	2	B	R	b	r
3	ETX	DC3 XOFF	#	3	C	S	c	s
4	EOT	DC4	$	4	D	T	d	t
5	ENQ	NAK	%	5	E	U	e	u
6	ACK	SYN	&	6	F	V	f	v
7	BEL	ETB	'	7	G	W	g	w
8	BS	CAN	(	8	H	X	h	x
9	HT	EM	)	9	I	Y	i	y
A	LF	SUB	*	:	J	Z	j	z
B	VT	ESC	+	;	K	[	k	{
C	FF	FS	,	<	L	\	l	|
D	CR	GS	-	=	M	]	m	}
E	SO	RS	.	>	N	^	n	~
F	SI	US	/	?	O	_	o	del

ascii-table.com