BIOMETRICS Fall 2007
Assignment 2
Due: October 9, 2007

Problem 2.13

In many pattern classification problems one has the option either to assign the pattern to one of the c classes, or to *reject* it as being unrecognizable. If the cost for rejects is not too high, rejection may be a desirable action. Let

$$\lambda(\alpha_i|\omega_j) = \begin{cases} 0 & i = j \quad i, j = 1, \ldots, c \\ \lambda_r & i = c + 1 \\ \lambda_s & otherwise, \end{cases}$$

where λ_r is the loss incurred for choosing the $(c + 1)$th action, rejection, and λ_s is the loss incurred for making any substitution error. Show that the minimum risk is obtained if we decide ω_i if $P(\omega_i|\mathbf{x}) \geq P(\omega_j|\mathbf{x})$ for all j and if $P(\omega_i|\mathbf{x}) \geq 1 - \lambda_r/\lambda_s$, and reject otherwise. What happens if $\lambda_r = 0$? What happens if $\lambda_r > \lambda_s$?

Problem 2.24

Consider the multivariate normal density with mean $\boldsymbol{\mu}$, $\sigma_{ij} = 0$ and $\sigma_{ii} = \sigma_i^2$, that is, the covariance matrix is diagonal: $\boldsymbol{\Sigma} = diag(\sigma_1^2, \sigma_2^2, \ldots, \sigma_d^2)$.

(a) Show that the evidence is

$$p(\mathbf{x}) = \frac{1}{\prod_{i=1}^{d} \sqrt{2\pi}\sigma_i} \exp\left[-\frac{1}{2}\sum_{i=1}^{d}\left(\frac{x_i - \mu_i}{\sigma_i}\right)^2\right].$$

(b) Plot and describe the contours of constant density.

(c) Write an expression for the Mahalanobis distance from $\mathbf{x}$ to $\boldsymbol{\mu}$.

Problem 2.27

Suppose we have two normal distributions with the same covariances but different means: $N(\boldsymbol{\mu}_1, \boldsymbol{\Sigma})$ and $N(\boldsymbol{\mu}_2, \boldsymbol{\Sigma})$. In terms of their prior probabilities $P(\omega_1)$ and $P(\omega_2)$, state the condition that the Bayes decision boundary *not* pass between the two means.

Problem 2.14 (6000 level)

Consider the classification problem with rejection option.

(a) Use the results of Problem 2.13 to show that the following discriminant functions are optimal for such problems.

$$g_i(\mathbf{x}) = \begin{cases} p(\mathbf{x}|\omega_i)P(\omega_i) & i = 1, \ldots, c \\ \frac{\lambda_s - \lambda_r}{\lambda_s} \sum_{j=1}^{c} p(\mathbf{x}|\omega_j)P(\omega_j) & i = c+1. \end{cases}$$

(b) Plot these discriminant functions and the decision regions for the two-category one-dimensional case having

- $p(x|\omega_1) \sim N(1, 1)$,
- $p(x|\omega_2) \sim N(-1, 1)$,
- $P(\omega_1) = P(\omega_2) = 1/2$, and
- $\lambda_r/\lambda_s = 1/4$.

(c) Describe qualitatively what happens as λ_r/λ_s is increased from 0 to 1.

Problem 2.25 (6000 level)

Fill in the steps in the derivation from Eq.59 to Eq.60-65.